MAGNETISM AND ELECTRICITY

MAGNETISM AND ELECTRICITY

A MANUAL FOR STUDENTS IN ADVANCED CLASSES

BY

E. E. BROOKS, B.Sc. (Lond.), A.M.I.E.E.
HEAD OF THE DEPARTMENT OF PHYSICS AND ELECTRICAL ENGINEERING IN THE MUNICIPAL TECHNICAL SCHOOL, LEICESTER

AND

A. W. POYSER, M.A. (Trin. Coll. Dubl.)
HEAD MASTER OF THE GRAMMAR SCHOOL, WISBECH

WITH 413 ILLUSTRATIONS

LONGMANS, GREEN AND CO.
39 PATERNOSTER ROW, LONDON
NEW YORK, BOMBAY, AND CALCUTTA
1912

PREFACE

Owing to the enormous progress made in scientific research during the last twenty years, and to the consequent developments in electrical theory, this volume has been prepared to replace Poyser's *Advanced Magnetism and Electricity*, originally published in 1892. Practically, the whole of the subject-matter has been rewritten on modern lines; indeed, little of the old book remains except its experimental form. This has been retained because it is important that a beginner should learn to recognise that all theory is based upon a groundwork of experimental fact.

The present work is intended to afford such a range of general reading in the subject as is desirable for the majority of students, before they begin to specialise either in pure science or in the various branches of electrical engineering. They are assumed to have an elementary knowledge of algebra, geometry, trigonometry, and mechanics.

The scope of the volume is that required for the pass examinations of the Universities, and for the examinations of the Board of Education. The chapters marked with an asterisk need not be read for the Lower Examination of the latter body.

The sequence adopted is based upon a long experience in teaching. We freely admit that several alternatives are possible, each possessing its own advantages; and that room exists for wide differences of opinion as to the most logical form of treatment.

Again, we are conscious that many important subjects have been inadequately treated, but we hope that the various references and allusions will be regarded as indicating directions for further study.

Although the electron has received a considerable amount of attention, we have thought it undesirable (in the present state of uncertainty as to the true nature of a positive charge) to base an elementary treatise entirely upon its properties.

On the other hand, experience has shown that it is exceedingly desirable for a student to acquire the habit, at an early stage, of thinking out problems in terms of the properties of lines of force (electric and magnetic), and of applying the various mechanical analogies, which may be employed—within due limits—to invest mathematical expressions with a tangible physical meaning.

It will be seen that we have introduced K and μ into certain formulæ, where they are usually omitted as being equal to unity. If they are suppressed, the equations are incorrect as regards dimensions, and the student may encounter difficulties later which will not arise if they are used throughout.

Of the 413 illustrations, nearly three hundred have been engraved from our own drawings; the remainder have been taken by permission from well-known books, of which acknowledgment is made in the text.

We have to acknowledge most valuable assistance from Mr. Francis Hewson, of the Postal Telegraphs, for revising and correcting the chapter on Telegraphy and Telephony.

E. E. B.
A. W. P.

May 1912.

CONTENTS

ELECTROSTATICS

CHAPTER I

ELECTRIFICATION

Electrical Attraction.—**Exp. 1.** Warm and dry[1] a glass rod and a piece of silk. Rub the glass with the silk and then present it to any light bodies, *e.g.* pieces of paper, pith, or bran. Observe that they are attracted towards the rod.

Thus by friction an additional and curious property has been imparted to the rod, due to what is called *electricity;* and the rod itself is said to be *electrified*, *excited*, or *charged.*

Thales, a Greek philosopher (600 B.C.), was the first to record that this power was possessed by rubbed *amber*, the Greek for which—ἤλεκτρον (electron)—is the origin of our word *electricity.*

Adopting the precaution mentioned in the foot-note, this property may be exhibited with a large number of bodies, such as sealing-wax, resin, shellac, or sulphur, rubbed with flannel; hot brown paper brushed with an ordinary clothes brush; ebonite rubbed with flannel or even with the dry hand.

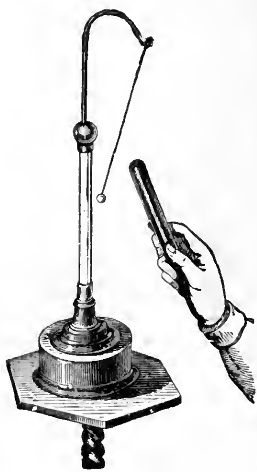


FIG. 1.

Electrical Repulsion.—**Exp. 2.** Suspend a small pith ball by a fine *silk* thread to a suitable support (Fig. 1). (Elder pith is best for this purpose. After cutting the pith into shape with a sharp knife, it is advisable to press it slightly between the fingers in order to remove any projecting points.)

Place this apparatus, which is called the *pith-ball pendulum*, before a fire to dry the silk. Bring an electrified rod near the ball. Observe that attraction

[1] To exhibit this property all rods, rubbers, and apparatus must be warm and dry, and it is therefore advisable to place them in front of an ordinary coal fire or a gas reflecting stove for some time before use.

first takes place, but after contact with the rod the ball is violently repelled, nor will it again approach the rod (unless indeed it has been in contact with the metallic or wooden support).

Two lessons may be learnt from this experiment—

1. That a body becomes charged by contact with an electrified body.

2. That when two bodies are charged by contact, they repel one another.

Two States of Electrification.—**Exp. 3.** Bend a piece of wire, as shown in Fig. 2, and suspend it by a silk thread from a suitable support. Electrify a glass rod with silk, and place it across the stirrup.

1. Electrify another glass rod with silk, and hold it near the suspended rod. Repulsion takes place.

2. Repeat this experiment with a rod of sealing-wax or of ebonite rubbed with flannel. Notice attraction.

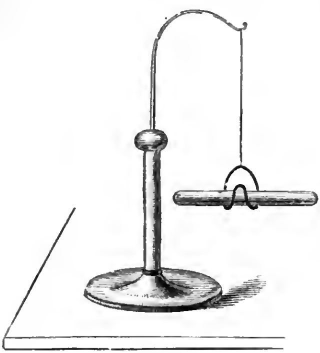


FIG. 2.

This experiment teaches us that there are two different states or kinds of electrification—

(1) That developed on glass rubbed with silk;

(2) That on sealing-wax rubbed with flannel.

Electrification developed on glass rubbed with silk was once called vitreous (*vitrum*, Lat., glass); that developed on sealing-wax or resin rubbed with flannel, resinous. These terms have, however, been discarded, as observers soon found that vitreous electricity could be developed on resinous substances, and *vice versâ*, by merely altering the material of the rubber.

Vitreous electricity is now called positive (or +) electricity.

Resinous electricity is now called negative (or –) electricity.

We also learn from Experiment 3 that

(*a*) bodies whose electrifications are of opposite kinds mutually attract one another;

(*b*) bodies whose electrifications are of the same kind repel one another.

Conductors and Insulators.—**Exp. 4.** Rub a brass rod, held in the hand, with warm silk. Bring it near a pith-ball pendulum, or better still, near an electroscope (see p. 5), and observe that the ball is unaffected.

Exp. 5. Mount the brass rod on a glass handle, or hold it with a sheet of india-rubber. Dry the handle and rub the brass. Present the rod to a negatively charged pith-ball pendulum and observe repulsion. It is therefore negatively charged.

Exp. 6. Repeat the last experiment, but, before bringing it to the pendulum, touch it with the finger. There is, now, no sign of electrification

For many years it was thought that only a certain number of bodies—which were called **electrics**—were capable of being electrified. All other bodies were called **non-electrics**, because they did not exhibit any signs of electrification even after violent friction. This distinction was erroneous, as *all* bodies are capable of being electrically excited, though in different degrees, if proper precautions are adopted. The reason of this difference in the action of various bodies escaped the early observers from the fact that, with non-electrics, the electricity is discharged to the earth through the hand and body of the experimenter, but that with electrics it is not so discharged.

Bodies, such as brass, which allow electrification to spread readily over them and to carry it away to other bodies, are called **conductors**, while those bodies, such as glass, sealing-wax, ebonite, &c., which do not allow the electricity to escape as soon as it is developed, are called **non-conductors, insulators**, or **dielectrics.**

Exp. 7. Insert a wooden rod into a cork, and then pass the cork into a glass tube; rub the tube close to the cork, and then present the end of the rod to any light body. Notice attraction.

Exp. 8. Fasten a key or a metal ball to a cotton thread, and then tie the free end round the wooden rod. Electrify the tube as before, and observe that light bodies are attracted to the key (Fig. 3).

Exp. 9. Repeat the last experiment with silk instead of cotton. The light bodies are not attracted.

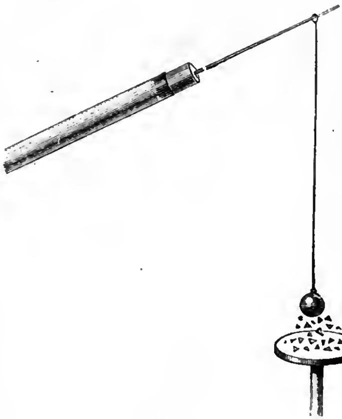


FIG. 3.

Wood, cotton, and metal are therefore *conductors;* silk is a *non-conductor*.

Different bodies have different conducting powers. It must be remembered that all bodies offer some resistance to the passage of electricity, although with good conductors the resistance is almost inappreciable, while with the best non-conductors it is so great that practically no electricity passes from one point to another.

The best conductors are the metals (of which silver stands first), then follow charcoal, acids, water, the human body, cotton. The best non-conductors are dry air, fused quartz, glass, paraffin, ebonite, shellac, sulphur, gutta-percha, resin, silk, wool, porcelain, oils. These substances are given in their approximate order of conduction and insulation respectively. Dry wood, marble, paper, straw occupy an intermediate place, and are therefore sometimes called partial conductors.

Rods made from fused quartz insulate remarkably well, and are not hygroscopic. This material can be drawn out into very fine fibres, suitable for delicate suspensions in measuring instruments, which give much better insulation than silk. (See its use in an electrometer, p. 87.)

Conduction is affected by temperature, *e.g.* glass loses its power of insulation when it is made very hot.

Electroscopes.—An instrument which will detect the presence, and determine the kind of electrification of a body is called an **electroscope.** The first electroscope was made and used by Gilbert in the year 1600, and merely consisted of a straw balanced on a fine point.

We have already used a pith-ball pendulum as an electroscope.

For rough experiments a pith-ball electroscope is sometimes employed. This consists of two small balls of elder pith, suspended by *cotton* threads from one end of a brass wire, the other end being terminated by a knob. A hole, somewhat larger in diameter than the wire, is bored through a cork, which fits the mouth of a glass vessel. The wire is passed through the hole in the cork, and then fastened in its place with hot shellac.

The Gold-leaf Electroscope.—Another useful instrument is that known as the gold-leaf electroscope (Fig. 4). A metal rod is supported by a paraffin-wax plug fitting into the neck of a wide-mouthed glass vessel without a bottom and standing on a wooden base. The upper end of the rod is fitted either with a metal ball or a plate, the latter being more generally useful. To the lower end is soldered[1] a flat strip of metal, about half an inch wide, which carries two gold leaves (each about half an inch wide and two inches long). The leaves are thus parallel and hang very close together. On opposite sides of the interior of the vessel are generally placed two strips of tinfoil, which touch the wooden base and which are themselves touched by the leaves when too great a divergence is produced. These strips, besides preventing the leaves from becoming broken by contact with the sides of the vessel, have another important function, which will be explained in later experiments.

[1] Soldering is easily performed as follows:—Make a soldering iron by pointing a stout copper wire, and then inserting the other end into a handle. Now place some zinc in a bottle, pour over it a small quantity of dilute hydrochloric acid, and allow the action to go on for some time; the solution is then known as "killed spirits." Having heated the "iron," dip it for an instant in the killed spirits, and then hold the bright point in a small piece of solder. This gives the point a shining silver-white surface, and the operation is technically called "tinning." The easiest method of soldering the brass disc to the copper wire is to make both pieces bright and clean, then place a little killed spirits on them, and a small piece of solder on the disc. Hold the iron in a Bunsen's flame, an inch or two from the "tinned" end. When the end has become sufficiently hot, touch the solder on the disc with it, also holding the wire on the required place. The heat melts the solder and, by careful manipulation, the solder flows over the two surfaces. The w re is held motionless until the solder is set. Afterwards wash with plenty of water to prevent rusting.

A later pattern is shown in Fig. 5. The containing vessel is a light rectangular metal box, having two of its opposite sides fitted with glass. The rod is supported by means of a plug of paraffin wax, which fits in the neck, and its lower end terminates in a fixed brass plate. *One* leaf—which for ordinary purposes may consist of dutch metal or of aluminium foil instead of gold-leaf—is attached to the upper part of the fixed plate.

(Since the discovery of radio-activity the electroscope has become extremely important, and, in some of its later forms, it is an exceedingly sensitive and accurate instrument of measurement.)

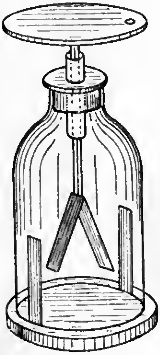


FIG. 4.

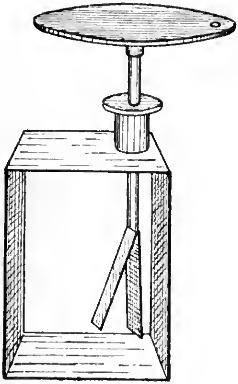


FIG. 5.

Uses of a Gold-leaf Electroscope.—The electroscope is used (1) to indicate the *presence* of a charge, and, roughly, the *amount of charge* on a body; (2) to determine the *kind of charge*. (See also p. 33.)

(*a*) *Experiments to indicate the existence of a charge and to estimate its amount.*

Exp. 10. Rub a rod of sealing-wax, ebonite, or shellac with flannel, and bring it gradually over the cap of the electroscope. Notice that the leaves diverge; if the body be slightly charged the divergence is small, if it be highly charged, the divergence is greater.

Exp. 11. Gently strike the disc with fur, and observe the divergence of the leaves.

Exp. 12. Grind roll sulphur to powder in a mortar, and notice the divergence of the leaves when a small quantity is dropped on the disc.

(*b*) *Experiments to determine the kind of electrification of a body.*

The following rule, which will be easily understood after the chapter on induction has been read, must be learnt. If, when the leaves are divergent, the approach of an electrified body causes them to diverge *more*, the leaves and the approaching body are similarly electrified; if they diverge *less*, the leaves and the approaching body are either oppositely electrified, or else the approaching body is an uncharged conductor.

Take care to observe the *first* movement of the leaves as the body approaches the electroscope.

Exp. 13. Repeat Exp. 11. Bring up a negatively charged rod, and observe that the divergence of the leaves is greater. The disc is therefore negatively electrified.

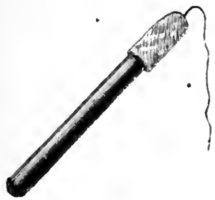


FIG. 6.

Exp. 14. In a similar manner prove that the sulphur in Exp. 12 is negatively electrified.

Exp. 15. Place a flannel cap (Fig. 6), to which a silk thread is attached, over one end of an ebonite rod, both having been previously warmed. Rub the cap round the rod, and then, by means of the silk thread, place it on the disc. (*a*) Bring up a positively charged rod, and observe that there is a further divergence of the leaves. The flannel is therefore positive. (*b*) Bring up a negatively charged rod, and observe that the leaves collapse. (*c*) Place the hand over the disc, and again notice that the leaves diverge less than at first.

Repulsion is the only sure Test of Electrification.—From the last experiment we learn that there is increased repulsion between the leaves, when they and the approaching body are similarly electrified; and that both an oppositely charged body and an uncharged conductor cause a partial, or total, collapse of the leaves. In order, therefore, to ascertain the kind of charge we are testing, it is essential to rely solely on repulsion.

Simultaneous and Equal Development of Both Kinds of Electricity.—**Exp. 16.** (1) Repeat Exp. 15, by means of which the flannel cap is proved to be charged positively. (2) Charge a gold-leaf electroscope negatively. Observe that when the rod is brought near, there is an increased divergence of the leaves. The rod is therefore negatively charged. (3) Discharge the rod (by passing it through a flame), and also the cap. Show that they are neutral by bringing each in turn near the electroscope. Replace the cap, and rub again. Without removing the cap, present them to an uncharged electroscope. The leaves remain at rest.

This experiment conclusively proves that (1) positive and negative electrifications are produced simultaneously, and (2) the positive electrification is exactly equal in amount to the negative.

As this fact is so important, it may be advisable to demonstrate it with greater accuracy.

Exp. 17. Insulate a metal can, connect it by a fine wire to the cap of an electroscope some little distance away, and place a pad of flannel at the bottom of the can. Test an ebonite rod to make sure that it is uncharged, and then rub the flannel gently with the end of it. Observe that no effect is produced on the leaves while the rod remains on the pad, but that they diverge when it is removed. Again introduce the rod, and if no leakage has occurred along it, notice that the divergence disappears.

Bodies not absolutely Positive or Negative.—**Exp. 18.** Charge a gold-leaf electroscope negatively. Excite a hot glass rod with fur and bring it near the electroscope. Notice that there is increased divergence, proving that the glass is negatively electrified.

Thus we learn that the kind of electrification developed by friction depends not only on the body rubbed, but also upon the rubber.

The following list has been prepared so that if any two bodies be chosen, the one standing first becomes positive, the other negative:—

+ Fur	Paraffin-wax
Flannel	Ebonite
Ivory	The hand
Glass	Metals
Cotton	Sulphur
Paper	Celluloid
Silk	Rubber tubing

It must be remarked, however, that the results of experiments are somewhat uncertain with those substances which stand close together on the list, as a slight difference in chemical composition, or in their physical properties, may alter their behaviour. The order of the last seven substances on the list is given from experiments made at the time of writing. For instance, an ebonite rod (first cleaned with emery cloth), which becomes negatively charged when rubbed on the coat sleeve, became positively charged when flicked with a piece of rubber tubing, and also when drawn through the dry hand. This piece of rubber tubing became negative to every other substance on the list, whereas another specimen (a photographer's rubber squeegee), was found to be positive to metals, sulphur, and a pocket comb, its position on the list lying between "the hand" and "metals."

Exp. 19. Test as many of these bodies as possible by means of a gold-leaf electroscope, electrified from a known source.

Discharging an Electrified Body.—**Exp. 20.** Rub an ebonite rod with flannel. Show that it is electrified by bringing it near an uncharged gold-leaf electroscope. Pass the rod through a Bunsen's or spirit-lamp flame. Test as before. There is no divergence of the leaves, showing that the rod is completely discharged.

Exp. 21. Charge a gold-leaf electroscope. Hold a lighted taper above, but not in contact with, the disc. Observe that the leaves collapse; the electroscope is therefore discharged.

See also the experiments on the discharge by points, pp. 22 and 82.

Electrification by Pressure and Cleavage.—Electrification may be produced by many methods other than friction, *e.g.* by chemical action, by heat, by mere contact of dissimilar metals (all of which will be treated of in due course), and by pressure and cleavage.

(*a*) If a piece of insulated cork be pressed on india-rubber, gutta-percha, amber, or metals, it becomes positively electrified.

(*b*) If sulphur be placed on a piece of india-rubber, then broken by a hammer and allowed to fall on the disc of a gold-leaf electroscope, the leaves diverge with negative electricity.

(*c*) Lumps of sugar broken in the dark emit a feeble light, due to the recombination of the two electrifications.

Pyro-Electricity.—When certain minerals are cooled or heated, electricity is produced. Such electricity is called pyro-electricity. It is best studied with a suitable crystal of tourmaline (Fig. 7), suspended by a fine wire over a metal plate, heated by a spirit lamp. In a short time, the ends will be oppositely charged. If now the plate be removed, and the crystal discharged by passing a flame over it, it will be found that as it cools, the end, which when heated was positive, becomes negative, and that which was negative becomes positive. The points at which the free electricity is at a maximum are called the poles; that which was positive when the temperature was rising is called the analogous, and that which was negative, the antilogous pole. In Fig 7, A is the analogue, and B the antilogue.

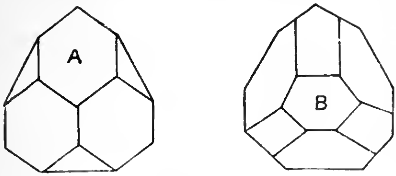


FIG. 7.

EXERCISE I

1. If a hot sheet of paper be brushed with an ordinary clothes brush, it will cling to the wall of a room. The drier the air is, the longer it will cling. Why is this?

2. Describe an experiment to demonstrate the existence of two kinds of electrification.

3. How would you show that these two kinds are always produced in equal amounts?

4. Is it possible to electrify a metal rod by friction? If so, show how it can be done.

5. What will be the electrical state of a silk glove after being drawn off the hand?

CHAPTER II

INDUCTION—LINES OF ELECTRIC FORCE

Exp. 22. Place two insulated brass spheres in contact. (Two spherical bedstead knobs, two or three inches in diameter, supported on ebonite penholders are excellent, and may be easily made at a cost of a few pence.) Bring a positively charged rod near. Negative electricity is found on the sphere next the rod, and positive on the remote one (Fig. 8). Prove the truth of this statement by removing the sphere remote from the rod—taking care to touch the insulating support only—and testing its charge. Remove the rod, and then test the charge on the other sphere.

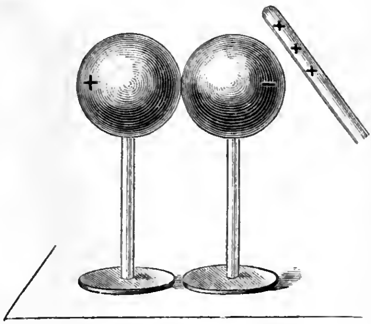


FIG. 8.

When we bring an electrified body near, but not in contact with, an insulated conductor—the two brass spheres mentioned above form one conductor when they are placed in contact—we find that an action takes place across the dielectric (in this case, air), with the result that the near side of the conductor becomes charged with the opposite electrification and the remote side with the same kind of electrification, which is present on the electrified body.

Such electrical action is called **induction**. The electrified body producing the action is called the *inducing body*, and the charges produced by the action are called *induced charges*.

Before doing any further experiments on induction, we must describe an exceedingly useful instrument, called a proof-plane, which we may use when dealing with large charges of electricity.

A Proof-Plane (Fig. 9) merely consists of a small conductor, mounted on an insulating handle. An excellent one is made from a disc of metal, having the edges well rounded off, fastened to a rod of sealing-wax or of ebonite.

Sometimes the conductor is a small brass ball suspended by a silk thread. It is then commonly called a **carrier ball**.

If such an instrument be brought in contact with a charged body, it will receive part of its charge, which can therefore be removed, and the kind of electrification tested, without incurring the danger of fracturing the gold leaves of an electroscope.

Exp. 23. Take an insulated, unelectrified, cylindrical conductor with rounded ends. Bring a negatively charged rod near it, and touch the other end with a proof-plane. (*a*) Prove that the charge on the proof-plane is negative (use a negatively charged electroscope, and observe greater divergence of the leaves). (*b*) Touch the end near the rod with a proof-plane, and show that the divergence of the leaves of a positively charged electroscope is increased, when the proof-plane is brought near. (*c*) Test a point midway between the two ends. There is no divergence of the leaves of an unelectrified electroscope.

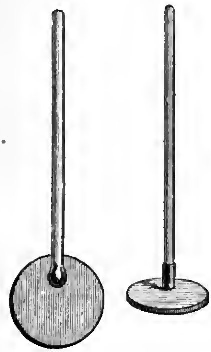


FIG. 9.

Exp. 24. Repeat Exp. 22, and show that the induced charges are equal in amount by bringing the two spheres into contact, and then testing that there is no residual charge. (Especially good insulation is required, or the experiment may fail on account of unequal leakage.)

From experiments similar to these, we infer that a charged body acts inductively in *all* directions, any insulated conductor near it acquiring *two* induced charges, that of opposite sign being nearer the charged body, and that of the same sign farther away.

Induction on a Conductor connected with the Earth.—We are now in a position to understand the effect of placing a conductor, when under inductive influence, in connection with the earth.

Exp. 25. Bring a positively electrified rod near an insulated conductor. Touch the conductor with the finger or, indeed, with any uninsulated conductor. Remove the hand or conductor, and then the inducing rod, and test the remaining charge. We find that it is negative; the positive has therefore disappeared.

The reason is this:—the human body and the floor of the room are conductors, so that the positive electrification induced in this experiment has escaped through them to the earth and is practically lost. The conductor, however, is not discharged, but retains a negative charge, which becomes sensible when the inducing-rod is removed.

Theory of Electrostatics.—Some working theory must now be given to co-ordinate the experimental facts mentioned in this and the previous chapter. It may be well to point out, however, that theories must not be regarded as final explanations, but merely as aids to thought, so that we can form mental pictures of facts. As examples of this statement, we may mention that—

(1) Until quite recently it was impossible to form any definite idea regarding the meaning of the term *electric charge*, and in order to express the facts in an orderly manner, it was usual to speak vaguely of hypothetical "electric fluids." We now know that a negative electric charge is an assemblage of small particles (known as "electrons"),

which have an existence as definite and real as that of the atoms of matter, although they are very much smaller in size (see p. 488), and which are able to move with great freedom in the bodies we call good conductors, and with much less freedom, or with practically no freedom at all, in the best insulators. The nature of a positive charge is as yet an open question, and need not be discussed here.[1]

(2) The facts of induction used to be explained by supposing that electric charges attracted or repelled each other across the intervening dielectric, and that all neutral conductors contained an inexhaustible supply of the two kinds of electricity in equal quantities. On this supposition, the explanation of inductive action was easy, for, if an electrified body be placed near an insulated uncharged conductor, it may be regarded as attracting one kind of charge and repelling the other; and, if the conductor be "earthed," the repelled charge (*i.e.* the free charge) passes away, and leaves the attracted charge behind, which was then said to be "bound" by the inducing charge. This hypothesis, although it leads to perfectly correct results in most cases, *does not, however, bring into sufficient prominence the part played by the surrounding dielectric.* A more correct and clearer idea of the actions involved can be obtained by introducing the conception of *lines of electric force.*

Lines of Electric Force.—From what has been said, it is obvious that, if a small body containing (say) a unit positive charge be brought into the neighbourhood of another charge, the small body will be acted upon by a mechanical force, which has at any point a definite magnitude and direction. The direction in which the positive charge moves, or tends to move, is (for convenience) arbitrarily defined as being the *direction of the line of force at that point.*

Such lines of force will, therefore, extend from any charged body to an oppositely charged body, and the space around the charge—or a number of charges—containing such lines, is called *an electric field.*[2] We can represent such a condition by regarding the charged body as the centre of a diverging number of lines of force (always the same number for the same charge), which terminate on the surface of a conductor, or of surrounding conductors, *in a charge of the opposite kind.*

We also suppose (1) that these lines are in a state of tension, always tending to contract in length, and (2) that they repel one another laterally. All attractions or repulsions may then be regarded as due to the endways pull or the sideways repulsion of these lines of force.

[1] All substances contain these electric charges, and we may regard all electric generators—from the old frictional machine to the modern dynamo—as being different devices for obtaining the same result, viz. the separation of electric charges. When a charge is at rest upon the surfaces of a body, it is commonly known as *frictional* or *static electricity;* when the charges are allowed to rejoin each other by flowing through conductors, their properties, whilst in this state of motion, are included in the general term *voltaic* or *current electricity.*

[2] The *strength* of the field will be defined later as being measured by the number of lines of force per square centimetre.

A charge is present on a conductor whenever the lines of force pass from it or into it; in fact, from this point of view, charges are merely the *ends* of lines of electric force.

Let us consider an insulated positively charged body. Lines of force pass from it to adjacent conductors, *e.g.* the table, the ceiling, and the walls of the room, and end there in a negative charge, which, although widely distributed, is necessarily equal in total amount to the original positive charge. Now place an insulated uncharged conductor near, so that it is under induction. As it is in an electric field, some lines of force must of necessity pass through it. But inside a conductor such a state of strain cannot exist, and *there* the field completely disappears, leaving the conductor charged with equal amounts of opposite sign, simply because as many lines are leaving it as are entering it.

The real meaning of the terms *free* and *bound* now becomes evident, for let A (Fig. 10) be a positively charged body acting inductively on the body BC. The induced charge at B is connected by lines of force to A, and its tendency is simply to get as near as possible to A. Hence, connecting the body with the earth will have no effect on the charge at B, but that at C will be removed because it is linked to the earth by lines of force.

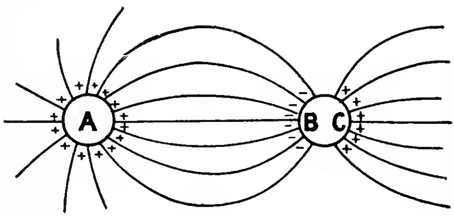


FIG. 10.

Beginners are apt to think that a charge has some mysterious tendency to flow to earth; they must, however, remember that this occurs only when the opposite charge to which it is linked is there already.

We also learn from Fig. 10 that, although the induced charges at B and C are necessarily equal in amount, they are each less than the inducing charge on A, for only a portion of the total lines from A pass through the conductor BC. We further see that the charges at B and C will increase in amount as the distance is decreased, and *vice versâ*, but, under ordinary conditions, they can never become equal to that at A, because even when the conductors are close together, some of the lines from A will take an independent path to earth.

Faraday's Ice-Pail Experiment.—If *all* the lines from A (Fig. 10) could be compelled to pass through the second conductor, each of the induced charges would be numerically equal to that on A. This fact was first demonstrated by Faraday in his historical "ice-pail"[1] experiment.

Exp. 26. Insulate a metal vessel A, Fig. 11. Connect its outside, by means of a wire, to the disc of a distant electroscope E. Charge a metal ball C,

[1] So called because an ice-pail was originally used.

suspended by a dry silk thread, and lower it into the vessel. Owing to induction, the leaves immediately diverge. Observe that, when the ball has reached a short distance from the top of the vessel, the leaves are at their greatest divergence, and that they do not alter when the ball is allowed to descend lower. We see from the diagram that, as soon as the ball reaches a sufficient depth, practically all its lines of force pass through the metal vessel on their way to the opposite charge on the walls of the room. Hence, their ends inside the vessel must constitute a negative charge, and on the outside there must be an equal positive charge. For instance, if the ball has a charge of +10 units, there must be −10 units inside the vessel and +10 units on the outside (partly on the vessel and partly on the electroscope).

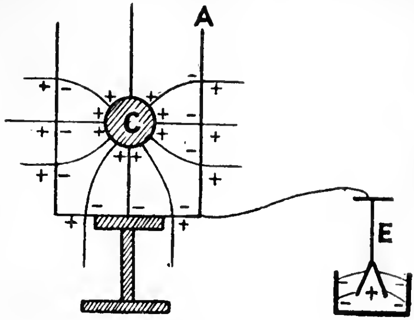


FIG. 11.

Bring the ball in contact with the interior of the vessel, and observe that the leaves remain divergent to the same extent as before. Remove the ball, and show that it is completely discharged. At the moment of contact, the charge inside and that on the ball just neutralise each other, leaving the leaves outside unaffected.

Whence, we conclude that the induced charges are each exactly equal to the inducing positive charge.

We may arrive at this result in a slightly different manner.

Exp. 27. Again charge the ball positively and lower it into the vessel. Touch the electroscope with the finger. Observe that the leaves collapse, owing to the removal of the positive induced charge. Now remove the finger, and touch the vessel with the ball. Withdraw the ball, and notice that the leaves show no evidence of electrification, proving that the induced negative and the inducing positive charges were equal in amount.

Faraday confirmed this result by using four such vessels (Fig. 12), insulated from each other by blocks of shellac. Precisely similar results occurred, each pail becoming inductively charged with opposite electricities, all equal to the charge on the inducing body.

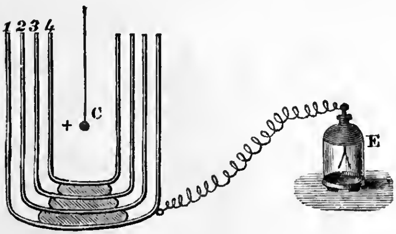


FIG. 12.

Thus, when the electrified ball was lowered into pail (4), inductive action took place through the series, pail (1) producing the same result as that obtained in the previous experiments with one pail.

Evidently, the lines of force diverging from C pass through the whole arrangement, each vessel becoming charged, negatively on its inner side and positively on its outer side.

Inductive Process of Charging a Gold-leaf Electroscope.—From what has been said, it will be easily understood that we can electrify a body by induction with a charge *opposite* to that of the inducing body. In fact, this is the common method of charging an electroscope.

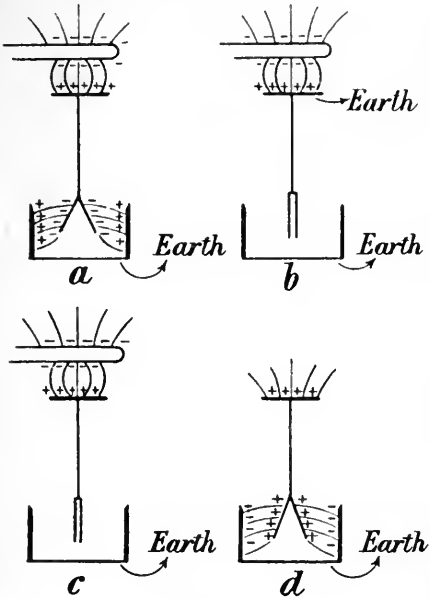


Fig. 13.

In Fig. 13, *a*, we have a negatively charged rod brought near the disc. The lines of force pass from the disc to the rod, and from the leaves to the earth-connected strips at the base.

In *b*, the disc is touched by the hand. The leaves collapse because this really connects the leaves to earth, and that portion of the field disappears.

In *c*, the earth connection, *i.e.* the hand, is removed.

In *d*, the rod is withdrawn, and part of the + charge on the disc spreads on the leaves, both of which send out lines of force to the earth-connected strips.

The Electrophorus.—By means of this instrument, a series of charges may be conveniently obtained from a single original charge. It was invented by Volta in 1775, and is essentially the same in principle as the powerful influence machines to be subsequently described.

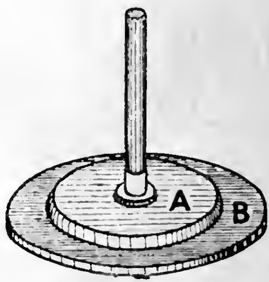


Fig. 14.

It consists of (1) a non-conducting plate B (Fig. 14), which can be easily excited by friction. This is most conveniently made of ebonite (although originally such plates were made by melting together resin and shellac, and pouring the mixture into a shallow tin).

(2) A metal disc A, provided with an insulating handle. This disc must be smaller in diameter than the plate B.

There is sometimes (3) a brass or tinfoil base called the sole, but this is not essential.

Method of Using the Electrophorus.—**Exp. 28.** (1) Warm the plate B until it is quite dry. Rub or strike it with warm flannel or fur. This, of course, develops a negative charge on its surface (Fig. 15, *a*).

(2) By means of the insulating handle, place the metal disc on the plate. On account of slight irregularities on the surfaces the two plates touch only at a few points. Fig. 15, *b*, shows the action, with the air space exaggerated for clearness. The charge on the plate does not pass on to the disc, which is merely under induction in the electric field produced by the plate.

(3) Touch the disc with the finger. The induced negative charge on the upper surface then disappears (Fig. 15, *c*).

(4) Remove the finger, and raise the disc by the handle. It will be charged positively (Fig. 15, *d*), and probably a spark can be obtained from it.

These operations may be repeated many times without again exciting the plate, the energy represented by the induced charges being obtained at the expense of the operator, who does work in separating the disc and the plate against the pull of the lines of electric force.

Exp. 29. Place a pith-ball on the disc. It will remain there quietly during operations 2 and 3, but, on raising the disc, it will jump off, thus indicating the change of distribution which occurs.

Exp. 30. If a spark can be obtained from the disc, bring it over a gas jet from which gas is escaping. The latter will be ignited by the spark.

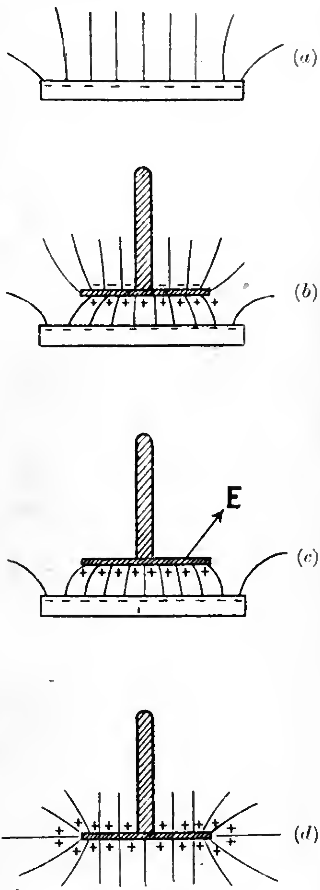


FIG. 15.

Further Properties of Lines of Force.—Several consequences of the properties of lines of force must be kept in view. In the first place, the *ends* can move freely on a conductor; and when the lines pass through a conductor, they may arrange themselves differently on opposite sides. For example, let us imagine that a charged body is placed inside a hollow uncharged sphere, but not at the centre. Inside the sphere, the lines of force will have the distribution shown in Fig. 16; they are generally curved, although at their ends they are perpendicular to the surface. On the outside (assuming no other conductors are near) they are radial and uniformly distributed, although the total number is unaltered. Again, if a number of charged bodies were placed inside, the actual distribution of the field inside would be rather complicated, but the charge on the outside would be the algebraic sum of the internal charges, and the field outside would be radial and symmetrical as before.

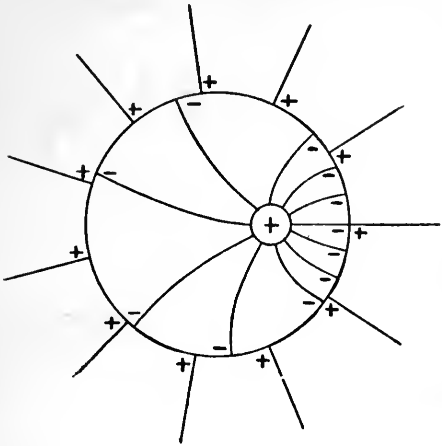


Fig. 16.

The student should notice that the external charge may be altered or removed without affecting the distribution inside.

In the second place, although lines of electric force are in general curved in space, they always end at right angles to the surface of a conductor. This is a necessary consequence of the tension along them and the freedom of movement at the ends.

As a mechanical analogue, think of a piece of wood, floating on the surface of water, with a string attached to it, which is kept under slight tension. Then, the wood will always move until the pull is normal to the surface of the water. (The tension is assumed to be insufficient to raise the wood.)

It is shown later (see p. 41), how the actual path of lines of force may be obtained in simple cases, but it must be remembered that their direction at any point (more correctly, the tangent to the curve at that point) is simply the direction of the mechanical force acting on any small charged body placed at that point. From which may be deduced the important corollary, *two lines of force cannot intersect*, for if they did, there would be two directions of the resultant force at that point, and this is impossible.

Exercise II

1. If a glass rod, strongly electrified by rubbing with silk, is held at some distance from a pith ball hung by a silk thread and having a slight positive charge, the ball is repelled by the rod. But if the rod is brought sufficiently near to the ball the latter is attracted. Explain this.

2. A metal pot A is placed on an insulating stand; a smaller metal pot B is put inside A but insulated from it. A positively electrified metal ball is hung by

a silk thread inside B without touching it. The pot B is now connected for a moment with the earth, the ball is then removed ; next, the pot A is connected for a moment with the earth ; lastly, B is taken out from A without connecting either A or B with the earth. What is finally the kind and degree of electrification of the two pots as compared with the ball ?

3. A glass rod which has been rubbed with silk is held just below the spout of a metal funnel from which shot drop one by one, without hitting the glass rod, into a cup of the same metal as the funnel. State and explain the result which may be observed (1) if the funnel and the cup are each connected with a separate electroscope, (2) if they are both connected with the same electroscope.

4. You have two metal pots on separate insulating stands ; also a metal ball carried by an insulating stem ; also a wire connected with the earth. Suppose the ball, or one of the pots, to be slightly electrified. Describe and explain a process by the repetition of which you can electrify two pots more and more strongly, one positively, and the other negatively.

5. A cake of shellac is rubbed with catskin. Show how to obtain from the shellac either a + or a − charge on an insulated conductor. How can you ascertain whether a given charge is + or − without changing its amount ?

(Lond. Univ. Matric., 1900.)

6. Water escapes from a small earth-connected metal jet directed vertically downwards breaking into separate drops immediately upon leaving it. Near the jet, with its centre in a horizontal line with it, is a positively electrified sphere. The drops fall into an insulated can, and this is found to become more and more strongly electrified. Explain this.

If the insulation of the sphere and can were perfect, the drops would after a time cease to fall into the can. Explain this, and show where they would fall. (Lond. Univ. Matric., 1905.)

7. Describe Faraday's method of showing that, when a charged body is introduced into a nearly closed conductor, the charges produced by electrostatic induction are equal in magnitude to the inducing charge.

(Camb. Local, Senior, 1907.)

8. What is meant by a line of electric force ? Prove that a line of electric force must cut the surface of a conductor normally. (B. of E., 1904.)

CHAPTER III

DISTRIBUTION—COULOMB'S LAW

It follows as a necessary consequence of the properties of lines of force, that a charge is in general confined to the outer surface of a conductor. The charge is linked to an equal charge of opposite sign by lines of force under tension, and as this opposite charge is usually exterior to the conductor—the table, the walls of the room, &c.—the original charge will naturally be pulled to the outer surface of the conductor carrying it. This fact can be easily demonstrated in various ways.

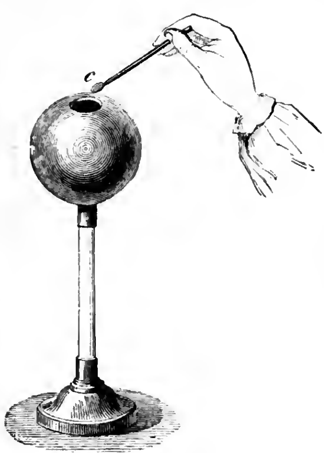


Fig. 17.

Exp. 31. Take a *hollow* insulated metal sphere, having a circular aperture at the top of about one to one and a half inches in diameter (Fig. 17). This can be cheaply made from a bedstead knob insulated on an ebonite penholder. Having charged the sphere with positive electricity, apply a proof-plane, *c*, to the interior, and bring it in contact with an uncharged electroscope; no action takes place; there is, therefore, no electrification inside the conductor. Now touch the outside with the proof-plane, and, on presenting it to the electroscope, observe that the leaves diverge.

Exp. 32. (Commonly called Cavendish's or Biot's experiment). Charge an insulated metal ball by means of an electrical machine (to be described later). Place two hemispherical envelopes, furnished with glass handles, on the outside (Fig. 18). (Again, the spherical conductor may be made by mounting a bedstead knob on an ebonite penholder. For the hemispheres, cut a slightly larger knob in two, and mount each half, by means of sealing-wax, on ebonite penholders. They answer quite as well as the more expensive apparatus.) After contact with the sphere remove the hemispheres, and present each of them to an uncharged gold-leaf electroscope. Notice the divergence of the leaves. Now bring the sphere near the electroscope; there is no divergence of the leaves, thus proving that, although the sphere was originally charged, the electricity has passed to the outer surface, *i.e.* to the two hemispheres.

Exp. 33. (Faraday's butterfly-net experiment). Make a conical muslin bag and fasten it to a brass ring supported on a glass stem (Fig. 19). To the apex of the bag attach two silk threads, by means of which the bag can be turned inside out. After charging the bag (*a*) test the inside by means of a proof-plane and an electroscope. There is no action. (*b*) Touch the outside with the proof-plane, and present it to the electroscope; the leaves immediately diverge. Discharge both proof-plane and electroscope, and then (*c*) turn the bag inside out, and again show that there is no electrification on the inside.

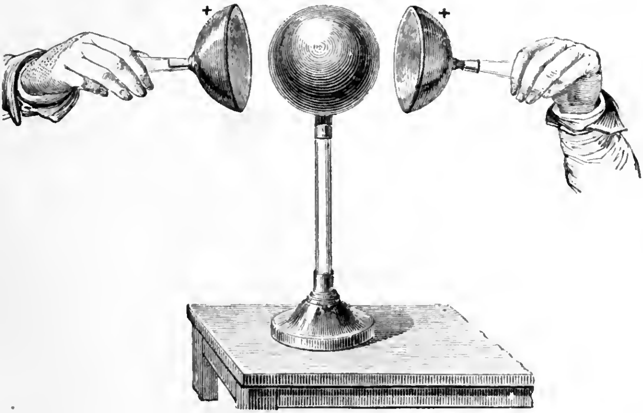


FIG. 18.

It may be mentioned as a further illustration of this fact, that Faraday had a room built, each side of which measured twelve feet. He describes it as follows: "A slight cubical wooden frame was constructed, and copper wire passed along and across it in various directions, so as to make the sides a large network, and then all was covered in with paper, placed in close connection with the wires, and supplied in every direction with bands of tinfoil, that the whole might be brought into good metallic communication, and rendered a free conductor in every part. This chamber was insulated in the lecture-room of the Royal Institution. . . . I went into the cube and lived in it, and used lighted candles, electrometers, and all other tests of electrical states. I could not find the least influence upon them, or indication of anything particular given by them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of its outer surface."[1]

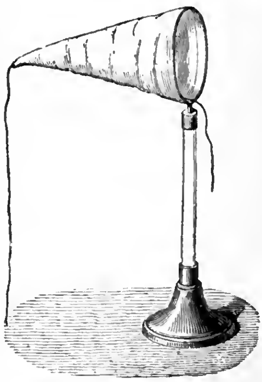


FIG. 19.

[1] *Experimental Researches*, 1173, 1174.

Exp. 34, *as an illustration of Faraday's room.* Take a cubical tin box—say a large biscuit tin—with a small hole (about two inches square) cut in each of two opposite sides. Insulate the box—blocks of paraffin-wax are excellent for this purpose—and place an electroscope inside, so that any movement of the leaves can be seen through the holes. Now connect the box to an electrical machine, and charge it until long sparks can be drawn from it. While the lid is on, notice that there is not the slightest divergence of the leaves. When the lid is off, there is usually, as might be expected, a slight divergence which would, however, be diminished by using a deeper box.

From the facts proved in this chapter we can easily understand why delicate instruments are often covered with gauze or muslin during experimental work.

Several points connected with this subject are worthy of mention.

When we bring a charged conductor into contact with an insulated uncharged conductor, the charge is, as a general rule, shared between them. Indeed there is only one case in which a charge can be completely transferred from one conductor to another, and it appears from the preceding experiments that this occurs when the uncharged conductor is hollow and the charged conductor is lowered into it to a sufficient depth before contact.

Again, it follows that a charge may exist on the inside of a hollow conductor, if the conditions are suitable. As previously stated, the charge is usually on the outside because it is linked to an opposite charge on surrounding bodies. If, however, an earth-connected conductor be supported inside it without contact, it is equivalent to partly putting the earth inside, and now a great part of the lines of force will pass from the inside of the vessel to earth, and a charge may be taken (by means of a proof-plane) from either the inside walls, or from the earthed conductor. These charges will, of course, be opposite in sign.

Electric Surface Density.—When the fluid theory was in vogue, electricity was considered to accumulate to certain depths on the surface of conductors, and electricians used the term *electric density* to indicate this accumulation. It must, however, be carefully remembered that *electric density at a point is the charge per unit of area in the neighbourhood of the point*, and we must rid our minds of the idea of depth. If a charge of Q units be uniformly distributed over an area A, the density (ρ) is given by

$$\rho = \frac{Q}{A}$$

By means of the following experiments, although they are not sufficiently accurate as exact quantitative measurements, we may show that the distribution of electricity varies on differently shaped conductors, and that the density becomes greater as the conductors become pointed.

Exp. 35. Charge an insulated metal sphere with positive electricity. (*a*) Touch any point on the surface with a proof-plane, and bring it in contact with an uncharged gold-leaf electroscope. Notice the amount of divergence. (*b*) Discharge both the proof-plane and the electroscope. (*c*) Touch a different point on the sphere with the proof-plane, and touch the electroscope as before. Notice that there is an equal divergence of the leaves.

Exp. 36. Electrify an insulated pear-shaped conductor (Fig. 20). (*a*) Touch the rounded end of the conductor with a proof-plane, and bring it in contact with the disc of a gold-leaf electroscope. Observe the amount of diver-

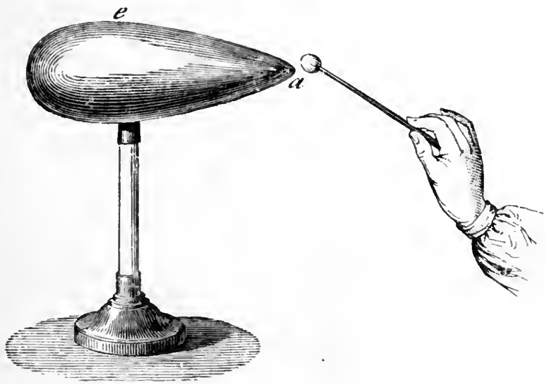


FIG. 20.

gence of the leaves. (*b*) After discharging both the proof-plane and the electroscope, touch the pointed end *a*, and notice that we obtain a greater divergence than before.

Exp. 37. Charge a hollow can, placed on a dry glass tumbler or on a cake of paraffin-wax. Touch (*a*) the middle of one side, and (*b*) the edge with a proof-plane, and observe that a greater divergence of the leaves of an electroscope is obtained in the latter case.

From these experiments we, therefore, learn that the electricity is of equal density on the sphere, but of unequal density on the pear-shaped conductor and on the can.

Electrical Density on differently shaped Conductors.—We have learnt that density varies on conductors of different shapes. We may represent this variation to the eye by dotted lines at various distances from the conductors (Fig. 21).

Coulomb performed many quantitative experiments on distribution. He found that on a cylinder, having rounded ends, 30 inches long and 2 inches in diameter, if the density at the middle was represented by 1, that at the ends was 2·3, while at 1 inch and 2 inches from the ends, it was 1·8 and 1·25 respectively.

Riess gave the density at the middle of the edge of a cube

$2\frac{1}{2}$ times as great, and that at the corners 4 times as great, as that at the middle of a face.

It is advisable to point out that the distribution indicated in

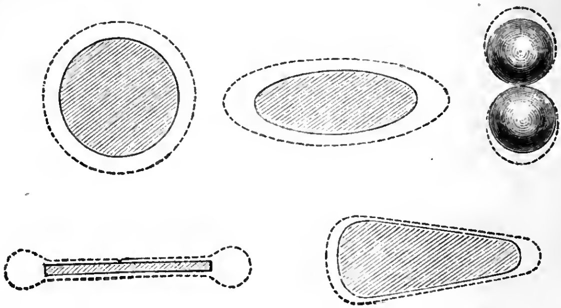


FIG. 21.

Fig. 21 is correct only when the conductors are remote from the influence of other electrified bodies.

Action of Points.—From what has been said regarding the density of electricity on various conductors, it will be seen that the charge tends to accumulate at the edges, corners, and points. This means that the lines of force naturally tend to concentrate in these places, and in fact when their number per unit area exceeds a certain limiting value, the charge will be detached, the lines terminating then on dust or air particles, which move as if repelled by the body. It will be shown later that this tension is proportional to the *square* of the surface density (p. 46), and thus it increases very rapidly near a point. Hence, a point on a strongly charged conductor acts as a leak, and the effect may be great enough to practically discharge it.

Exp. 38. Electrify an ebonite rod, or other non-conductor, by friction. Pass the point of a sharp needle over the surface two or three times without contact. Show, by means of an uncharged electroscope, that the non-conductor is almost completely discharged.

Coulomb's Law.—The earliest application of measurement to electrical forces is due to Coulomb, who, about the year 1785, measured the attractions and repulsions of electric charges by means of his **torsion balance**. He established the law that, when the charges are on bodies so small that they may be regarded as points in comparison with the distance between them, the attraction or repulsion varies directly as the product of the charges and inversely as the square of the distance between them. His form of instrument is now practically obsolete. At the best, it gives very rough results, and a much more correct method of verifying the inverse-square law will be given later. In view, however, of its historical importance, we append a brief description of the instrument and the method of using it.

It consists of a light rod of shellac, p (Fig. 22), having at one end a small disc, n, and suspended horizontally within a cylindrical glass vessel, A, by a very fine silver wire. The upper end of the wire is fastened to a brass button, t, in the centre of a graduated torsion head, e, which is capable of moving round the tube, d ; a being a fixed index which shows the number of degrees the head is turned. A glass rod, i, passes through the aperture, r, and is terminated in a gilt pith-ball, m (the carrier ball). A scale, oc, graduated in degrees, is fixed round the glass cylinder on a level with the pith-ball.

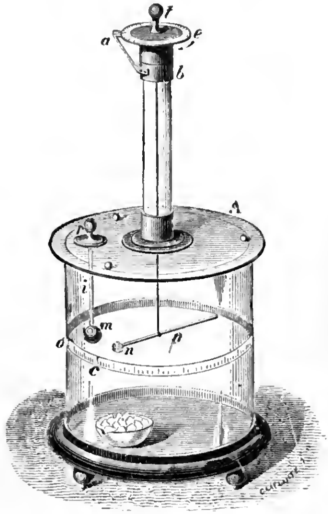


FIG. 22.

Method of using the instrument. The torsion head is turned until the disc, n, and the pith-ball, m, are in contact. The glass rod, i, is removed, the ball, m, charged, and then quickly replaced. When m and n touch, n receives part of the charge of m, and is therefore repelled. This causes the wire to become twisted. The force of repulsion becomes smaller as the distance between m and n increases, while the *force of torsion* becomes greater. Hence at a certain distance these two forces balance each other. Now, when this is the case, as *the force of torsion is proportional to the angle of torsion*, the number of degrees on the scale e is read.

In one of Coulomb's experiments the angle between m and n was 36°, *i.e.* the torsion on the wire was 36°. The torsion head was then turned until m and n were 18° apart (*i.e.* the distance was halved). This was accomplished by turning the disc through 126° in the opposite direction, which, together with the twist of 18° at the lower end of the wire, made a total twist of 144°. To bring m and n 9° apart (*i.e.* to make the distance one quarter of the original distance) it was necessary to turn the disc through 567° ; in this case, therefore, the total torsion was 576°, whence

the distance being	1 :	$\frac{1}{2}$:	$\frac{1}{4}$
the force of repulsion was	36 :	144 :	576
i.e. ,, ,,	1 :	4 :	16

Now these numbers are obtained by squaring the distances and then inverting ; proving that *the force of repulsion varies inversely as the square of the distance.*

By a somewhat modified experiment, the force of attraction between two oppositely electrified bodies was proved to follow the same law.

Coulomb also proved by means of this instrument that, the distances remaining constant, *the force of attraction or repulsion between two small electrified bodies is proportional to the product of the quantities with which they are charged.*

A charge was given to m. When n was brought in contact with m, repulsion ensued, and the angle (say 60°) between them was observed. Afterwards half the charge on m was removed by touching it with an insulated uncharged ball of equal size. It was then found that when m was again introduced (without contact with n) that the torsion head had to be twisted 30° to bring m and n to their original angular distance of 60°. The torsion on the wire is therefore $60° - 30° = 30°$, *i.e.* half the original torsion. Thus the repulsive force is halved on halving the charge on m.

The student should carefully notice the conditions under which the inverse-square law holds good.

It is essentially a *point-source* law, but beginners are apt to apply it indiscriminately to all cases of attraction. It would be absurd to use it to calculate the force between two charged spheres, say 1 inch in diameter and 1 inch apart, although it would give a fairly exact result if the spheres were 1 foot apart, and a still more accurate one at greater distances, the *shape* of the conductors being then immaterial.

Coulomb's law may be written in the form

$$p \propto \frac{qq_1}{d^2} \qquad (1)$$

where p = the force of attraction or repulsion,
q = quantity of one charge,
q_1 = quantity of the other charge,
and d = distance between them.

The first step towards obtaining a system of electrical units may now be taken by using this law to define *unit quantity of electricity* or *unit charge*.

We may define unit charge—as was done years ago by a committee of the British Association—in such a way that the above expression becomes an equality when the force is measured in dynes (see p. 573), and the distance in centimetres. We then have

$$\text{Force of attraction or repulsion (in dynes)} = \frac{qq_1}{d^2} \qquad (2)$$

Evidently this means that unit charge is that charge which at unit distance (1 centimetre) from an equal charge—both concentrated at points—attracts or repels it with a force of one dyne. It is also evident that, in numerical calculations, we need not concern ourselves about positive or negative signs, because, if the signs are unlike, the force is one of attraction ; and, if they are alike, it is one of repulsion.

In one respect our definition is incomplete, for experiment shows that the force depends, not merely upon the magnitude of the charges and the distance between them, but also upon the nature of the surrounding medium. For example, if the air be replaced by, say, carbon disulphide, it will be found that the force is less than half what it was before, although the charges and the distance apart are unaltered. Hence, in defining the unit of charge, it is necessary to state that the force is measured in air.

In order that equation (2) may be of general application, we must write

$$p = \frac{qq_1}{Kd^2} \qquad (3)$$

where K is a number—called the *dielectric constant* of the medium—whose value depends upon the nature of the surrounding dielectric, and which, for air, is taken as unity. (The reason of our putting K in the denominator will not yet be understood by the student, but it may be remarked in passing, that, if it were placed in the numerator, the values would be almost always fractional.)

Equation (3) may now be used as the basis of an extensive mathematical theory leading to quite correct results; the only objection is that the method completely disguises the real nature of the actions involved, and hence it is advisable to consider the matter from an alternative point of view, viz. that of the *electric field.*

Let us consider a simple case. Imagine two insulated spheres placed near each other, charged positively and negatively respectively. Some lines of force will pass from one to the other, and some will extend from the spheres to the surrounding objects or to the walls of the room. Now, if a charged pith-ball be placed between the spheres, we know that it will be attracted by one sphere and repelled by the other, *i.e.* it will move in a certain direction. Instead, however, of regarding this action as being caused by the attraction and repulsion of the charges, we may regard it as being due to the fact that the pith-ball is placed in an *electric field,* without concerning ourselves about the charges, which, as we have mentioned, are merely the terminations of the lines of force on the conductors forming the boundaries of the portion of the dielectric between them. This can be expressed by saying that, when an electric charge is placed in an electric field, a *mechanical* force is exerted on the conductor, which carries the charge, tending to move it along a line of force.

If, however, the nature of the surrounding dielectric is altered, without any other change being made, experiment shows that the mechanical force is altered in amount, although the strength of the field is the same as before. Hence, we are obliged to distinguish carefully between two quite different quantities—(1) the strength of an electric field at a given point in it, and (2) the electric force at the same point.

Definition.—The strength of an electric field *at a given point in it is measured by the number of lines of force per square centimetre at that point, the area being supposed to be at right angles to the lines of force.*[1]

The student will easily understand that this definition applies equally well when the field is not uniform in strength, for, in that case, it is understood that by *the strength of a field at a given point* is

[1] See remarks on magnetic fields, p. 133.

meant the number of lines of force which *would* pass through a square centimetre, if the field had everywhere the same strength that it actually possesses at that point. It is advisable to notice that an electric field has a definite direction as well as a definite strength, *i.e.* it is a vector[1] quantity.

Definition.—The electric force *at a given point in an electric field is measured by the mechanical force in dynes exerted upon* **an** *electric charge of unit strength placed at that point.*[2]

The three quantities—electric field, electric force, and charge—are linked together by the very important relation:—

Strength of electric field = K × Force on unit charge (4)

where it is understood that K is to be taken as unity for air (or more strictly for a vacuum).

We are now in a position to understand the real meaning of Coulomb's equation. For, imagine a charge of q units placed at a point in space distant from other bodies. According to our ideas, a definite number of lines of force must radiate uniformly from it. If we suppose that the *unit* charge possesses n lines of force, then nq lines start from the point in question. (It will be convenient to regard both the value of n and the magnitude of the unit charge as undetermined at this stage, except in so far that they must be in accordance with the definition given above).

Now consider any other point P at distance d from the charge q. As the lines of force are spread over the surface of a sphere of radius d, we have

$$\text{Field strength at P} = \frac{nq}{4\pi d^2},\quad 4\pi d^2 \text{ being the area of a sphere,}$$

but $$\text{Field strength} = \text{K} \times \text{Force on unit charge.}$$

$$\therefore\ \text{Force on unit charge} = \frac{\text{Field}}{\text{K}} = \frac{nq}{\text{K}.4\pi d^2}$$

Now, suppose that any charge q_1 of opposite sign is placed at P,

then $$\text{Force on } q_1 = \frac{nq}{\text{K}.4\pi d^2} \times q_1,$$

but the force due to the two charges q and q_1 is one of attraction,

$$\therefore\ \text{Attraction in dynes} = \frac{nqq_1}{\text{K}.4\pi d^2} \qquad (5)$$

[1] Any quantity, *e.g.* a mass or a volume, which is completely defined by a number only, is called a scalar quantity. A force or a velocity cannot be completely defined in that way; we require in addition to know the ***direction*** in which it acts. Such a quantity is called a vector. In this work it will be assumed that the student is familiar with the theorem known as the "parallelogram of forces," which shows how vectors may be added together or resolved into components.

[2] In all ordinary cases, the charge is carried by some conductor, and it is the mechanical force on the conductor that we really measure.

This is evidently the true form of the law discovered experimentally by Coulomb. It is true whatever we may decide to take as the unit charge, and from it we are in a position to define unit charge in several different ways. But, as we have already remarked, this has been done in such a way that

$$\text{Attraction (or repulsion) in dynes} = \frac{qq_1}{Kd^2} \qquad (6)$$

Comparing equations 5 and 6, and remembering that in both of them $K = 1$ for air, we see that they become identical if we make $n = 4\pi$. Hence, the accepted definition of unit charge also implies that such a charge must be regarded as possessing 4π lines of force.[1]

The student should also notice that, in the accepted system of units, the field strength at distance d from a charge q is $\frac{q}{d^2}$ (for at that distance a total number of lines $4\pi q$ is spread over an area $4\pi d^2$), and also that the force on unit charge at that distance is $\frac{q}{Kd^2}$.

It would be a great convenience if some generally accepted notation could be adopted for the fundamental ideas mentioned in the last few paragraphs. The student will find that similar conceptions are required in dealing with magnetism, but in that case it fortunately happens that certain symbols have received an international significance. No such agreement, however, has been arrived at in statics, and hence much confusion is felt by the student, who finds the idea *force on unit charge* expressed in many different ways by different writers (*e.g.* intensity of electric force). There is some objection to every plan, but we shall in future adopt the following terminology :—

F = Field strength or number of lines of force per square centimetre.
U = Force on *unit* charge (dynes).
K = Dielectric constant (numerically identical with specific inductive capacity).
p = Mechanical force, *i.e.* attraction or repulsion (dynes).

Hence, we have

$$F = KU, \qquad (7)$$

$$p = \frac{qq_1}{Kd^2} \qquad (8)$$

The *total* number of lines of force passing through a given area

[1] This definition of unit charge leads to troublesome constants in certain equations of frequent use in electrical engineering calculations. If it were possible to begin *de novo*, most probably the definition would be altered.

will be called (in accordance with the accepted usage in magnetism) the **Flux.** In a uniform field

$$\text{Flux} = \text{F} \times \text{Area}. \qquad (9)$$

Tubes of Force.—The phenomena of electric (and magnetic) fields may also be represented in terms of the properties of *tubes of force.* Consider a small area on a charged conductor, carrying a small portion q of the total charge. Evidently $4\pi q$ lines of force emanate from this area, and, after passing through some dielectric medium, end finally in an equal and opposite charge on another body. If we think of these lines as a single bundle, their bounding surface traces out a "tube of force," and the whole electric field can be divided up in this way into a number of such tubes, which will completely fill the dielectric space traversed by the field. If we choose the original area so that it carries one unit charge, the result is a "unit tube," and the attractions and repulsions of electric charges may be regarded as due to the tendency of such tubes to contract in length (due to the tension along them), and to expand laterally. We do not consider it necessary to develop this point of view, because the conception of lines of force is universally used by electrical engineers in actual calculations, and experience has shown that it is sufficient for all practical requirements, besides being more convenient in practice than methods based upon the conception of tubes of force.

The Inverse-square Law for Point Charges.—As this law forms the basis of the mathematical theory of electrostatics, a proof of its correctness is of considerable importance. For this purpose, the method of Coulomb cannot be regarded as satisfactory—at the best it affords merely a rough verification.

An indirect, but much more satisfactory, proof may, however, be obtained as follows:—We know as a fact that there is no electric force inside a hollow conductor, and in 1773 Cavendish[1] proved mathematically that this is a necessary consequence of the inverse-square law for point charges, and is inconsistent with any other rate of variation with the distance. Laplace, Bertrand, Clerk-Maxwell, and others have also given proofs of this theorem, for which the student must be referred to more advanced works. Now, the fact itself can be verified experimentally with great exactness, and such experiments constitute the real proof of the law.

It should be noticed that similar facts are met with in connection with gravitation. If the earth were a hollow sphere, there would be no gravitational force inside the cavity, and a body placed therein would have no weight—a result which depends upon the inverse-square law of gravitational attraction.

This resemblance appears to have been first noticed by Priestley, who also investigated the properties of hollow conductors, and who, in 1767, suggested (but without proof) that they might be due to an inverse-square law such as that known to hold good in the case of gravitation.

Another way of regarding the question is instructive. Consider a uniformly charged hollow sphere, and think of the charge as being made up of an infinite number of very small point charges symmetrically distributed upon it. Each of these point charges possesses a definite number of lines of force, and, in the absence of all disturbing conditions, these lines would radiate from it symmetrically in straight lines. Imagine such a system of lines to be drawn from every one of the point charges, then, if the resultant of all these superposed lines be taken (as for instance in Fig. 28), it will be found that there is no field within

---

[1] Cavendish did not publish his results, and Coulomb's work (about 1785) was performed independently.

the sphere, and that the field outside is radial and of the required strength. Now, in assuming that the lines of force radiate in straight lines from each point charge, we have really assumed the inverse-square law. Any other law would involve a different distribution, and the resultant field would not vanish inside the sphere.

Exercise III

1. An electrical machine is placed in an insulated chamber which is lined inside with tinfoil. The rubber of the machine is connected with the tinfoil. What will be the effect upon an electroscope placed outside and connected with the chamber when the machine is in action? Explain your answer.

2. A wire is fastened by one end to the inside of a deep insulated metal jar, and by the other end to an electroscope. When the jar is electrified, the leaves of the electroscope diverge; but no charge is given to a proof-plane put in contact with the side of the jar. Explain these results.

3. A hollow metal vessel is insulated, connected by a wire with a gold-leaf electroscope, and charged with electricity. The leaves diverge. An uncharged metal ball is lowered into the vessel without touching it, (1) by a silk thread, and (2) by a wire. What is the effect on the gold leaves in each case?

4. Two insulated metal spheres, charged respectively with $+5$ and -5 units, are placed one metre apart. What is the direction of the resultant electric force exerted on a small $+$ charge at a point one metre distant from the centres of each of the spheres?

5. The force of attraction between two small balls was 8 dynes, when they were placed 6 centimetres apart. What is the charge on each, if the $+$ charge was twice the $-$ charge?

6. Define unit (electrostatic) quantity of electricity, and find the force exerted between two equal small spheres with centres 8 centimetres apart, one charged with $+16$ and the other with -20 units of electricity. If the two spheres are momentarily joined by a thin copper wire without being moved, find the force between them. (Lond. Univ. Matric., 1903.)

7. Electric charges of 10 and 5 units are given to two bodies which are at a distance of 50 centimetres apart. At what point on the straight line joining the charges is the electric force zero? (C. of. P., Senior, 1906.)

8. Equal electrical charges of 10 units each are placed on small conductors at two opposite corners of a square of 10 centimetres side. Calculate the electric force at either of the remaining corners. (C. of P., Senior, 1908.)

9. Two small spherical pith-balls, each 1 decigram in weight, are suspended from a point by threads 50 centimetres long, and are equally charged so as to repel each other to a distance of 20 centimetres. Find the charge on each in electrostatic units ($g=980$). (B. of E., 1901.)

10. Define unit charge of electricity. Two charged conducting spheres repel each other with a force equal to the weight of a milligram when placed at a certain distance from each other. If the charge on one of the spheres is doubled and the distance between the spheres is also doubled, what is the amount of the repulsion? (B. of E., 1903.)

11. Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milligrammes. What is the charge on each, in electrostatic units? (B. of E., 1892.)

12. A small pith-ball weighing 1 decigram, suspended by a silk fibre and charged with positive electricity, is repelled when a charged glass rod is brought near it. If the direction of the electric field of the glass rod near the ball is horizontal and its magnitude equal to 20 C.G.S. electrostatic units, when the deflection of the fibre is 45°, what is the charge on the ball?

(B. of E., 1906.)

13. Two copper spheres, each 1 millimetre in diameter, are suspended from

the same point by silk fibres 1 metre long, and when equally charged are at a distance of 1 centimetre from centre to centre. Determine the charge on each sphere, the density of copper being 8·9 and the acceleration of gravity 980.

(B. of E., 1907.)

14. A short ebonite rod, with a small electrified knob at one end, is mounted so as to turn freely about its centre in a horizontal plane. In a horizontal line with this centre, and at distances from it of a quarter and half a metre respectively, are placed insulated balls that are also charged. The rod makes ten vibrations in a given time, but makes thirty vibrations in the same time if the balls are interchanged. Compare the charges on the two balls.

(B. of E., 1893.)

CHAPTER IV

POTENTIAL

Exp. 39. Attach one end of a long fine wire to the disc of an electroscope, and the other to an ebonite rod, so that the wire can be moved about and yet remain insulated. Charge an insulated cone-shaped conductor, and by means of the rod bring the end of the wire in contact with it. Notice that it is quite immaterial *where* contact is made; whether it be at the pointed or blunt end, a certain deflection is obtained at the moment of contact, and this remains unaltered in amount as the wire is moved over the surface of the conductor.

Now in Experiment 36, when a proof-plane was used in the ordinary way, a larger charge was obtained from the pointed end than from any other part of the conductor, and we therefore infer from these experiments that, although the charge is not uniformly distributed over the surface of the conductor, there is some condition which is constant at all points upon it.

This fact can be still more clearly brought out by using a hollow insulated metal can. We know that normally the charge is entirely on the outside, and yet, when the end of the wire connected to the electroscope is brought into contact with the can, exactly the same deflection is obtained whether the contact is made inside or outside.

These results are expressed by saying that the **potential** of a charged conductor is the same at all points, and is quite independent of the distribution of the charge upon it.

As the idea of potential often presents some difficulty to beginners, it is useful to give a simple analogy. Imagine a tank filled with water, so shaped that the water is deep in some parts and shallow in others. This is the analogue of a non-symmetrical, charged body. Imagine further that the electroscope is represented by a tall, graduated glass jar, and the proof-plane by a little bucket fastened to the end of a rod. Evidently if we scoop up the water in different places in turn and empty the contents in each case into the graduated jar, we shall be likely to find more water in some parts than in others, because in some places there may not be sufficient depth to fill the bucket.

This is the result obtained in Experiment 36.

If, however, we join the tank to the graduated jar by a pipe or tube, the water will at once rise to a definite level in the jar, and the reading will be the same wherever the connection is made, provided that it is below the surface of the water.

This is the result obtained in Experiment 39.

From this analogy it will be seen that the relation between the ideas expressed by the terms *potential* and *charge* in electricity is similar to the relation between the terms *height* or *level* and *quantity* in the case of liquids. Again, in the science of heat, *temperature* is not the same thing as *amount of heat*, but it is the relative condition of a body which determines the direction in which heat flows when the body is connected to another body. So, *potential* is not *charge* or *quantity of electricity*, but is a relative condition which determines the direction a charge tends to flow when the charged body is put in conducting communication with another body.

Although it is convenient to speak of the *temperature* or the *height* of a body, the expressions really mean the *difference of temperature* or *difference of level* between that body and some arbitrary zero—the freezing-point of water in the first case, and the height above the earth's surface or the sea-level in the second case. Similarly, in electrical theory, we are really *always* concerned with the *difference of potential* between two bodies (usually written Potential Difference, or P.D. for brevity), and when the term *potential* of a body is used alone it is understood that it really signifies the P.D. between that body and some other body whose potential is taken as zero. Although the state of a body infinitely distant from all electrified bodies would be the ideal realisation of zero potential, in practice it is sufficient to regard the potential of the earth as zero.

With conducting bodies, we may summarise the previous statements as follows:—

(1) If a flow takes place between two bodies when they are connected by a conductor, a P.D. must have existed between them *before* they were connected.

(2) The effect of such flow is to equalise their potentials, hence

(3) Two bodies in conducting communication are necessarily at the *same* potential.

(4) If no flow takes place when the two bodies are connected, they must have been at the same potential *before* they were connected.

As corollaries to the above, we may note—

(5) A body connected to earth is at zero potential, whether charged or not.

(6) All points on the same conductor must necessarily be at the same potential, for if not, a flow would instantly take place inside its substance tending to equalise the potentials. If two points on it are kept permanently at different potentials by some external means, the result is a continuous flow or *current*, a case studied later as "Voltaic" electricity.

So far, we have said nothing about the *direction* of flow, and, strictly speaking, a flow must be regarded as involving a passage of equal amounts of positive and negative charges moving in opposite directions. If we find that when two bodies, A and B, are connected, a positive charge passes from A to B, then we may either say that A is at a *positive potential with respect to B*, or that B is at a negative potential with respect to A. Such statements are definite and satisfactory, but it has become customary, by analogy with the use of the terms *high* or *low* in the case of gravitational level, to speak of A as being at a *higher* potential than B, when we really mean that it is at a positive potential with respect to B. Hence, an isolated positively charged conductor is at a *higher* potential than the earth, and a similar negatively charged conductor is at a *lower* potential than the earth. These terms are sometimes convenient, and are perfectly intelligible, but the student must remember that they have no real physical meaning.

An Electroscope Measures Differences of Potential.— In explaining the result of Experiment 39, an electroscope was likened to a tall, graduated glass jar for measuring quantities of a liquid, and it follows from that experiment that when the cap of an electroscope is connected to a body by a wire, the deflection really measures the P.D. between that body and the tinfoil strips at the base of the instrument, *i.e.* the earth.[1]

If this statement is correct, no deflection should be obtained when the cap and strips are connected together by a conductor, *i.e.* when they are at the same potential.

Exp. 40. Place an electroscope on an insulating stand, connect the cap to the tinfoil at the base by a thin wire. Hold a strongly charged rod over the disc, and notice that there is no deflection. (If the rod be brought near the side of the glass vessel, it may be possible to produce a deflection, although cap and base are at the same potential. Strictly speaking, it is understood that only the cap is to be exposed to external electrification, and to satisfy this condition the glass vessel may be enclosed in a wire cage, or more conveniently, vertical tinfoil strips may be pasted on the outside, making contact with the base, space merely being left for the strips to be visible. Such devices are, however, unnecessary for ordinary purposes.)

The fact that the deflections measure differences of potential may be well shown as follows. If any source giving a P.D. of about 200 volts is available (*e.g.* electric lighting mains), connect one terminal to the cap and the other to the base of the instrument. The result is a perfectly definite and steady deflection, whether the supply is direct or alternating. (Care must be taken not to "short circuit" the mains; a very short piece of very fine wire or "fuse" should always be in the circuit as an additional security.)

[1] This is the great advantage obtained by having such strips, or having metal sides to the vessel as in the more exact type of instrument. An electroscope will work well when the leaves are inside a glass flask or bottle, but it is then uncertain what the deflection really measures. Made as above, its deflections measure the P.D. between the cap and the earth, and the readings have a definite quantitative meaning.

Exp. 41. Insulate an electroscope and charge the base—conveniently by induction. Observe that the leaves diverge. Touch the cap and notice that the divergence is increased.

In the first instance—assuming that the base is positively charged—both the base and the leaves and cap are at a *positive* potential, but not at the *same* potential, that of the base being the higher, hence a deflection, which is a measure of the P.D. between them. When the cap is connected to earth, its potential falls to zero, so that the P.D. between it and the leaves is increased—hence the increased divergence. (The student will learn later that the P.D. of the *base* was also lowered by touching the *cap; i.e.* both potentials are altered, but it would needlessly complicate the question to enter into details now. As stated, the nett result is that the P.D. is increased by earthing the cap.)

Difference of Potential Defined Quantitatively.—There is another way of regarding these matters, which is very helpful and instructive. We may express it briefly by remarking that when we say a difference of potential exists between two bodies, it is really equivalent to saying that they are *connected by lines of electric force.* If they are not connected by lines of electric force, there is no difference of potential between them.

If an electric charge be placed between the bodies it will be in an electrical field, and there will be a mechanical force acting on it and tending to move it in one direction or the other, according to the sign of the charge. Again, if we carry the charge from one body to the other against a force, a definite amount of work must be done—which, as in the case of gravitation, is quite independent of the path traversed, as long as we begin and end at the same points. Thus, charged bodies are capable of either *doing work* or *requiring work to be done on them*, and it is this idea of work that is involved in the definition of potential.

Definition.—*The difference of potential between any two points is measured numerically by the work in ergs done in moving a charge of unit strength from one point to the other against the force.* If the charge is moved in the reverse direction, an equal amount of work is done, but in this case the work is done *by* the charge, instead of *on* it.

It must be remembered that although it is convenient to measure a difference of potential in terms of work, yet potential and work are not the same thing. The difference of *level* between the floor and the ceiling of a room is numerically equal to the work done when unit mass is carried from one to the other, but obviously this difference of level is not the same thing as work.

Although we are always concerned with differences of potential, it is frequently convenient to speak of the *potential at a point* without further qualification, just as we are always concerned with differences of level, although we often speak of the *height of a point.* In both cases, the term evidently means the difference between the point in question and some accepted point of zero potential or zero level.

The *absolute potential at a point* is usually defined as being the P.D. between that point and "infinity." This is convenient as a definition, but there is no way of ascertaining its actual value in any given case.

As we have already mentioned, we usually take the earth as a standard, and hence by *potential* we really mean the P.D. between a body and the earth. The fact that the earth has a very variable potential does not introduce any practical difficulty.

Electrical Potential at a Point near a Charged Body.— If we have a charge of Q units at a point in space, the potential it produces at any other point at a distance r centimetres is equal to $\frac{Q}{Kr}$.

The following elementary proof of this formula may be given :—

Let us consider that M (Fig. 23) is a point charged with Q units of positive electricity.

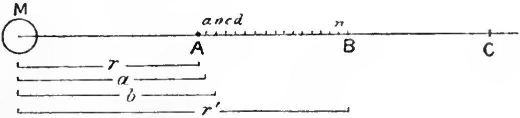


FIG. 23.

It is required to find the potential at A at distance r. By definition, this will be numerically equal to the work done in bringing a unit positive charge from "infinity" to A against the repulsive force due to Q. (It will not affect our argument if we write "earth" in place of "infinity." Such an alteration would merely imply that the value obtained is relative to that of the earth.)

For convenience we shall determine first the P.D. between the point A (at distance r), and the point B at distance r', *i.e.* the work done in moving unit charge from B to A.

Divide the distance $r'-r$ into a very large number of equal parts, Aa, ab, bc . . . . nB.

Then, Force on unit charge at $A = \frac{Q}{Kr^2}$ (p. 27).

Similarly, Force on unit charge at $a = \frac{Q}{Ka^2}$

(a being the distance between M and the *point* a, Fig. 23).

Similarly, Force on unit charge at $b = \frac{Q}{Kb^2}$

" " " $n = \frac{Q}{Kn^2}$

" " " $B = \frac{Q}{Kr'^2}$

Now, as the spaces between A and a, a and b, &c., may be made as small as we please, there will be no material difference if we call $r \times a$ the mean between r^2 and a^2,[1]

$$i.e. \text{ the mean force between } r \text{ and } a = \frac{Q}{Kra}$$

$\therefore$ work done in moving the unit charge from a to A

$$= \frac{Q}{Kra}(a-r) = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{a}\right)$$

Similarly, the work done in moving it from b to a

$$= \frac{Q}{K}\left(\frac{1}{a} - \frac{1}{b}\right),$$

$$\text{and from } c \text{ and } b, \text{ the work done} = \frac{Q}{K}\left(\frac{1}{b} - \frac{1}{c}\right), \text{ \&c.}$$

$$\text{finally, from B to } n = \frac{Q}{K}\left(\frac{1}{n} - \frac{1}{r'}\right)$$

whence, adding these equations, the work done in passing from B to A

$$= \frac{Q}{K}\left\{\left(\frac{1}{r} - \frac{1}{a}\right) + \left(\frac{1}{a} - \frac{1}{b}\right) + \left(\frac{1}{b} - \frac{1}{c}\right) + \text{\&c.} + \left(\frac{1}{n} - \frac{1}{r'}\right)\right\}$$

$$= \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r'}\right)$$

$$i.e. \ V_A - V_B = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r'}\right) \qquad \text{(i.)}$$

where V_A, V_B are the potentials at A and B respectively.

If B is at an infinite distance from M, then r' is infinite, so that

$$\frac{1}{r'} = 0,$$

whence, from equation (i.), $$V_A = \frac{Q}{Kr} \qquad \text{(ii.)}$$

[1] A numerical example will perhaps make this statement clear. Suppose each of the small spaces between A and B = ·01 centimetre, and that $r = 10$ centimetres, then $a = 10·01$ centimetres,

then $r^2 = 100$

and $a^2 = 100·2001$

so that the mean between r^2 and $a^2 = \frac{200·2001}{2} = 100·10005$

but $r \times a = 10 \times 10·01 = 100·1$

so that the difference between the mean of r^2 and a^2 and $r \times a$ is only ·00005. If the distance between A and B had been still smaller (say ·001 centimetre) the difference between the two results would have been only ·0000005.

Students acquainted with the calculus will obtain this result by writing

$$\text{Potential at distance } r = \int_r^\infty \frac{Q}{Kr^2}dr = \frac{Q}{K}\int_r^\infty \frac{1}{r^2}dr = \frac{Q}{Kr}$$

Of course, it will be noticed that if the medium is air, K = I, and

$$\therefore \text{ Potential at distance } r \text{ in air} = \frac{Q}{r}.$$

Application to an Isolated Sphere.—The lines of force from a charged sphere, which is placed at a distance from other conductors, are uniform and radial. Hence, they diverge as if from a point charge at the centre of the sphere, and the potential at the surface will be $\frac{Q}{r}$, where r is the radius of the sphere.

Potential at a Point due to a number of Charges.—If there are a number of charges Q_1, Q_2, Q_3, &c., at distances r_1, r_2, r_3, &c., respectively from the point A (Fig 24), then

$$V_A = \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \&c. = \Sigma\frac{Q}{r}$$

where Σ is the sign of the summation of the series.

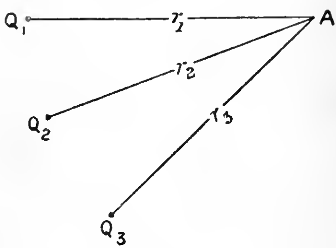


FIG. 24.

Example.—As an example of the kind of problem depending upon this section, we append the following:—

Within a spherical vessel of brass 1 centimetre thick, the external diameter of which is 14 centimetres, a brass ball 8 centimetres in diameter is hung by a silk thread so that the centres of the two spheres coincide. If the ball is charged with 36 units of positive electricity, and if the potential of the vessel is 7, what is the potential of the ball? (B. of E., 1896.)

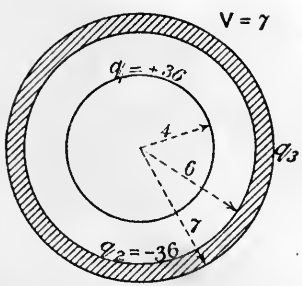


FIG. 25.

As the potential of the outer vessel is +7, we have (Fig. 25) $7 = \frac{q_3}{r} = \frac{q_3}{7}$, *i.e.* $q_3 = 49$.

Also, if there is a charge of +36 units on the enclosed ball, there must be a charge of −36 on the inner side of the outer sphere, for there the ends of the lines from the former must terminate,

$\therefore$ If V = potential of the ball

$$V = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}$$

$$= \frac{+36}{4} + \frac{-36}{6} + \frac{+49}{7}$$

$$= 9 - 6 + 7$$

= 10 static units of potential.

Equipotential Surfaces.—Let a charge Q be placed at a point in space, distant from other bodies or charges. It may, for convenience, be supposed to be carried by a conductor of negligible dimensions. We have shown that the potential at any point at distance d from this charge is $\frac{Q}{d}$. Now all points situated at this distance d must have the same potential, *i.e.* the potential has the same value all over the surface of an imaginary sphere of radius d. Such a surface is known as an **equipotential surface.** Evidently *any* sphere having the point charge at its centre is an equipotential surface; and this will also be true when the charge is carried by a conducting sphere of any radius, provided no other bodies or charges are near.

The characteristic property of such surfaces is that a charge may be moved about on them without doing work against electric force, from which it follows that they are everywhere at right angles to the lines of force. It will be seen that they are analogous to horizontal surfaces in relation to gravitation, for a body can be moved anywhere on the level without doing work against gravity.

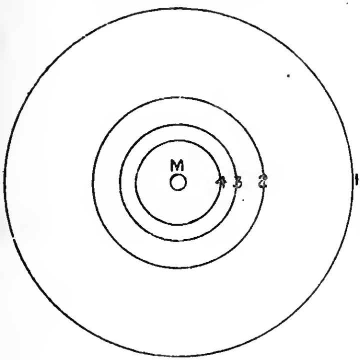


Fig. 26.

The surface of a conductor, of any shape whatever, is obviously an equipotential surface, as was proved in Experiment 39.

We may draw a system of equipotential surfaces, due to a charged sphere, so that there is a *unit difference of potential* between each surface, *i.e.* so that one erg of work is done by carrying a unit of positive electricity from one surface to the next, as follows:—

Let M (Fig. 26) be a small sphere charged with twelve units of positive electricity.

$$\text{Now } V = \frac{Q}{r} \quad \therefore \; r = \frac{Q}{V}$$

To obtain the surface where $V = 1$ with this charge,
we, therefore, have $r = 12$
Where $V = 2$ we have $r = 6$
,, $V = 3$,, ,, $r = 4$
,, $V = 4$,, ,, $r = 3$, &c.

Thus the distances between the surfaces, between which there is unit difference of potential, become greater and greater as we recede from the charged body.

The charge may be thought of as the summit of a mountain, and the equal potential lines drawn round it as indicating equal decreases in height as its slopes are descended. If the charge is negative, it may be thought of as a hole in the ground with sloping sides. The lines shown in Fig. 26 would then resemble the "contour lines" frequently drawn on maps to connect points at the same level.

When the charged body is not spherical, or when several charges are present, the equipotential surfaces or lines are very complicated.

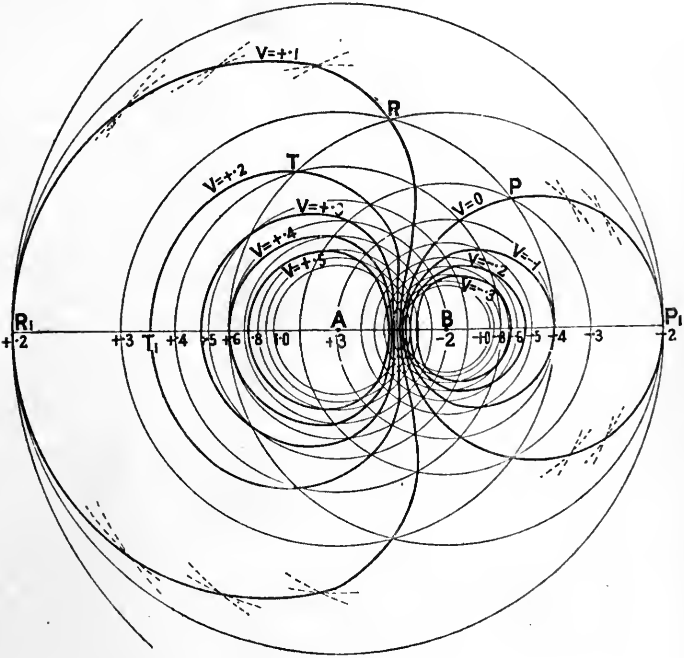


FIG. 27.

Fig. 27 is a reduced copy of a diagram drawn to show how their form may be obtained in a comparatively simple case, viz. around two charges (supposed concentrated at points) of + 3 and − 2 units respectively, placed 5 centimetres apart.

The first step[1] is to draw, *for each charge separately*, a series of equipotential lines, differing in potential by a convenient amount (in the figure, $\frac{1}{10}$ of a unit was used). These will form a series of concentric

[1] The student is advised to follow the instructions on an imperial sheet of cartridge paper.

circles around each charge. For instance, $V = +1$ at a distance of 3 centimetres from the charge of $+3$ units at A, and hence a circle of radius 3 centimetres is described around A as centre. Similarly $V = -1$ at a distance of 2 centimetres from the charge of -2 units at B, and hence a circle is described around B with a radius of 2 centimetres. Outside these, other circles are described, corresponding to potentials $+\cdot9$, $+\cdot8$, $+\cdot7$, &c., and $-\cdot9$, $-\cdot8$, $-\cdot7$, &c. Within the first two circles, others corresponding to potentials 1·1, 1·2, &c., might be drawn, but these lie so close together that it is difficult to show them properly without confusing the diagram. Hence they are omitted, but it must be remembered that this is done only for convenience.

At the point on the axis where the two first circles touch, the potential is $+1$ due to one charge, and -1 due to the other, the resultant being zero. The next two circles cut in two points, and as they represent potentials of $+\cdot9$ and $-\cdot9$ respectively, the resultant is again zero at their points of intersection. The method of drawing the equipotential line $V=0$ is now obvious. It is only necessary to draw a line through the intersections of the successive four-sided figures as shown in the diagram. When the point P is reached, the student may be doubtful what to do next. Now P_1 is evidently a point on the line, because there the potentials are $+2$ and -2 respectively. Hence, a freehand line might be drawn from P to P_1, but in the present case, in order to obtain greater accuracy, two other points, indicated by the intersections of the dotted arcs, have been determined. These correspond to potentials $+\cdot25$, $-\cdot25$, and $+\cdot225$, $-\cdot225$ respectively, the radii being calculated by writing $V = \frac{Q}{d}$, or $+\cdot25 = \frac{+3}{d}$, from which $d = 12$ centimetres; and similarly $-\cdot25 = \frac{2}{d_1}$ from which $d_1 = 8$ centimetres.

Again, in tracing the line $V = +\cdot1$, no points of intersection are obtained between R and R_1, and therefore three intermediate points, indicated by dotted arcs, have been inserted.

When drawing the line $V = +\cdot2$, it is uncertain what to do next when the point T is reached. In this case, it is sufficient to find where the line cuts the axis. Let this be at distance d from the charge $+3$ units, then we have

$$+\cdot2 = \frac{3}{d} - \frac{2}{5+d}$$

On solving this quadratic and taking the positive value, it is found that $d = 8\cdot66$ centimetres.

In the same way, points on the axis corresponding to $V = -\cdot2$, and $-\cdot3$ can be determined.

Sometimes it is desirable to find the point of intersection between

A and B. For instance, let the line $V = +\cdot 5$ cut the axis at distance d from A, and between A and B, then

$$+\cdot 5 = \frac{3}{d} - \frac{2}{5-d}$$

or $d = 2\cdot 4$ centimetres.

Construction for Lines of Force.—The chief difficulty in drawing a diagram of lines of force is due to the fact that we have to represent on a plane surface what is really a distribution of lines in space. For our present purpose, it will be best to simplify the problem

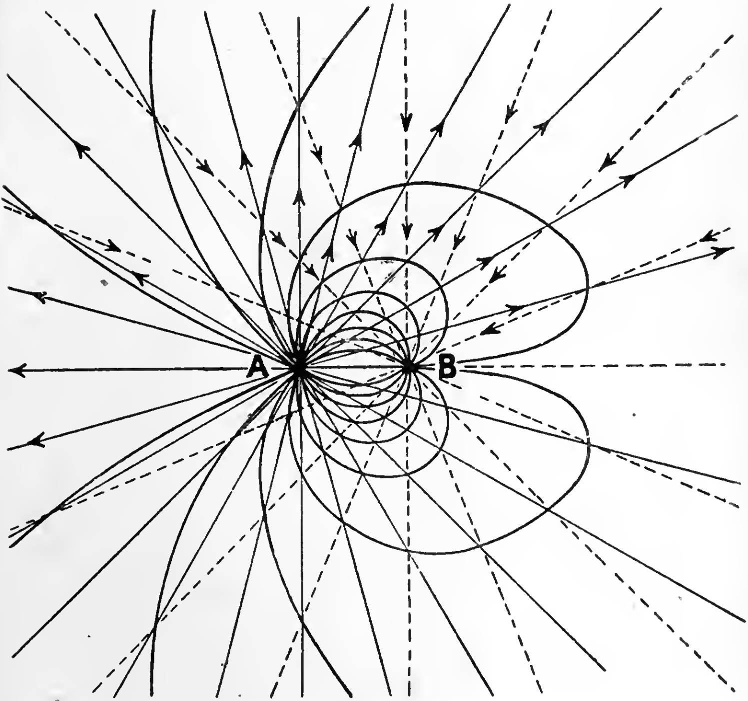


FIG. 28.

by assuming that all the lines are confined to one plane, viz. the plane of the paper, although this necessarily involves a loss of accuracy.

With this assumption, Fig. 28 has been drawn, showing the distribution of the lines of force in the case of the two charges for which the equipotential lines have just been obtained. As these charges are

$+3$ and -2, there are $4\pi \times 3$ lines diverging from A, and $4\pi \times 2$ lines from B, but it is sufficient to draw any convenient number of lines in each case, provided we make their ratio 3 : 2. For instance, 24 radial lines have been drawn from A, and 16 (drawn as broken lines) from B. All the lines diverging from A should be marked with arrows pointing *from* A (because a + charge would move *from* A), and all lines diverging from B, with arrows pointing *towards* B. Four-sided (or triangular) figures are produced by the intersection of these radial lines, and it will be evident on inspection that, at each intersection, the resultant, or actual line of force, will pass through that point roughly in the direction of the diagonal. If the lines were very much more numerous, each quadrilateral would be approximately a parallelogram, and the resultant would be its diagonal; but, as it is, we are only certain that the resultants pass through the opposite corners of each space; thus, although their general direction between A and B can be quickly traced, no pretence is made to great accuracy in the details. From the method of procedure adopted, it will be evident that the figure applies generally to *any* two charges, whatever their actual value, provided they are in the ratio 3 : 2, and + and − respectively. This means that we obtain the general appearance of the field, but not the correct number of lines per square centimetre. If the actual number of lines of force be drawn, the figure may be made quantitatively correct. For instance, as we have used 24 and 16 lines, Fig. 28 may be regarded as strictly applying to charges of $+\frac{24}{4\pi}$ and $-\frac{16}{4\pi}$, apart from the error caused by drawing them on a plane surface.

There will be a point on the axis, at which the field is zero. Remembering that the field at distance d from a charge Q is $\frac{Q}{d^2}$, we see that, if d be the distance of this point from B, its position will be determined by the equation

$$\frac{3}{(5+d)^2} - \frac{2}{d^2} = 0, \text{ which gives } d = 22{\cdot}3 \text{ centimetres.}$$

This point is too far to the right to be shown on the diagram.[1]

Electrical Capacity.—When we charge an insulated conductor with a certain quantity of electricity, we raise the potential of the conductor, but the extent to which it is raised depends upon the *capacity* of the conductor.

Definition.—*The capacity of any conductor is measured by the quantity of electricity with which it must be charged in order to raise its potential from zero to unity.* Thus, a small insulated sphere would

[1] Similar methods apply in the case of lines of force due to magnetic poles.

require a small quantity of electricity to raise its potential from zero to unity, *i.e.* it has small capacity. On the other hand, a large insulated sphere must be charged with a large quantity of electricity in order to raise its potential from zero to unity, *i.e.* it has a large capacity. We therefore learn, that the potential of a conductor depends both upon its amount of charge and upon its capacity; in fact, if C be the capacity of a conductor, Q the quantity of electricity with which it is charged, and V the potential, then it follows from the definition that

$$C = \frac{Q}{V} \text{ or } Q = VC$$

By aid of this formula, we are able to obtain the potential of a number of conductors, placed at considerable distances apart, whose capacities are C_1, C_2, C_3, &c., and whose initial potentials are V_1, V_2, V_3, &c., respectively, when they are joined by fine wires (the capacities of which we may neglect). They all acquire the same potential, V, so that we have

$$V(C_1 + C_2 + C_3 + \&c.) = V_1C_1 + V_2C_2 + V_3C_3 + \&c.$$

$$\therefore V = \frac{V_1C_1 + V_2C_2 + V_3C_3 + \&c.}{C_1 + C_2 + C_3 + \&c.}$$

Unit Capacity.—*The capacity of a sphere is measured by its radius in centimetres;* for suppose we have an insulated sphere, removed from the influence of other charged bodies, of radius r, and charged with a quantity of electricity, Q, then

$$\text{the potential (V) at the surface} = \frac{Q}{Kr}$$

whence, substituting this value in the formula $C = \frac{Q}{V}$ we have

$$C = Q \div \frac{Q}{Kr}$$

$$= Kr$$

If r is 1, C is also 1; hence *the unit of electrostatic capacity is the capacity of a sphere of one centimetre radius* in air (or in a vacuum), when placed at a sufficiently great distance from other conductors or charged bodies.

Subdivision of Charges on Spheres.—We are now in a position to understand the subdivision of charges on spheres when they are put in conducting communication with each other. The quantity of electricity taken by each will depend upon its capacity.

I. *Spheres of equal capacity*, i.e. of equal radius.

(*a*) If we charge an insulated sphere with a certain quantity of electricity, and then bring another insulated sphere of equal size in contact with it, each one will contain half the original charge; if, on separating them, another be brought in contact with either, it will receive half *its* charge, *i.e.* a quarter of the charge originally imparted to the first sphere.

(*b*) If two equal insulated spheres be charged, one with ten units of positive electricity, and the other with twenty units of positive electricity, and then placed in contact, each will have half the sum of the two charges, *i.e.* fifteen units.

(*c*) Similarly, if one of them has originally twenty units of positive electricity, and the other ten units of negative electricity, after contact each has $\frac{+20-10}{2}=5$ units of positive electricity.

2. *Spheres of unequal capacity*, i.e. of unequal radius.

If a large insulated metal sphere of radius r has a charge of Q units, and a smaller insulated sphere of radius r_1 be brought in contact, and afterwards separated; or if the two spheres be merely connected by a fine long wire (whose capacity we may neglect) and then charged with Q units, the quantities (q and q_1 respectively) contained by each may be easily ascertained, for

$$Q=q+q_1 \qquad (1)$$

and the two conductors will be at one potential;

$$\therefore\ V=\frac{q}{r}=\frac{q_1}{r_1}$$

$$\text{whence } \frac{q}{r}=\frac{q_1}{r_1} \qquad (2)$$

Substituting the value of q_1 from equation (1) we have

$$\frac{q}{r}=\frac{Q-q}{r_1}$$

$$\text{whence } q=\frac{Qr}{r+r_1}$$

$$\text{and similarly } q_1=\frac{Qr_1}{r+r_1}$$

This relation is true, and the proof is the same, for any two conductors of capacity C and C_1 respectively, irrespective of shape,

$$\text{and then } q=\frac{QC}{C+C_1} \text{ and } q_1=\frac{QC_1}{C+C_1}$$

Surface Density on Spheres.—(1) *One sphere.* We have already learnt that the density of electricity on any insulated conductor, when uniform, varies directly as the quantity and inversely as the surface, *i.e.* $\rho = \frac{Q}{A}$.

Now, the surface of a sphere, whose radius is r, is $4\pi r^2$, whence, on a sphere,

$$\rho = \frac{Q}{4\pi r^2}$$

(2) *Two spheres joined by a long fine wire.*

Let q and q_1 be the charges on the spheres of radii r and r_1 respectively, and let Q be the total charge,

Then $Q = q + q_1$, and by equation (2) on p. 44,

$$\frac{q}{q_1} = \frac{r}{r_1}$$

i.e. the quantities are directly as the radii.

The density will, however, be inversely proportional to their radii, for from the equation given above

$$\rho = \frac{q}{4\pi r^2}$$

$$\rho_1 = \frac{q_1}{4\pi r_1^2}$$

$$\therefore \quad \frac{\rho}{\rho_1} = \frac{q}{q_1} \times \frac{r_1^2}{r^2}$$

$$\text{but} \quad \frac{q}{q_1} = \frac{r}{r_1}$$

$$\therefore \quad \frac{\rho}{\rho_1} = \frac{r}{r_1} \times \frac{r_1^2}{r^2} = \frac{r_1}{r}$$

Relation between Density and Potential on Spheres.—The relation between density and potential on a sphere may be easily obtained, for

$$V = \frac{Q}{r}$$

$$\text{and } \rho = \frac{Q}{4\pi r^2}$$

whence, by substitution, we have $V = 4\pi r\rho$.

Field and Force very close to a Charged Conductor.—As the density is the number of unit charges per square centimetre, and as unit charge has 4π lines of force proceeding from it, it follows that $4\pi\rho$ lines of electric force must emerge from one square centimetre of any charged conductor, whatever its shape may be, provided ρ be understood to mean the surface density at the place in question.

Again, as the field strength at any point is, by definition, the number of lines per square centimetre, it follows that very close to the surface—before the lines appreciably diverge—

$$\text{Field} = F = 4\pi\rho, \text{ and consequently}$$

$$\text{Force on unit charge} = U = \frac{4\pi\rho}{K} \text{ dynes.}$$

The latter result holds good as long as we are considering the force on an independent charge, close to the surface, but not actually in contact with it.

Force on the Conductor itself.—Evidently there must be an outwards pull due to the tension of the lines of force, and it is required to determine its amount. Here, it must first be pointed out that when a charge Q is placed in a field of strength F, the field due to Q must in reality be superposed upon F, producing a resultant field. According to our definitions, however, the force acting on Q is $Q \times \frac{F}{K}$, from which we see that in such calculations the field due to Q itself must be ignored, *i.e.* we must not regard F as being the resultant field. Hence, if we use this method to find the pull per unit area on a surface charged with ρ units per square centimetre—in which case the actual field is obviously a resultant—it is evident that we must think of the charge ρ as being under the influence of a field partly due to itself and partly due to the remainder of the charge on the body; and it is this latter component which must be used to determine the electric force $\frac{F}{K}$

Now, according to the point of view suggested on p. 28, if we consider the charge ρ alone, we must regard it as made up of point sources, each possessing a radial and symmetrical field. If so, half of its total lines must radiate outwardly, and the other half inwardly, the forces due to the two halves being obviously in opposite directions. Hence (assuming the charge to be positive), the external field due to ρ is $+2\pi\rho$, and the internal field is $-2\pi\rho$. But we know that the resultant field inside the conductor is zero, and, therefore, the component due to the other charges on it must be just equal in amount but opposite in sign to $-2\pi\rho$. Moreover, this component is superposed upon both halves alike, and thus it follows that the resultant external field is $4\pi\rho$, as already deduced by simple reasoning. It will be seen that the force on the charge ρ is the same as it would be on an independent charge of that value placed in a field of strength $2\pi\rho$, *i.e.* the radial pull is $2\pi\rho^2$ dynes.

This important result may be stated more generally by saying that when a charge Q forms the bounding surface of an electric field, in which the electric force is U, then the force acting on Q is $Q \times \frac{U}{2}$.

Exercise IV

1. What is the difference of potential between the points A and B (Fig. 23), if M be charged with 72 units of positive electricity; the distance of A being 8 centimetres, and that of B 12 centimetres?

2. Charges of +10, +15, −5, −4 units of electricity are placed at the corners A, B, C, D respectively, of a square, whose side is 10 centimetres long. Find the potential at the middle point of CD.

3. Twenty units of + electricity are placed at the middle points of the sides of an equilateral triangle, the sides of which are 9 centimetres long. Find the potential at the centre of the inscribed circle.

4. Charges of 10, 20, 30 units of + electricity are placed at the corners A, B, C respectively, of a square, whose sides are 10 centimetres long. Find the potential at the corner D, and at the centre O. Find the amount of work necessary to be done in order to bring a + unit from D to O.

5. Find the quantity of electricity which must be given to an insulated sphere of 6 centimetres diameter, so that its potential may be raised from zero to 15.

6. Three insulated metal spheres placed at considerable distances apart are charged with electricity till their potentials are 2, 5, 7 respectively. If their radii are 2, 3, 4 respectively, find the potential of the whole system when they are connected by a fine wire.

7. If the radii of the spheres were 4, 5, 6 centimetres respectively, and their initial potentials were 6, 7, 8 respectively, find the potential of the whole system when joined by a wire.

8. Two insulated metal balls, one being 1 centimetre radius, and the other 1·5 centimetre radius, were each charged to a potential 70. Find the force of repulsion between the two balls when placed half a metre apart.

9. Two insulated brass balls are joined by a long fine wire; one has a diameter of 3 inches, and the other a diameter of 1 inch. A charge of 48 units of + electricity is given to them. How will the charge be distributed?

10. A large insulated metal sphere is charged with 20 units of + electricity; another sphere of one-ninth the radius of the first is brought in contact. How is the charge distributed when they are separated?

11. Two insulated metal balls are connected by a fine wire; one has a radius of 5 centimetres, and the other a radius of 8 centimetres. They are charged, and on testing the larger one, it is found to have a charge of 16 units. What was the total charge?

12. To what potential must we charge an insulated sphere of 7 centimetres radius, so that its surface-density may be represented by 2?

13. Two insulated brass balls are connected by a long fine wire. They are charged to a potential 40. If the diameter of one is fourteen times that of the other, compare their densities.

14. How much energy is expended in carrying a charge of 50 units of electricity from a place where the potential is 20 to another where it is 30? What is meant by saying that the potential of a conductor is 20? (B. of E., 1894.)

15. Two similar deep metal jars are placed on the caps of two similar electroscopes at some distance apart and the caps are connected by a fine wire; a positively electrified ball is lowered into one of the jars without contact. Explain the effect as to potential and divergence on both sets of leaves, and also that which occurs on breaking the wire connection by means of a silk thread and then removing the ball without allowing it to touch the jar. (B. of E., 1898.)

16. A conducting sphere, of diameter 6, is electrified with 105 units; it is then enclosed concentrically within an insulated and unelectrified hollow conducting sphere formed of two hemispheres, of thickness $\frac{1}{4}$ and internal diameter 7. The outer sphere is then put to earth. Determine the potential of the inner sphere before and after the outer sphere is earth-connected. (B. of E., 1899.)

17. A hollow metal vessel is insulated and charged to potential V, and the following operations are successfully performed: (1) an insulated metal ball is lowered into the jar without touching it, (2) the ball is momentarily earth-connected, (3) the jar is momentarily earth-connected, and (4) the ball is removed to a distance. State the changes of potential of the jar and the ball at each stage. (B. of E., 1901.)

18. Two insulated spheres having radii of 3 and 1 centimetres respectively are placed a long way apart; a charge of 15 units is given to the larger sphere: what charge must be given to the smaller in order that the larger sphere may neither gain nor lose charge when the two spheres are connected by an insulated wire? (B. of E., 1903.)

19. Give a careful freehand drawing of the lines of force due to a charge of 4 units of positive electricity at A, and one of 1 unit of negative at B, if the distance between A and B is 2·5 centimetres. (B. of E., 1904.)

20. The charge and potential of an isolated sphere are each numerically equal to 10. Draw as correctly as you can the equipotential surfaces for potentials 2, 4, 6, 8. (B. of E., 1902.)

21. A brass ball, 7 centimetres in radius, is suspended concentrically inside a spherical brass vessel of internal radius 9 centimetres and external radius 10 centimetres. If the charge on the ball is 56 units and the potential of the outer vessel 5, what is the potential of the ball? (B. of E., 1904.)

22. The difference of potential between two points A and B in a uniform electric field is 100, A and B being 4 centimetres apart and lying upon the same line of force. A body charged with ten units of positive electricity is placed upon the line AB. What force does it experience? (Lond. Univ. Matric., 1906.)

23. Two insulated metal spheres of equal size are equally charged and placed so that the distance between their centres is about three times the diameter of either. Draw diagrams to represent the lines of force round the spheres when their charges are of the same and of opposite sign, respectively. Represent upon other diagrams the effect of placing an earth-connected metal sphere, of the same radius, midway between the two spheres. (Lond. Univ. Matric., 1908.)

24. Explain what is meant by *electrical potential*, and describe some experiment which proves that the potential of a body can be altered without altering its charge.

25. Water is allowed to escape, drop by drop, from a tube connected with an insulated vessel. The knob of a charged Leyden jar is placed near the end of the tube from which the water drops, but so that the water does not touch it. What will be the effect on an electroscope connected with the water-vessel? On what does the ultimate electrical condition of the water-vessel and electroscope depend?

26. One pole of a battery of many cells is earth-connected, and a long insulated wire projects from the other end. Two insulated metal balls, of 1 inch and 5 inch diameter respectively, are put one after the other in contact with the end of the insulated projecting wire. What are the comparative quantities and densities of the electricities on the two balls?

27. A large, strongly electrified metal ball is brought towards a similar unelectrified ball supported by a dry glass stem, as near to it as possible without a spark passing between the balls. The balls remaining at this distance, the unelectrified one is touched with the finger, and immediately there is a spark between it and the other ball. Explain this.

28. A gold-leaf electroscope is put inside a tin can, which is hung up by silk

cords so as to be insulated. On holding a strongly electrified glass rod below the can, no divergence of the gold leaves takes place; but on touching the cap of the electroscope with the finger (without touching the can) the leaves diverge. Explain these results.

29. An insulated conductor, rounded at one end and pointed at the other, is charged. Two small equal spheres supported by insulating stands are made to touch the two ends and then removed. Will electricity pass from one sphere to the other if they are connected by a wire, (1) when they are in contact with the conductor, (2) when they are removed to a distance from it?

30. Two equal soap-bubbles, equally and similarly electrified, coalesce into a single larger bubble. If the potential of each bubble while at a distance from the other and from all other conductors was P, what is the potential of the bubble formed by their union? (N.B.—Volume (v) of a sphere is proportional to *cube* of radius (r), or $v=\frac{4}{3}\pi r^3$.)

31. Three metal balls, the diameters of which are respectively 5, 7, 12 inches, are all connected together by a fine wire, but are otherwise insulated. If the smallest ball has a charge of 10, what are the charges on the other balls?

32. Two insulated and widely separated metallic spheres receive charges of positive electricity, which raise their potentials to 4 and 5 respectively. The densities of the charges being in the ratio 4 : 9, compare the radii of the balls.

33. An insulated brass sphere of 4 centimetres radius is brought into a region where the potential is 5. It is then brought into earth-connection and removed. What is its free charge?

34. An insulated brass sphere was brought into a region where the potential was 10, touched with the finger and then removed. It was found to have 40 units of negative electricity on it. What was its radius?

35. How much work has to be spent in charging a sphere from potential 0 to 12, the diameter being 4 centimetres?

36. Find the strength of the electric field at a point just outside an isolated sphere of 10 centimetres radius charged with 10 electrostatic units.

(B. of E., 1903.)

37. Explain carefully how the distribution on a conductor may be tested. If a wire attached to an electroscope is made to touch different parts of an irregularly-shaped conductor charged with electricity, how would you expect the indications of the electroscope to vary?

38. In what respects is electrical potential analogous to temperature?

The end of a wire connected to a gold-leaf electroscope is put through a hole in the side of a hollow charged conductor. Describe and explain what happens when

(1) it is held there without being allowed to touch the conductor,
(2) it is withdrawn,
(3) it is again inserted and allowed to touch the inside of the conductor,
(4) it is withdrawn,
(5) it is made to touch the outside of the conductor.
(6) Would the result of (2) be different if the electroscope were badly insulated?

CHAPTER V

CONDENSERS AND CAPACITY

FROM the definition of capacity (p. 42) we obtained the expression

$$Q = VC$$

for any charged conductor. As this equation is extremely important, it may be advisable to point out its analogies with familiar facts. In the first place, the idea of *capacity* in electrical theory is, in some ways, analogous to its ordinary meaning, and from this point of view the above equation simply amounts to the statement that the quantity of liquid contained in a vessel depends upon its "capacity" and the height to which it is filled. Ordinary vessels have a definite height, *i.e.* there is a definite *height* to which they may be charged, whereas conductors may be charged electrically to a *potential* limited only by leakage and by tendency to form brush discharge; hence, it is convenient to regard a conductor as similar to a vessel of indefinite depth, but of perfectly definite cross section. If now we agree to define the *capacity* of any vessel as being numerically equal to a quantity of liquid required to fill it to unit depth, the above equation follows at once.

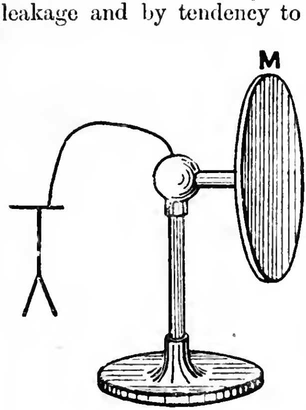


FIG 29.

There is, however, one great fundamental difference between these two cases, for the capacity of a vessel is definite and unalterable, whereas the capacity of an electrical conductor is largely dependent upon its position with reference to neighbouring bodies.

Exp. 42. Obtain two similar metal plates on insulating stands. These may conveniently be circular and have a diameter of about 10 or 12 inches. Connect one plate, M, to an electroscope (Fig. 29), and charge the system by induction (or by any other convenient method) until a large deflection is obtained. Bring the other insulated plate, N (not shown in the figure), within half an inch or so, and notice that there is practically no alteration in the deflection. (Really there is a slight decrease, but so small that it is scarcely noticeable.) Connect N to earth by touching it with the finger, and notice that the deflection is greatly reduced. Remove N to a distance, and notice that the deflection returns to its original value.

During these experiments, the charge on the first plate remains unaltered. Let this be Q. The deflection measures its potential V, and as $Q = VC$, we infer:—

(1) That the presence of the second plate does not appreciably alter the capacity of the first as long as the former is insulated.

(2) When the second plate is earth-connected, its presence *increases* the capacity of the first, for the potential falls although its charge is constant.

Exp. 43. Use the same arrangement, but connect N to earth by a wire. Charge M until the deflection is the same as at first. Then we know that V is the same as at first, and, as C has been increased, Q must be greater. Demonstrate this by removing N. At once the divergence increases until the leaves touch the metallic sides and discharge themselves.

Hence, we find that the capacity of a conductor is considerably increased by the mere presence of an adjacent earth-connected conductor. Further experiments will show, however, that the essential

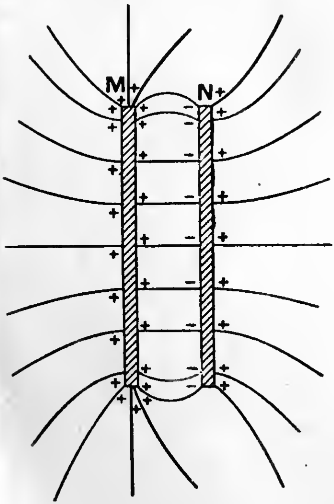


Fig. 30.

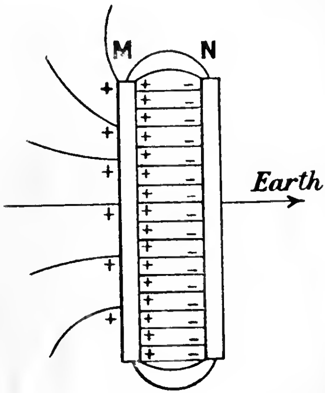


Fig. 31.

condition is not earth-connection, but the presence of an opposite charge on the second plate, for the effect will be unaltered if the latter is kept insulated and charged oppositely in any convenient way.

These actions become easily intelligible, if we consider the diagrams of the electric field shown in Figs. 30 and 31. In Fig. 30 both plates are insulated, M being charged positively and N uncharged. Evidently N is under induction, but, from its shape and position, it scarcely alters the distribution of the field due to M. In Fig. 31 the second plate, N, is earthed. The distribution of the field is now completely altered. The lines from M proceed to the earth, and as N becomes, in effect, a portion of the earth, the lines are therefore concentrated largely between the plates, although we know from experiment that some lines still diverge from M. The closer the plates are together, the greater will be the concentration of the field between them, and the greater becomes the charge which may be concentrated on M for a given P.D. between the plates.

Such an arrangement, which in practice takes many different forms, is known as a *condenser*.[1]

Limit to the Charge on Condensers.—When a dielectric is traversed by lines of force, there is a mechanical strain in it, and when this exceeds a certain value (depending on the nature of the dielectric), the insulation breaks down, a spark or brush discharge passing across it. For this reason, air condensers are used only for standards of capacity; in ordinary cases a more rigid dielectric, *e.g.* glass, mica, or paraffined paper, is used, which enables the plates to be brought much nearer together without undue risk of puncture, and which has a further influence—to be discussed later—depending on the value of K.

The Leyden Jar.—The earliest form of condenser was the well-known Leyden jar, and is made as follows:—Obtain a wide-mouthed glass bottle about 6 or 7 inches high. First, paste tinfoil on the inside of the bottle, leaving $1\frac{1}{2}$ or 2 inches of glass uncovered at the top (Fig. 32). This may be done by cutting a strip of tinfoil of the necessary width and length, and after pasting or gumming the bottle, rolling it up and dropping it inside the bottle; now unroll the foil, taking great pains to secure a smooth surface (a piece of wood, enlarged at one end and covered with linen, is useful for this purpose); and then place a circular piece over the bottom. Next, cover the exterior with tinfoil, leaving a similar margin at the top. The efficiency of the jar is increased, and the risk of puncture is decreased, by making thorough contact (avoiding air bubbles, &c.) between the coatings and the glass. Coat the glass margin with shellac varnish. Fit a wooden stopper into the mouth, having first passed a stout brass wire through the centre. Now solder a brass knob to the end

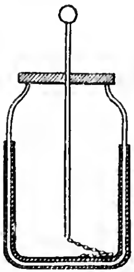


Fig. 32.

[1] In applying the equation $Q=VC$ to condensers, it must be understood that *V is the difference of potential between the conductors.* When one of them is earthed, V is the potential of the other, but in order to simplify the notation employed in this work, V will (unless otherwise stated) always be used in the more general sense, *i.e.* it will denote the P.D. between the conductors.

of the wire outside the jar, and to the inner end fasten a chain, which must be of sufficient length to lie on the bottom of the jar.

A Leyden jar is said to be charged with the kind of electricity accumulated on its inner coating.

In the following experiments, it is assumed that an electrical machine of some kind is available, although for convenience, such machines are dealt with in the next chapter.

The Discharging Tongs or Discharger consists of two curved brass rods terminated in knobs, and joined by a hinge attached to one or two insulating handles (Fig. 33). It is used to discharge a condenser with safety

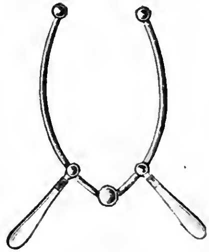


FIG. 33.

Exp. 44. Charge a Leyden jar by holding the outer coating in the hand and presenting the knob to the prime conductor of a frictional machine, or to one of the terminals of an influence machine. Place one knob of the discharging tongs on the outer surface, and bring the other knob near the knob of the Leyden jar. Notice that a sharp crack is heard and a spark seen, due to the neutralisation of the two opposite charges. This method of discharging is called *instantaneous discharge.*

Exp. 45, *to illustrate the opposite electrical conditions of the two coatings of a Leyden jar.* Charge the jar positively.

(1) Holding the outer coating, draw a figure with the knob on a cake of dry vulcanite.

(2) Place the jar on an insulator, and, taking hold of the knob, draw another figure with the outer coating.

(3) A mixture of red lead and flowers of sulphur is then shaken through a muslin bag, from a height above the cake. By the friction between the red lead and the sulphur, the red lead becomes positively and the sulphur negatively electrified. The red lead, therefore, seeks the lines traced by the exterior coating of the jar, and the sulphur the lines traced by the knob.

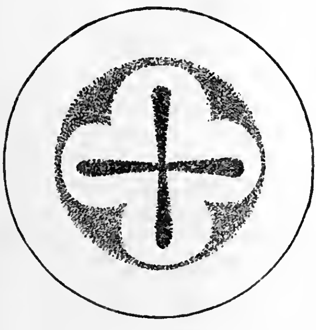


FIG. 34.

Fig. 34 represents the result of an experiment, in which the circle was drawn with the knob, and the cross with the outer coating of the jar. The selection of the red lead for the negative cross and of the sulphur for the positive circle is due to both attraction and repulsion, *i.e.* the sulphur was attracted by the positive circle and repelled by the negative cross. As there is less repulsion in the space between the arms of the cross than at the ends, the particles of sulphur arranged themselves as shown. Such figures are known as **Lichtenberg's figures.**

Franklin's Plate or Fulminating Pane is another form of condenser consisting of sheets of tinfoil fastened on a plate of glass. The tinfoil sheets are smaller than the glass; in fact, if the glass plate be approximately 12 inches by 10 inches, a two-inch margin may conveniently be left between the edge of the tinfoil and that of the glass. The coatings are, however, much more conveniently made with *metallic paint* instead of tinfoil.

Seat of Charge.—**Exp. 46.** Take a glass vessel, B (Fig. 35), having two movable metallic coatings, C and D. Place the parts together so as to form a Leyden jar. After charging it, place the jar on a non-conducting stand. Remove the inner coating by means of an ebonite rod. Show that it is charged by bringing it over the cap of an electroscope, then touch it and notice that the spark obtained (if any) is barely perceptible. Hence, it is only very feebly charged. Now lift out the glass vessel by holding its edge between the fingers. Place it on a non-conductor, or better, retain it in the left hand whilst testing the outer coating for charge. This also will be found to be very feebly charged. Replace the glass, then the inner coating (by means of the ebonite rod), and finally discharge the jar by means of the discharging tongs. Notice that the spark is almost as strong as if the parts had not been separated.

Experiment 46 was devised by Franklin, and clearly demonstrates the important fact that the essential portion of a condenser is the dielectric; the coatings themselves being mere appurtenances, which

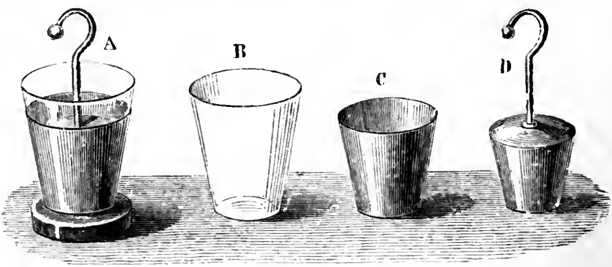


FIG. 35.

enable us to produce conveniently an electric field in the dielectric. Hence, *the energy of charge resides in the dielectric.*

This statement holds good generally. For instance, if we charge an isolated conductor, as in many previous experiments, the energy of the charge is in the dielectric (the air) around it. In the case of a charged Leyden jar, *practically* the whole energy resides in the glass, although a very slight portion, negligible in comparison, is contained in the air around the knob.

All this amounts to saying that electrostatic energy is *always* energy in an electric field, and is therefore located in the medium in which the field exists, *i.e.* in the dielectric.

Residual Charges.—If a condenser be charged slowly to a given potential, and then discharged, *i.e.* brought to zero potential, we find that after a short time it will acquire a potential of the same sign as at first, but smaller in amount, so that it can be again discharged, and so on. The rise in potential after each discharge is owing to absorption of the electricity by the dielectric, and subsequent conduction to the surface after the primary discharge has occurred.

This phenomenon has been carefully investigated. Clerk-Maxwell accounted for the presence of such charges on the hypothesis that the dielectric consisted of heterogeneous particles of unequal conducting powers. Their presence is, however, usually explained on the supposition of the electric strain; the molecules of the glass, being acted upon by the stress of the opposite charges, are strained, and are therefore unable to recover their original form and volume immediately. The discharges after the first are due to what are called *residual charges.*

Exp. 47, *to show the presence of residual charges.* Charge a Leyden jar slowly, and then discharge it. After allowing it to stand for a short time, place one knob of the discharger on the outer coating and bring the other to the knob of the jar. Observe that a second discharge occurs. If the jar be quite dry, a third, fourth, and even fifth discharge may be obtained.

Leyden Battery.—When a powerful discharge is required, a number of jars is generally used, having all their inner coatings in metallic connection, and also all their outer coatings. Fig. 36 shows

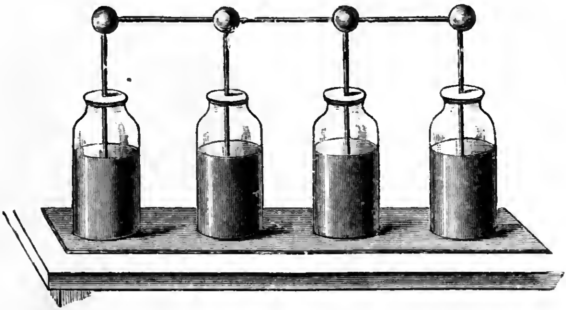


FIG. 36.

this arrangement. The jars stand on a conductor, *e.g.* a piece of tinfoil, thus placing all their outer coatings in conducting communication, while their inner coatings are joined by means of a wire passing through holes in the knobs.

The jars are, however, generally placed in a box lined with tinfoil, which is connected with a hook or handle on the outside of the box. The inner coatings are connected as shown in Fig. 36. Such an arrangement is called a Leyden battery.

Universal Discharger.—This discharger (Fig. 37) consists of two movable brass arms, provided with universal joints and supported on glass legs. The object, through which the discharge is to be passed, is placed on a small table, between the two knobs terminating the arms.

Exp. 48. Place a small quantity of gunpowder on the table of the discharger; fasten one end of a piece of *wet* string to that arm unconnected with the battery, and the other to one knob of the discharging tongs. Having charged the battery, discharge it, and observe that the gunpowder is ignited. The wet string, which is merely a bad conductor, is necessary to increase the time required for the discharge—without it, the powder is scattered mechanically before its temperature is raised to the ignition point.

Exp. 49, *to show the disruptive effect of the discharge of the Leyden battery.* Take a block of shellac through which a wire has been passed, one end cut off flush with the surface and the other terminated in a loop. Place a thin sheet of glass on the upper surface of the shellac. Insulate another wire, pointed at one end and terminated with a knob at the other, and then place it vertically opposite the wire in the shellac cake. Attach a chain or wire to the loop, and

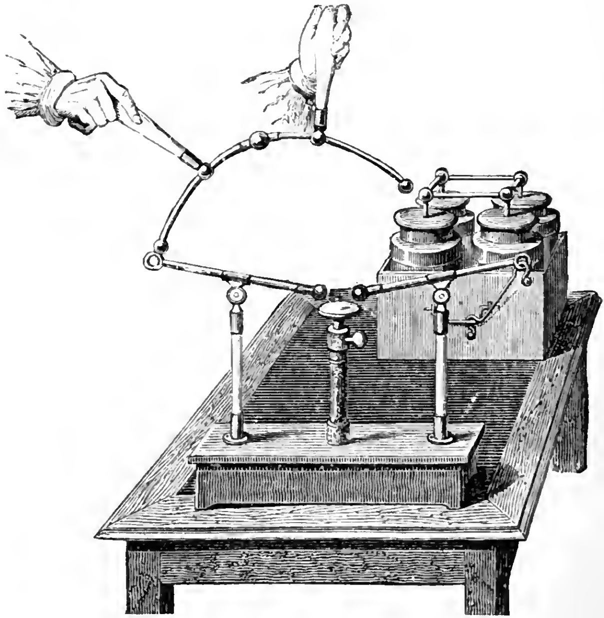


FIG. 37.

then connect it with the outer coating of a Leyden battery. Charge the battery, and by means of the discharging tongs connect the knob of the upright wire with one of the knobs of the battery. A discharge occurs, which pierces the glass.

Exp. 50. Instead of the glass, place a sheet of cardboard on the shellac cake. (1) Let the two wires be opposite each other, and, after the discharge, observe that the perforation is frayed on both sides of the sheet, as though it were pierced from the middle outwards. (2) Let one wire be a little to the right or left of the other. Notice that the hole is nearer the negatively charged wire. This is known as *Lullin's experiment.*

Forms of Condenser used in Practice.—The Leyden jar type of condenser has recently become of great practical importance in connection with wireless telegraphy, &c. This is because—although its capacity is small for a given size—it is able to stand very great differences of potential.

As we have mentioned on p. 52, the risk of the discharge piercing the dielectric is decreased by making perfect contact between the metal coatings and the glass, and hence, in the most recent commercial form—known as the **Moscicki condenser**—the coatings are of chemically deposited silver.

When large capacities are required, and the difference of potential to be used is comparatively small (say, not more than about 100 volts), recourse must be had to mica or paraffined paper for the dielectric. The former is an excellent material, but, when of suitable quality, it is very expensive, and hence it is chiefly used for standards of capacity. Paraffined paper condensers are now employed in large quantities in ordinary telegraphy and telephony, and during recent years great improvements have been made in their manufacture. Perhaps the best method is that devised by Mansbridge. Paper tinned on one side only—what is known as "silver paper"—is employed, interleaved with plain paper as the dielectric. A machine carries two reels of tinned and two of plain paper, the suitably interleaved sheets being wound on mandrels into rolls of sufficient size to have the desired capacity. Connections are made to the two coatings by thin copper strips. When wound, the rolls are dried in ovens until the moisture is completely expelled, and are then placed in melted paraffin-wax in a good vacuum until completely saturated, (the reduced pressure is of great service in eliminating air bubbles). After removal, they are pressed into thin flat plates while still warm, and as soon as possible sealed up in air-tight metal cases.

To give the student some idea of the numerical magnitude involved, we may mention that the size most generally used has a capacity of 2 microfarads. It weighs about 7 ounces, and has a volume of about $7\frac{1}{2}$ cubic inches. Now, 1 microfarad = 900,000 static units of capacity (see p. 591), and hence, with this small bulk, a capacity is obtained equal to that of a sphere of radius 1,800,000 centimetres, *i.e.* about 22 miles in diameter. For the purpose of comparison, it may be remarked that the capacity of an extra large Leyden jar is not likely to be more than ·002 microfarad, *i.e.* about 1800 static units, which is the capacity of a sphere about 40 yards in diameter.

The Condensing Electroscope, invented by Volta, is an ordinary gold-leaf electroscope, provided with another disc (of the same diameter as that of the electroscope), to which is fixed a glass handle. The faces of the two discs are coated with shellac varnish, which forms the dielectric between the plates. The condensing electroscope is only useful when electricity from a weak but continuous source is to be tested.

Exp. 51. Take a compound bar made of zinc and copper soldered together, Hold the zinc end in the hand, and touch the disc of the electroscope with the copper. Connect the upper disc with the earth by touching it with the other hand. Remove the hand and the rod, and then lift the upper plate. This diminishes the capacity of the lower plate to such an extent that its potential rises, causing a divergence of the leaves. The charge will be found to be negative.

This experiment was devised by Volta to prove that electricity was developed by the mere contact of two dissimilar metals; and although the truth of his discovery was denied for a long time, the fact was established beyond doubt by the following simple experiment of Lord Kelvin. It is now known as the "Volta effect," and is further discussed on p. 478.

He suspended a thin strip of metal so that it would turn freely about a point A (Fig. 38), and then charged it with a known kind of electricity. Under it are placed two semi-circular discs or rings of dissimilar metals. No movement of the charged strip takes place while the two dissimilar metals are placed apart. When, however, they are placed in contact it immediately turns, being attracted by the oppositely electrified metal and repelled by the similarly electrified one.

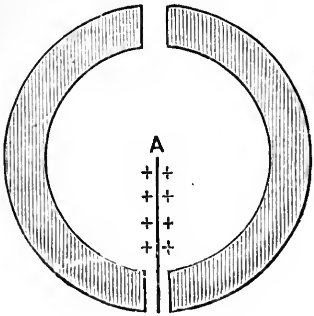


FIG. 38.

From this apparatus, the quadrant electrometer (see Chap. VII.) was afterwards developed by successive improvements, and the experiment can be much more easily performed with that instrument.

The following is a list of substances, which if any two are placed in contact, the first becomes positively, and the second negatively electrified:—

+ Sodium	Copper
Magnesium	Silver
Zinc	Gold
Tin	Platinum
Lead	− Graphite (Carbon)
Iron	

Another instance of the use of the condensing electroscope will be found on p. 197.

Energy of a Charged Body.—Consider an insulated conductor of capacity C, charged with Q units which raises its potential to V. Evidently it possesses *energy*, for it can attract or repel bodies, give a spark, and so on. As in the case of a condenser, the seat of this energy is the electric field; in fact, an isolated charged body is only a particular case of a condenser with widely separated coatings—the second coating being formed wherever its lines of force terminate on a conductor. Further, this energy is necessarily equal to the work done in charging the conductor, and we may regard this operation as equivalent to bringing a charge Q from an infinite distance up to it. If its potential had remained at value V during this operation, the work done would be QV ergs, but actually, its potential was zero at the beginning, and reached V only at the end of the operation. By considering the charge Q to be divided into small portions, which are brought up successively to the body, we can give the following simple analogy to illustrate the real nature of the problem. Suppose that a wall has to be built of height V feet, for which purpose a total weight of Q pounds of bricks is required. If this whole weight had to be lifted through the height V, the work done against gravity would be QV foot-lbs., but, as a matter of fact, only the upper bricks have to be lifted through this height, and the work really done in building, which is equal to the potential energy of the finished wall (with respect to gravity), is manifestly obtained by multiplying the mass by the height of the centre of gravity, *i.e.* $Q \times \frac{1}{2}V$.

The electrical case is exactly similar. If the charge be increased from zero to Q, the potential also increases from zero to V, so that the mean potential is $\frac{1}{2}V$, and the energy is $\frac{1}{2}QV$ ergs. Now $Q = VC$, whence, by substitution, we get

$$\tfrac{1}{2}QV = \tfrac{1}{2}V^2C = \tfrac{1}{2}\frac{Q^2}{C}$$

the three forms being numerically identical.

An exact proof is easily obtained by using the calculus, alternative methods being either cumbersome or unsatisfactory.

$$\text{We have Energy} = \int_0^V Q\,dV, \text{ where } Q = VC$$

$$\therefore \text{ Energy} = C\int_0^V V\,dV = \tfrac{1}{2}V^2C$$

$$\text{or Energy} = \int_0^Q V\,dQ, \text{ where } V = \frac{Q}{C}$$

$$\therefore \text{ Energy} = \frac{1}{C}\int_0^Q Q\,dQ = \tfrac{1}{2}\frac{Q^2}{C}$$

Two Conductors Sharing a Charge.—If a conductor, with charge Q and capacity C, shares its charge with another insulated uncharged conductor of capacity C_1, there will be a loss of energy, although the total charge is constant. As we have seen, the energy of the simple conductor is $\frac{1}{2}\frac{Q^2}{C}$, but for the two together it is $\frac{1}{2}\frac{Q^2}{C+C_1}$. Here again a simple analogy will be found useful. Consider a tank filled with Q lbs. of water to a depth V feet. Its gravitational energy is $\frac{1}{2}$QV lbs. If it be connected by a pipe to another tank, although no water is lost, the level is lower than before. In this particular case where the cross sections are equal, each will contain a charge $\frac{Q}{2}$ and the depth will be $\frac{V}{2}$. Hence, each contains $\frac{1}{4}$ of the original energy, and the other half has disappeared. It is also evident that loss of energy must occur if the second tank be charged either to a higher or a lower level (potential) than the first, the missing portion being the energy of the flow which takes place between them. There is only one case in which no such loss occurs, viz. when no flow takes place, *i.e.* when the levels are equal before connection.

Capacities of Simple Forms of Condensers.—There is one general method of calculating capacities. A charge Q is supposed to be given to the arrangement, the difference of potential between the coatings thereby produced is found by calculation, and then by means of the equation $Q = VC$, C is deduced.

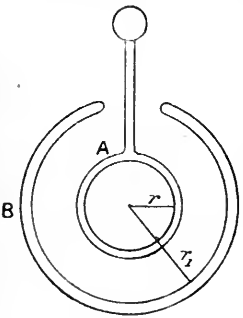


FIG. 39.

I. Spherical Condenser (Fig. 39).—This consists of an insulated sphere A of radius r, surrounded by an uninsulated spherical shell B, of radius r_1, and having a medium between them, whose dielectric constant is K. If A be charged with Q units of positive electricity, then the lines of force from A will terminate in a charge of $-Q$ units on the inner wall of B.

By the example given on p. 37, the potential of the inner sphere A is

$$V = \frac{Q}{Kr} + \frac{-Q}{Kr_1} = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

As the potential of the outer sphere B is zero, the P.D. between the coatings is also V, where:—

$$V = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$\text{Now } C = \frac{Q}{V}, \quad \therefore C = \frac{Q}{\frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r_1}\right)} = K \cdot \frac{rr_1}{r_1 - r}$$

To illustrate this by numerical examples, let $r = 9$ centimetres, and $r_1 = 10$ centimetres, then the capacity of A alone is 9 static units, but by surrounding it with an earthed conductor of 10 centimetres radius, and having air as the dielectric, this is increased to $\frac{9 \times 10}{10-9} = 90$ static units, *i.e.* the capacity of the condenser is the same as that of a sphere of 90 centimetres radius. Again, if the space is filled with turpentine, for which $K = 2{\cdot}2$, the capacity is $2{\cdot}2 \times 90 = 198$ static units.

II. Capacity of a Thin Circular Disc of Radius *r*.—It may be remarked that this is $\frac{2r}{\pi}$. It can be obtained by finding the capacity of an ellipsoidal conductor, and taking the limiting case when the length of its major axis becomes zero, but the proof is too difficult for insertion here.

III. Capacity of a Parallel Plate Condenser.—Let two similar plates M and N, Fig. 40, of any shape, and each of area A, be separated by a distance d. Let N be earthed, and M charged with $+Q$ units, and let us suppose that the whole charge is uniformly concentrated on the inner sides of the plate, *i.e.* the field is absolutely symmetrical, and is confined entirely to the space between the plates. These conditions are not exactly satisfied in any ordinary arrangement, especially near the edges of the plates, but they are approximately so when d is very small compared with the area of either plate.

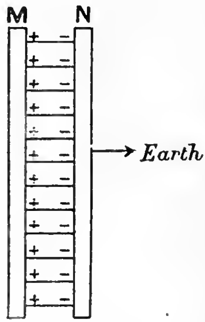


FIG. 40.

Under these circumstances, the charge on N will be $-Q$ units, and $4\pi Q$ lines will pass from plate to plate,

$$\therefore \text{ Field} = F = \frac{4\pi Q}{A}$$

$$\text{but } F = KU$$

$$\therefore \quad U = \text{force on unit charge} = \frac{F}{K} = \frac{4\pi Q}{AK}$$

Again, the P.D. between the plates $= V =$ work done in carrying unit charge from one plate to the other against the force,

$$\therefore \ V = \text{force} \times \text{distance} = U.d = \frac{4\pi Q}{AK} \times d$$

$$\text{whence } C = \frac{Q}{V} = \frac{Q}{\frac{4\pi Q.d}{AK}} = \frac{A}{4\pi d} \times K \text{ static units}$$

From this formula, we see that the capacity is directly proportional to the area of either of the plates, and inversely proportional to the distance between them.

This result holds good for condensers of any shape, if only the distance between the coatings is small compared with their area, and may, therefore, be applied to a Leyden jar, &c. For instance, let us apply it to the spherical condenser just considered, $r_1 - r$ being small compared with r or r_1. As the *average* area of either coating may be written $4\pi r r_1$, we have $C = \frac{A}{4\pi d} \times K = \frac{4\pi r r_1}{4\pi(r_1 - r)} \times K = \frac{r r_1}{r_1 - r}.K$, a result identical with that obtained on p. 60.

Effect of Changing the Dielectric.—It should be noticed that the capacity of a parallel plate condenser can be written in the form $C = \frac{A}{4\pi \frac{d}{K}}$. If, then $K = 2$, the capacity is exactly the same as it is in air, but with the plates at half the distance apart. That is, we may regard the effect of a medium, for which K is greater than unity, as equivalent to bringing the plates nearer together. This is true even if the dielectric does not occupy the whole of the space between the plates; for instance, if a slab of a dielectric of thickness t and dielectric-constant K be introduced between the plates of a condenser, its effective thickness in terms of air is $\frac{t}{K}$, and the effect on the capacity is the same as if the distance between the plates had been reduced by an amount $t - \frac{t}{K}$. For, if d be the distance between the plates, the air distance in second case is $d - t$, to which add $\frac{t}{K}$, therefore total is $d - \left(t - \frac{t}{K}\right)$

Exp. 52. Connect one of the insulated plates, M, used in Experiment 42, to an electroscope, and connect the other, N, to the earth. Charge M until a convenient deflection is obtained. Carefully introduce between the plates, but without contact, a large slab of paraffin-wax at least one inch thick.[1] Notice that the deflection decreases when the wax is inserted, but that it regains its former value when it is removed.

We know that the deflection is a measure of the potential between the plates, and hence we see that the effect of the wax is to reduce V. Q, of course, remains constant, so that by applying the formula $Q = VC$, we learn that the capacity has been *increased* by the insertion of the wax.

Influence of Dielectric on the Energy of a Charged Condenser.—As the energy of the system is $\frac{1}{2}QV$, it follows that

[1] Great care must be taken to ensure that the wax is uncharged. Paraffin-wax is so easily excited by friction, that merely picking it up from the table will probably charge it. It should be tested by means of an electroscope, and if charged, it should be passed through a flame, and then tested again.

some energy is lost on inserting the wax, which is regained on its removal. The question naturally arises, what becomes of it? The answer is simple. During insertion, there is a mechanical force acting upon the wax, tending to pull it into the strongest part of the field, and thus work is done at the expense of the energy of the system. When it is withdrawn, an equal amount of work is done by the experimenter in removing it against the force, and thus the energy is restored to the system. (In fact, a substance, for which K is large, behaves in an electric field somewhat in the same way as iron behaves in a magnetic field.)

During Experiment 52, Q remained constant, but the conditions might have been modified so that V was kept constant. For example, we might have left the plate M permanently connected with some source of constant potential V, then, on inserting the wax, a further flow into it would have taken place in order to keep V constant. In this case, it is evident that the effect of the wax is to increase the energy of the system.

These results may also be deduced by considering the equivalent expressions for energy $\frac{1}{2}\frac{Q^2}{C}$, $\frac{1}{2}V^2C$. The first shows that, when Q is constant, the energy is inversely as the capacity; and the second that, when V is constant, the energy is directly as the capacity.

Capacity of Condensers in Parallel and in Series.—Case I., in Parallel, *i.e.* such as the common arrangement of

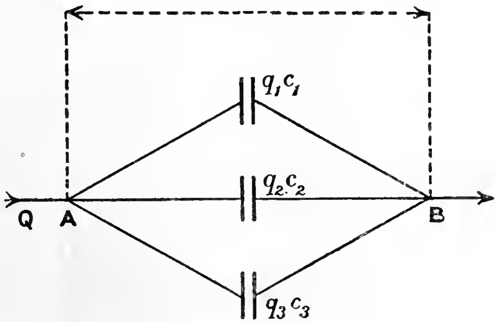


FIG 41.

Leyden jars, in which all the inner and all the outer coatings are respectively connected together.

Let any number of condensers of capacities c_1, c_2, c_3, &c., be connected in parallel, as shown in Fig. 41, and let Q be the total charge given to them, which produces a P.D. of V units between A and B, the common terminals of the system. Then the charges in the condensers will be different, but V will be the same for all of them.

If the charges be q_1, q_2, q_3, &c., we have

$$Q = q_1 + q_2 + q_3 + \&c.$$

If C be the total capacity, $Q = VC$,

and, of course, $q_1 = Vc_1$, $q_2 = Vc_2$, $q_3 = Vc_3$, &c.

$$\therefore \quad VC = Vc_1 + Vc_2 + Vc_3 + \&c.$$

$$\text{or} \quad C = c_1 + c_2 + c_3 + \&c.$$

Case II., in Series, such as the old cascade arrangement of Leyden jars, in which the inner coating of one is connected to the

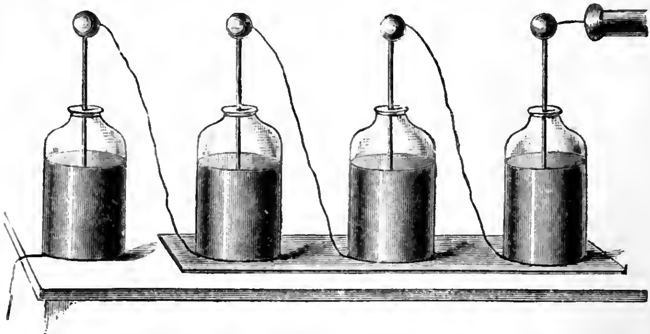


FIG. 42.

outer coating of the next, all the jars except one being insulated. (Fig. 42.)

As before, let a charge Q be given to the system, which produces at P.D. between A and B, the terminals of the system. In this case,

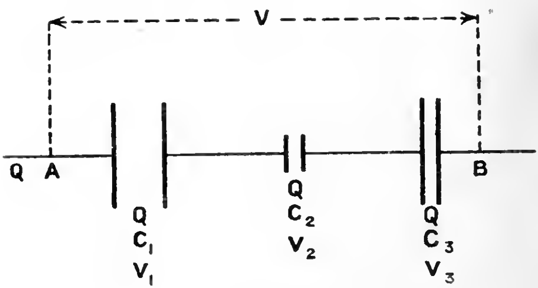


FIG. 43.

the P.D.'s between the coatings of the individual condensers will not be equal.

The charge Q is necessarily the same in all. This will be best grasped by supposing that the charge entering the jar on the right of Fig. 42 induces an equal negative charge on the outer coating of that jar, and an equal positive charge which passes to the inner coating of the second, and so on.

We have necessarily

$$V = V_1 + V_2 + V_3 + \&c.$$

$$\text{where } V = \frac{Q}{C},\ V_1 = \frac{Q}{c_1},\ V_2 = \frac{Q}{c_2},\ V_3 = \frac{Q}{c_3},\ \&c.$$

$$\therefore\ \frac{Q}{C} = \frac{Q}{c_1} + \frac{Q}{c_2} + \frac{Q}{c_3} + \ \&c.$$

$$\therefore\ \frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \ \&c.$$

$$\text{whence } C = \frac{1}{\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \&c.}$$

That is, the total capacity is less than the least of the component capacities, and, in the particular case where we have n condensers of equal capacity, *i.e.* when $c_1 = c_2 = c_3 = \&c.$, we have, from the general equation

$$\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \ \&c.$$

$$\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_1} + \frac{1}{c_1} + \ \&c. \ = \frac{n}{c_1}$$

$$\text{whence} \quad C = \frac{\text{capacity of one condenser.}}{\text{number of condensers in series.}}$$

These results are not of much practical importance in electrostatics, but they are often very useful in the application of condensers to electrical measurements with voltaic currents, for, by suitable combinations, it is possible to obtain a range of values, when, as is usually the case, only two or three condensers are available.

Why the capacity is reduced when "in series" will be understood by noticing that such an arrangement is equivalent to increasing the *thickness* of the dielectric.

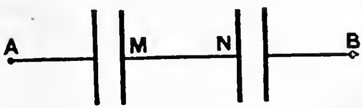


FIG. 44.

Consider two similar parallel plate condensers in series (Fig. 44). Evidently, the connecting wire MN may be shortened to any extent, and in the limit the two plates at M and N coincide. The single plate thus formed takes no active part in the result, and might be removed without altering the effect. We then have a single condenser with a dielectric of double thickness, and from the equation $C = \frac{A}{4\pi d} \times K$ (p. 61), this would have half the capacity of either of the original condensers.

Energy of Charged Condensers.—The expressions already given for energy apply to any combination of condensers, if C is understood to mean the capacity of the combination. In dealing with actual cases, the form most convenient for the purpose should be selected.

Suppose a constant charging source is available, which can produce a P.D. of V volts. Then

(*a*) A single condenser of capacity C will, when charged, possess energy $\frac{1}{2}V^2C$ ergs;

(*b*) If n similar condensers are connected *in parallel*, and charged from the same source, the capacity is nC, and the total energy is $\frac{1}{2}V^2 \times nC$. Evidently, each constituent condenser is charged to exactly the same extent as the single condenser in case (*a*), and the combination has, therefore, n times more energy, *i.e.* it is equivalent to one condenser of n times *greater* capacity;

(*c*) If the n condensers are arranged *in series*, the capacity is $\frac{C}{n}$, and the total energy is $\frac{1}{2}V^2\frac{C}{n}$, or $\frac{1}{n}$th the amount a single condenser would possess if used alone. The combination is now equivalent to one condenser of n times *smaller* capacity. As the total energy is equally shared by the constituent condensers, each must possess $\frac{1}{n^2}$ of the amount a single condenser would have. This is also evident if we notice that the P.D. between the coatings of each condenser is $\frac{V}{n}$, and therefore its individual store of energy is $\frac{1}{2}\left(\frac{V}{n}\right)^2 C$ or $\frac{1}{n^2} \cdot \frac{1}{2}V^2C$.

(*d*) When condensers of *unequal* capacity are joined up in any way, the total energy and its distribution can readily be found by using the expressions for capacity given on pp. 64 and 65.

Specific Inductive Capacity.—Cavendish first discovered that the capacity of a system depended upon the nature of the surrounding medium, although he did not publish his researches. Later, Faraday rediscovered the effect independently, and defined the **specific inductive capacity** of any medium A as being the ratio:—

$$\frac{\text{capacity of condenser when A is the dielectric.}}{\text{capacity of the same condenser when air is the dielectric.}}$$

Our previous results show that this quantity is numerically identical with K, the dielectric constant, although arrived at from a different point of view

Faraday's Experiments.—To determine the specific inductive capacity of a dielectric, Faraday used the apparatus represented completely in Fig. 45, and in section in Fig. 46. The outer coating consisted of a hollow brass sphere, made up of two halves, P and Q, which in every experiment was put in earth-contact. In the interior was a hollow brass sphere C, connected with a brass wire to the knob B, a thick layer of shellac, A, being used to insu-

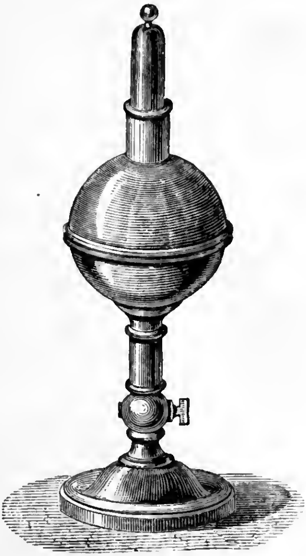


FIG. 45.

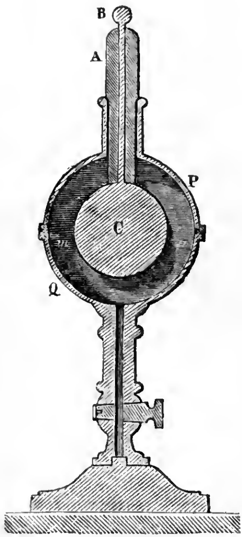


FIG. 46.

late the rod from the outer sphere PQ. The space *mn* contained the substance whose inductive capacity was to be determined. If the dielectric to be examined was solid, P and Q could be pulled apart to admit it. If, however, the substance was a gas, the space was first exhausted by screwing the foot to an air-pump, and the gas afterwards introduced. A stop-cock enabled the passage between *mn* and the base to be opened or closed at will.

Faraday employed in his experiments *two* exactly similar condensers, the space *mn* in *one* being filled with the dielectric—say shellac—to be examined, while that in the other was filled with air. The *air condenser* was then charged, and the potential (V) of its inner coating measured by means of a torsion balance.[1]

In a particular experiment Faraday obtained a torsion of 290°. This condenser was then made to share its charge with the other condenser, which was

[1] This method of measuring potentials is now obsolete.

filled with shellac. The potential (V_1) was again measured, a torsion being obtained of 113·5°.

Now, if the capacity of the shellac condenser had been equal to that of the air condenser, the torsion should have been 145°. As, however, the potential is less, the capacity must be greater. In fact, if C be the capacity of the air condenser, and C_1 that of the shellac condenser, then

$$V = \frac{Q}{C}$$

and, as both condensers are used together,

$$V_1 = \frac{Q}{C + C_1}$$

whence, from these equations, $VC = V_1(C + C_1)$

$$\therefore \; C_1 = \frac{V - V_1}{V_1} \cdot C$$

In the experiment mentioned above

$$C_1 = \frac{290 - 113 \cdot 5}{113 \cdot 5} \cdot C = 1 \cdot 55 \, C$$

The mean of a number of experiments gave $C_1 = 1 \cdot 5 \, C$.

For convenience, however, Faraday filled the lower hemisphere only with shellac, so that if K be the specific inductive capacity of shellac, and that of air be unity, we have

$$\frac{C_1}{C} = \frac{1 + K}{1 + 1} = \frac{1 + K}{2}$$

$$\text{but } \frac{C_1}{C} = 1 \cdot 5$$

$$\therefore \; \frac{1 + K}{2} = 1 \cdot 5$$

whence $K = 2$.

Since Faraday's time, much work has been done to determine the *specific inductive capacity*, or, as it is better termed, the *dielectric constant* of different substances, and many devices have been used to eliminate error. One of the chief difficulties which experimentalists have to contend with, arises from the fact that the capacity of the condenser (containing solid or liquid dielectric) is affected by *electric absorption.* If, for example, we charge a condenser, having ebonite as the dielectric, to a certain potential, we find that in a short time the potential has diminished. This is partly due to leakage, and partly to an absorption of the electricity by the dielectric. To restore the condenser to its original potential, a further charge must be added. Again, the potential falls from further absorption. Thus, the capactity of a condenser depends upon the *time* that the charge has been accumulating. and the results vary according as the charge is added slowly or instantaneously; methods of measurement, in which rapidly alternating currents are used, giving the lowest values. It is for this reason that *air* condensers are employed for the most accurate standards of capacity, as gaseous dielectrics are entirely free from such a defect. The condensers and methods employed in these researches are too complicated to admit of any satisfactory explanation in this work, but the principle embodied is explained on p. 91.

Again, the *temperature* at which the measurements are made should always be stated, otherwise the results may be seriously affected. This is especially the case with bodies which are liquid at ordinary temperatures. Thus, for water or ice within the range of ordinary temperatures, K has an enormous value of 78 to 80, but Fleming and Dewar have shown that at $-185°$ C., and a frequency of 100 cycles per second, it falls to between 2 and 3. (The abnornmally large dielectric constant of water is of great importance in connection with electrolysis.)

It is, therefore, not remarkable that the values obtained by different investigators are somewhat discordant, as shown in the following table, which has been prepared on the assumption that $K=1$ for air. (It would be more strictly logical to put $K=1$ for a vacuum, but as the values for gases are so nearly unity, the alteration would be quite inappreciable.)

Air	1·0
A vacuum	0·94
Hydrogen	0·9997
Carbon dioxide	1·0008
Liquid oxygen	1·478
Different kinds of glass	6 to 10
Ebonite	2·6 to 3·48
Mica	6·6 to 8
Shellac	2·74 to 3·73
Sulphur	2·24 to 3·84
Quartz	4·49 to 4·55
Paraffin-wax	1·99 to 2·29
Turpentine	2·1 to 2·3
Carbon disulphide	2·67
Water at 15° C.	80
Ice at $-23°$ C.	78
Ice at $-185°$ C.	Between 2 and 3
Alcohol at 15° C.	25
Alcohol at $-185°$ C.	3·1
Formic acid at 15° C.	62
Formic acid at $-185°$ C.	2·4

Exercise V

1. Two Leyden jars are exactly alike, except that in one the tinfoil coatings are separated by glass and in the other by ebonite. A charge of electricity is given to the glass jar, and the potential of its inner coating is measured. The charge is then shared between the two jars, and the potential falls to 0·6 of its former value. If the specific inductive capacity of ebonite be 2, what is that of glass? (B. of E., 1893.)

2. The inner coating of one spherical Leyden jar, whose surfaces have radii 12 and 14 respectively, is charged with 25 units of positive electricity, and the inner coating of another, with surfaces of radii 8 and 12, is charged with 5 positive units, the outer coatings of both being earth-connected. Their inner coatings are then momentarily joined by a fine wire; in which direction will electricity pass, the dielectric in both jars being air, and the distance between the jars considerable? Give full reasons for your answer. (B. of E., 1894.)

3. A Leyden jar consists of two concentric spherical surfaces of 5 and 6 centimetres diameter respectively, the intervening space being filled with air. The outer sphere is uninsulated, the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to the earth?
(B. of E., 1895.)

4. An insulated sphere of 2 centimetres radius is connected by a long thin wire with another insulated sphere, the radius of which is 6 centimetres, and which is surrounded by a third sphere of 8 centimetres radius concentric with it. The wire which connects the first and second spheres passes through a small hole in the third so as not to touch it. All the spheres are conductors. Calculate the capacity of the two connected spheres. (B. of E., 1896.)

5. A Leyden jar A, of capacity 3, is insulated and the outer coating is connected by a wire with the inner coating of another Leyden jar B, of capacity 2, the outer coating of which is uninsulated. If the inner coating of A be charged so that the potential is V, what is the potential of the inner coating of B?
(B. of E., 1899.)

6. What is meant by *the capacity of a condenser?* Calculate the capacity of a parallel plate air condenser of which each plate has an area of 400 square centimetres, the distance between the plates being half a millimetre. Be careful to state the unit in which you express your answer. (B. of E., 1906.)

7. Two Leyden jars are charged with quantities of electricity in the ratio of 2 : 3. If in the jar which receives the larger charge the tinfoil surface is twice as great and the glass is twice as thick as in the other, compare the quantities of heat produced by discharging them. (B. of E., 1903.)

8. Two Leyden jars, each having a capacity of 1000 centimetres, are charged in series to a difference of potential of 10 electrostatic units. Calculate the energy of discharge and state in what units it is expressed. (B. of E., 1907.)

9. The terminals of a condenser with mica as the dielectric are connected to a quadrant electrometer and the condenser is charged so that the scale deflection is 90. When a second condenser of the same dimensions as the first, but having paraffin-wax as the dielectric, is connected in parallel with the first, the deflection falls to 30 divisions. Compare the dielectric constants of mica and paraffin.
(B. of E., 1907.)

10. An insulated metal sphere A is positively charged. Another insulated sphere B of equal radius but uncharged is momentarily brought into contact with it and then removed. What will be the ratio of (1) the charge, (2) the potential, (3) the electric energy of A after contact to the value of each of those quantities before contact? (Lond. Univ. Matric., 1907.)

11. Explain what is meant by *specific inductive capacity*. Two plate condensers, A and B, are found to have the same capacity. The area of the plates

in A is four times as great as the area of the plates in B, and they are twice as far apart. Compare the specific inductive capacities of the dielectrics in A and B. (Oxford Local, Senior, 1901.)

12. An air condenser is formed of two circular metal plates, each of 5 centimetres radius, placed at a distance of 0·5 centimetre from one another. The collecting plate was charged to potential 4. What was its charge? (C. of P., Senior, 1905.)

13. A sphere of radius 40 millimetres is surrounded by a concentric sphere of radius 42 millimetres, the space between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 millimetres respectively, and the space between them is filled with paraffin of specific inductive capacity 2·5? (B. of E., 1892.)

14. What is meant by the electrostatic unit of capacity? The capacity of a spherical condenser is 0·0033 microfarad, the diameters of the inner and outer surfaces of the dielectric being 20 and 20·5 centimetres respectively. What is the specific inductive capacity of the dielectric? (1 microfarad = 900,000 electrostatic units of capacity.) (B. of E., 1908.)

15. Two insulated metallic plates are placed facing each other, and each of them is connected with a separate gold-leaf electroscope. If one plate is charged, the leaves of both electroscopes diverge. If now an unelectrified slab of sulphur is introduced between the plates without touching either, state and explain the effect on each electroscope.

16. A positively electrified metal ball is hung by a silk thread above a gold-leaf electroscope. Would the divergence of the leaves be altered (and if so—how and why) by putting (1) an unelectrified cake of resin, or (2) a metal plate held in the hand between the ball and the electroscope, so as not to touch either?

17. An electrified brass plate held over the cap of an electroscope causes the leaves to diverge. On touching the electroscope the leaves fall together. If, after removing the finger, an unelectrified dry glass plate is put between the electrified metal plate and the electroscope, without touching either, the leaves diverge again. Why is this? How must the electrified plate be moved to make the leaves collapse again?

18. Two insulated brass plates, a good way apart and connected by a fine wire, are electrified and then discharged. They are next electrified to the same degree as before, but before being again discharged, they are moved so as to be in contact with each other face to face. On now discharging the plates, more heat is produced than was produced in the previous discharge. Account for the difference.

19. Three equal similar Leyden jars are connected (1) *in series*, that is, so that the outside coating of the first is in contact with the knob of the second, the outside of the second in contact with the knob of the third, and the outside of the third earth-connected; (2) *abreast*, or with their similar coatings all connected together—and in each case the set of jars is charged as fully as can be by the same machine. What proportion does the heat produced by completely discharging all the jars in the first case, bear to the heat produced by discharging them in the second case?

20. A large insulated metal ball at a great distance from all other conductors is electrified, and then, by touching it with an earth-connected wire, is discharged. It is again electrified to the same degree as before, and then discharged by bringing it in contact with a wall of the room. Why does the second discharge produce less heat than the first?

21. The inside of a Leyden jar is connected with the prime conductor of an electrical machine, and the outside with the rubber of the machine, and also with a brass ball fixed at $\frac{1}{10}$ inch from the prime conductor. If the jar is at first without charge, ten turns of the machine are required to cause a spark to pass between the conductor and the ball. If now (the inside of the jar remaining, as

before, connected with the prime conductor) the outside is insulated and connected with the inside of an exactly similar jar, the outside of which is connected with the rubber and the brass ball; show how many turns of the machine would be required to make a spark pass.

22. Two equal insulated spheres, 8 centimetres in diameter, are placed some distance apart, and one of them is charged with 100 units of electricity. The two are then connected by a wire. Calculate the energy of the discharge which then takes place between them.

23. A sphere of 6 centimetres radius is suspended within a hollow sphere of 8 centimetres radius. If it be electrified to potential 6, and the outer coating be in earth contact, find the quantity of electricity with which it is charged.

24. Find the capacity of a spherical condenser, the radii of the two coatings being 6 and 8 centimetres respectively, the dielectric being paraffin whose specific inductive capacity is 2·9.

25. If the radii be 7 and 10 centimetres, and the dielectric be shellac (specific inductive capacity = 2), find the charge when the potential is 5.

26. A spherical condenser is formed by spheres of radii 10 and 8 centimetres respectively. The space between the coatings is filled with paraffin of specific inductive capacity 2. How much work must be done to charge the condenser so that there is a difference of potential of 10 electrostatic units between the coatings? (B. of E., 1904.)

27. Two Leyden jars, A and B, receive charges of $+50$ and -30 units respectively, their outer coatings being earth-connected and their knobs at potentials 20 and -10. Determine the energy of discharge of each jar after their knobs have been in contact with each other. (B. of E., 1900.)

28. An insulating conducting sphere of radius 15 centimetres is charged to a potential of 2000 electrostatic units, and is then caused to touch another insulated conducting sphere of the same size, but uncharged. Calculate the electrical energy of the two spheres before and after they have been brought in contact. How do you account for the difference in the energy in the two cases?
(B. of E., 1910.)

29. Explain what the meaning is of the term "lines of electric force," and what inference may be drawn from their distribution. How many lines of force approximately will there be per square centimetre cross-section in the space between two parallel plates, 10 centimetres diameter, of an air condenser charged with 250 electrostatic units of electricity. (Lond. Univ. Inter. B.Sc., 1904.)

30. Find the electrostatic capacity of a condenser formed by two concentric conducting spherical surfaces of radii 17 and 20 centimetres respectively, and find the energy required to charge it, if the surfaces differ in potential by 100 electrostatic units. In what way will the energy vary, if the space between the surfaces is filled by such a material as sulphur?
(Lond. Univ. Inter. B.Sc., Honours, 1904.)

31. Two large parallel plates 6 centimetres apart are charged—one positively, the other negatively—with 4 units of electricity per square centimetre. What is the intensity of field between them? (Lond. Univ. Inter. B.Sc., 1904.)

CHAPTER VI

ELECTRICAL MACHINES

An electrical machine is a particular form of **generator**, a term which includes all devices, from a battery to a dynamo, whose function is the continuous separation of electrical charges. In all generators, the nett result is to accumulate positive electrification at one point and negative electrification at another point, energy being necessarily expended in the process; and all electrical phenomena are due to the tendency of these charges to rejoin one another, an action which may be allowed to take place in various ways. The most characteristic property of the type of generator now under consideration is its power of producing extremely high differences of potential.

Such machines may be divided into two classes, depending upon (1) friction, (2) induction.

Machines depending on Friction.—The old-fashioned machine is a natural development of the method of obtaining charges by friction. It is now obsolete, but in view of its historical importance, it may be briefly referred to.

The Cylinder Machine is probably the simplest form of machine. It consists of (1) a *glass cylinder*, A (Fig. 47), supported on two wooden stems, B B, and capable of rotation, by means of the handle D, on a horizontal axis; (2) the cylinder is pressed upon by a *rubber*, E, made of leather stuffed with horsehair, to which is attached a flap of silk, F. The rubber should be covered with powdered amalgam,[1] and is supported on a wooden or glass stem, C (the latter is preferable, as a negative charge can then be collected from the rubber). (3) The *prime conductor*, G, which consists of a conducting cylinder, always insulated on a glass leg. The end nearest the glass cylinder carries a rod terminated by a cross piece, provided with a number of sharp brass points, which is called the comb.

Action of the Machine.—When the cylinder is turned, the friction of the rubber generates positive electricity on the glass and

[1] Electric amalgam is made by placing one part by weight of tin and two of zinc in a crucible; just fusing, and then adding six or eight parts of mercury. Stir while cooling, and then reduce the mass to powder. It may be mixed with lard and applied to the rubber; or the rubber may be first smeared with lard, and the amalgam sprinkled over it.

negative on the rubber. The positive charge on the glass is carried round until it comes opposite the metal comb. Induction is now set up, and negative electricity is discharged from the points, electrifying the air between them and the cylinder, which passes across to the glass and neutralises its positive charge, leaving the free positive on the conductor. Students are apt to imagine that the positive charges pass from the glass *to* the points. The row of points is merely a device for obtaining a charge from a non-conductor, and the above statement may be verified by touching the points with a small proof-plane while the machine is working freely, and transferring the charge obtained to an electroscope. It will be found

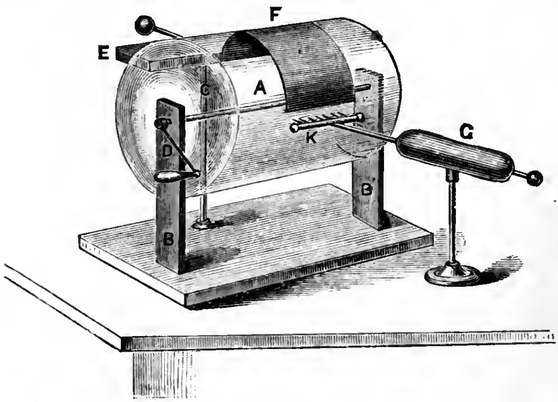


Fig. 47.

to be negative, although only positive charges can be obtained from other parts of the prime conductor (see Experiment 58).

If a negative charge is required, the rubber must be insulated and provided with a metal knob at the back, the prime conductor being placed in earth-communication. If both the rubber and the prime conductor are perfectly insulated, a point will soon be reached when sparks can no longer be obtained. This is because the normal working of the machine not only involves the separation of the charges by friction, but also the existence of a path or circuit through which they may flow to rejoin each other. In the usual case, a sufficiently conducting path is provided from the rubber through the stand and the body of the operator to the insulated prime conductor, or *vice versâ*, but when both are insulated the circuit is broken and the flow stops. But if the rubber and prime conductor were each provided with metal discharging rods arching over the machine until within sparking distance, both might be insulated on glass or ebonite pillars, and yet the action would go on freely.

Plate Machines have a circular disc of glass or ebonite instead of a cylinder. This change modifies the arrangement of the parts, but the action is the same. Electrically and mechanically considered, they may be regarded as a better type of machine than the cylinder machine just described.

Machines depending on Induction.—In this class of machines we require an initial charge of electricity which acts inductively on conductors placed near it.

A very large number of such machines have been devised, but here it will be sufficient to consider one or two typical forms.

The Voss Machine.—This machine consists of two parallel

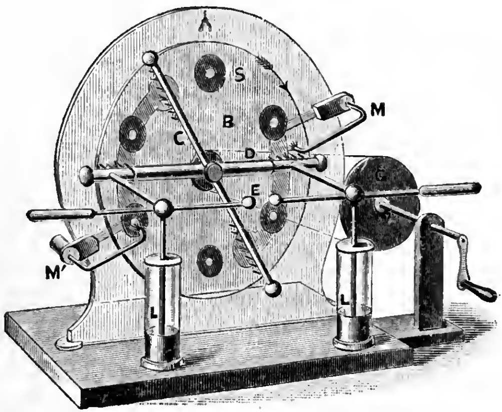


FIG. 48.

plates, one fixed, and the other capable of rotation on a spindle passing through its centre. In Fig. 48 the plate A is fixed, while B is capable of rotation in front of it. On the *back* of the fixed plate there are attached two pieces of tinfoil covered with paper (P, N, Fig. 49, which represents the *back* of the machine). These are called armatures. Metal rods, MM′, bent three times at right angles, pass from the armatures over the edges of both plates, each carrying a metal brush, which passes lightly over six or eight metal studs, S, fixed on the *front* surface of the rotating plate. Facing the front of the rotating plate are two horizontal brass combs, D, connected by metal rods with two knobs, E (between which the discharge occurs), as well as with the inner coating of two Leyden jars, LL. The outer coatings of these jars are connected together. There are two other combs connected by a conductor, C, in each of which the middle tooth is replaced by a metal brush which passes over the studs.

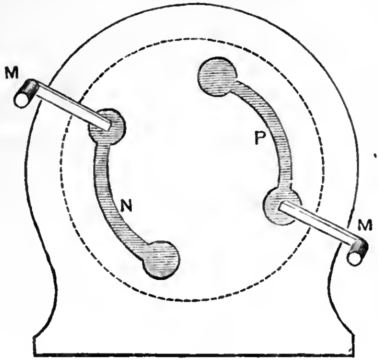


FIG. 49.

In this machine it is unnecessary to give an initial charge to the armatures, as practically there always exists some slight difference of potential sufficient to start the action, and the construction of the machine is such that the initial charge increases very rapidly.

The explanation of the action of the machine is somewhat difficult, but perhaps it can be most easily understood by the following method:—

Suppose we have two insulated conductors (Fig. 50), one, P, charged positively, the other, N, charged negatively. If we bring another insulated conductor near P, it will be acted on inductively, and if we make an earth-contact at the moment it is opposite P, it becomes charged negatively. If this negative charge be removed or used in any way, and then the conductor be brought near N, earth-contact being made as before, it becomes positively charged, and so on. Let the charged conductors—which we will call armatures — be fixed on the *back* of a stationary glass plate, and let another glass plate, bearing a conductor—say a metal stud—C (Fig. 51) rotate in front of it. It is clear that if we can (1) *maintain* (or better still, *increase*) *the charges on the armatures* P *and* N, and also (2) *arrange an earth-contact at the moment* C *is in front of* P *and* N, then the stud leaves P negatively charged, and N positively charged, so that, if it rotates in the direction of the arrow, we shall get a constant supply of negative electricity at A, and positive at B, and, if we arrange two collecting combs at these points, we have at once a practical machine.

FIG. 50.

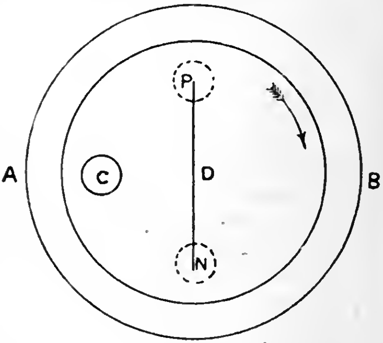


FIG. 51.

The latter of the two essential points, mentioned above—*the earth-contact*—can easily be arranged, as shown in Fig. 51, by using a neutralising rod, D, connected with metal brushes capable of touching the stud on the revolving plate, when it is just in front of P and N. It is evident that, when in action, it is immaterial whether this rod be insulated or not, for in the machine described on p. 75, the studs being at opposite ends of a diameter, whatever quantity of positive electricity is carried off by the brush opposite P, an equal quantity of negative electricity will be carried off, in the same time, by the brush opposite N, one charge neutralising the other.

Let us now consider the other essential essential point—*how to maintain the charge on the armatures* P *and* N. This is best done by connecting a conductor to the armature P, which terminates in a brush facing the rotating plate just before it reaches A, and another similarly with N and B. In Fig. 52 the course of the conductor is shown by dotted lines where it passes behind the plate, and by thicker lines, M, where it bends round the edges. Thus, when the charged stud, C, reaches the brush on M, its charge is shared with the armature, only a small free charge being retained. By this means the charge on the armatures is increased, however small it may be at first. Besides this passage of electricity to the armature from the *stud itself*,

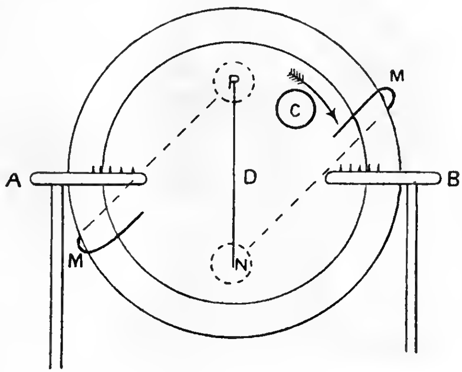


Fig. 52.

each portion of the glass, as it comes fresh from the induction of one armature, is put in practically metallic conduction with the opposite armature, and as the leakage from the armatures, *when once they are fully charged*, is small, so little electricity is removed from the plate by the brushes, that nearly the whole charge passes on to its discharging terminal, where it is collected in the usual way.

In the best mechanical form of the machine, the long conductor, M, from the armature, is avoided by making the armature itself extend further, and then fixing to this the short rod bent three times at right angles, shown in Fig. 48.

The Leyden jars, LL, are used to increase the capacity of the terminals. The machine will work, either with or without them; (*a*) *without jars*, the discharge is a steady flow of a diffuse brush type; (*b*) *with jars*, a greater charge is required to raise the potential difference between the terminals to the "sparking point," and so the discharge passes at longer intervals, but in a more concentrated form, giving a loud, bright spark. The outer coatings of the jars should be in metallic connection. Removing this connection produces much the same result as removing the jars themselves.

The Wimshurst Machine consists of two varnished glass or ebonite plates (preferably the former) arranged to rotate simultaneously in opposite directions, and usually carrying from twelve to sixteen narrow strips of tinfoil, known as "sectors." Both plates are exactly alike. Two neutralising rods lie obliquely across the plates, nearly at right angles to each other, one at the front and the other at the back. These rods terminate in brushes, which touch the plates as they revolve, and provide four earth-connections. Fig. 53 shows a simple design due to Messrs. Newton & Co. of Fleet Street. The discharging terminals are supported by glass rods, which also carry the collecting combs. To these combs, on the other side of the machine, are connected the inner coatings of two small Leyden jars. The outer coatings are uninsulated, as the jars stand on the wooden frame of the machine, although it is usual to provide a metallic connection between them, which may be removed at will.

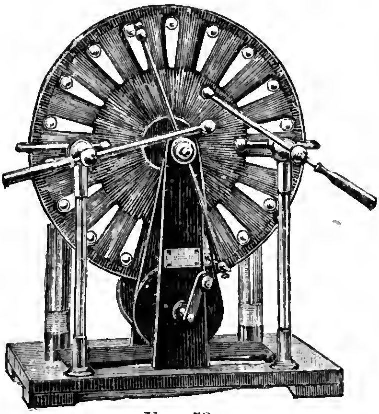


FIG. 53.

In this machine, the stationary inductive armatures of the preceding type disappear, for, as both plates rotate, it is possible to make the sectors themselves act as inductors during a part of their course. In each revolution, each sector twice receives a charge, and twice induces one; the former when it touches a brush, and the latter when it passes the position of the brush on the opposite plate. This will be understood from Fig. 54, which shows the distribution of charge when the machine is at work. The signs marked outside the circle refer to the charges on the plate at the back. Consider any sector, S, which has just passed one collector. It is under induction from the negative charges on the back plate which are descending towards the collector; when it touches the earth-connection E_1, the repelled negative charge is removed and it proceeds with a positive charge. It is now able to act inductively on the back plate, and when passing E_2—the position of the earthed brush on that plate—sectors in contact with E_2 receive a negative charge. The sector S then reaches the other collector, and is wholly or partially discharged; after which a similar sequence of operations recurs.

The presence of the metal sectors is not essential; the machine will work perfectly well without them, and will probably give longer sparks, but it will not be self-exciting, and it will be desirable to extend the brushes radially in order to cover a somewhat larger area. The greater the number of sectors, the more readily the machine excites, and when very numerous, it is practically always self-exciting. This is due to the fact that the greater the number,

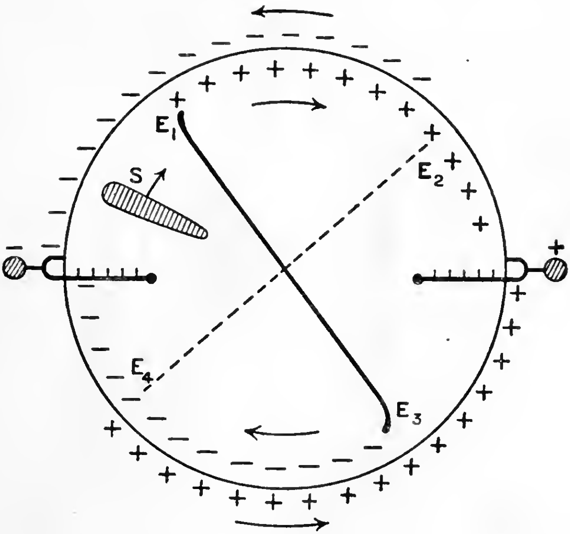


FIG. 54.

the greater the probability of differences of potential existing between them. At the same time, the length of spark obtainable diminishes, for the insulation across the plates is decreased, and it is obvious that a very large number of sectors would practically amount to a short circuit. On the whole, it is convenient to use a moderate number of sectors (say 12 to 16) on each plate.

It is interesting to notice that either the Voss or the Wimshurst machine will run as a motor, when connected to another machine used as a generator, the sequence of actions being reversed.

Experiments with Electrical Machines.—It is assumed that a Wimshurst machine is available, so constructed that the Leyden jars may be disconnected at will.

Exp. 53, *to illustrate the function of the accessory Leyden jars.*

(*a*) Work the machine *without the jars.* Notice that there is a faint continuous discharge between the terminals, scarcely visible in a lighted room unless the gap is very short, but appearing in the dark as a diffused bluish-violet glow.

(*b*) Place the jars in position, *with their outer coatings unconnected* by the metal wire or tinfoil strips usually provided. On working the machine, observe that the result is much the same as in (*a*), although probably some slight improvement in the way of concentration into distinct sparks may be noticed.

(*c*) *Connect the outer coatings metallically,* and observe, that when the machine is worked, the action is completely different—brilliant, sharply defined sparks being produced at regular intervals, which produce a much more striking and impressive effect.

The apparently feeble effect obtained in (*a*) is evidently due to the small capacity of the terminals. If it were possible to connect them to insulated conductors of great size, which would, of course,

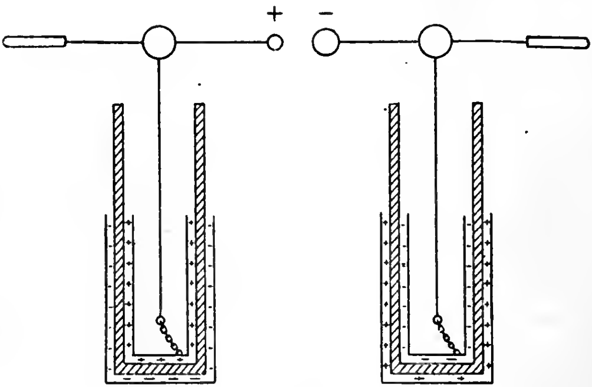


Fig. 55.

require large charges to give the potential necessary to break down the dielectric air, the discharges would occur less frequently, but the individual sparks would be more pronounced. Exactly the same effect, without the disadvantage of undue bulk, is produced by the addition of the Leyden jars.

It is perhaps less easy to understand why the metallic connection provided in (*c*) should make such a great difference to the result as compared with (*b*), for in both cases the outer coatings were uninsulated. Consider Fig. 55, which shows diagrammatically the state of the condensers during the charging process preceding the spark. On *each* outer coating two induced charges will as usual be produced, and the repelled charge (which is not shown in the diagram) will tend to pass away to earth. There is no difficulty about this, even if the jars are standing merely on wood, for the wood conducts quite well enough to permit the repelled charges to pass away readily, if sufficient time is allowed (really a very *short* time by ordinary standards), and during the charging process the time allowance is ample for the purpose. Meanwhile the P.D. between the terminals is steadily rising to the critical value at which the dielectric will give way, and just before that value is reached, the state of affairs would be as shown in Fig. 55.

It is now evident that the discharge of the inside coatings by the spark must simultaneously and suddenly release a practically equal quantity on the outer coatings. Moreover, unless ample facilities are provided for the escape of these outer charges, the main spark will be retarded and largely suppressed. Now, wood does not conduct sufficiently well to permit the instantaneous discharge required, whereas a metallic communication answers admirably. We see, therefore, that the total flow takes place in a kind of circuit, each spark between the main terminals involving an equal and oppositely directed discharge between the outer coatings. These facts may be illustrated as follows :—

Exp. 54. Remove the metal connection between the outer coatings of the jars, and connect the outer coatings to a pair of insulated auxiliary discharging terminals. When these terminals are widely separated, the main spark is feeble and more or less of the type obtained in (*a*) Experiment 53. When they are brought within some small distance—depending on the power of the machine —the main sparks are bright, sharp, and crackling, and each is accompanied by a second spark between the auxiliary terminals, due to the discharge of the outer coatings.

This second spark, which may be of considerable length, is noteworthy, because it is obtained from conductors which are already connected by the wooden stand. These charges would certainly leak through the wood, if more than the briefest time were allowed, but under the circumstances the charges on the outer coating—both of which are at zero potential until that instant—are released so suddenly that the P.D. between them rises above the sparking value *before* the badly conducting wood can act effectively.

Exp. 55. Connect the outer coatings by a long *iron* chain, the rustier the better. Notice that each spark between the main terminals is now accompanied by flashes of light at each link—due to bad contact—but that the chain may be handled freely without shock. Now break the chain in the middle and hold the broken ends, one in each hand. Notice that, at each spark, a shock is felt, whose magnitude depends upon, and is capable of regulation by, the distance between the main terminals.

Action of Points.—**Exp. 56.** Work the machine in the dark ***without jars*** to obtain a brush discharge, and slowly separate the terminals. Observe that at first the glow fills the whole space, with more brilliant flickering roots emanating from the positive side, and that at greater distances it breaks up into a well-defined ***positive brush*** (Fig. 56) separated by a dark space from a feeble negative glow.

Exp. 57. Hold a blunt piece of wire near each terminal in turn. Notice that when it is near the *negative* terminal, a well-defined positive brush will form upon it, but when it is near the *positive* terminal, the result will be a minute point of light, which is equally typical of a negatively charged point.

FIG. 56.

In this way it is easy to determine the kind of charge on the terminals, without using a proof-plane and an electroscope.

Exp. 58. Look at the points upon the collecting combs whilst the machine is at work. Notice that the positive brushes proceed from the points on the *negative* side of the machine, and that negative points of light are present on the *positive* side of the machine. (See Experiment 57.)

Exp. 59. Attach a blunt-ended wire, about $\frac{1}{8}$ inch thick, to each terminal in turn. In both cases notice that it reduces the sparking distance; and in the dark, especially when the hand or other conductor is held near, that a *positive* brush forms upon it when it is on the positive side, and a negative point when it is upon the negative side. Notice also that the hand, if it be held sufficiently near, feels as though a wind was blowing upon it.

Exp. 60. Demonstrate the existence of the stream of escaping charged particles of air or dust by using the piece of apparatus—known as the electric whirl—Fig. 57. It need scarcely be mentioned that the whirl rotates in a direction opposite to that in which the points are turned.

Another very striking illustration is as follows:—

Exp. 61. Place an electroscope at a distance of 15 to 20 feet from a machine. (A light metal can may with advantage be placed upon the cap, but, as this is merely to increase the effective area of the cap, it is not absolutely necessary.) Work the machine in the usual way to get good sparks, and notice that the electroscope is unaffected. Now separate the terminals beyond sparking distance, attach a bent wire to either terminal, with the free end pointing towards the distant electroscope. Work the machine, and, in a short time, notice that the leaves will begin to diverge, and that the divergence remains unaltered when the machine stops. When the point is on the positive terminal, the charge found on the electroscope is positive and *vice versâ*.

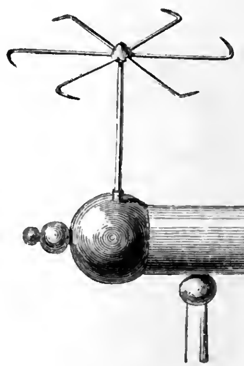


FIG. 57.

Exp. 62. Hold a lighted candle near the wire. Notice that, whether the wire be positive or negative, the flame is blown about as by a wind; but notice, in the latter case, that one part of the flame—when fairly close to the wire—is distinctly *attracted* by the wire.

This is due to the fact that a flame contains charged particles or ions (see p. 318), there being a very large excess of positively charged ions. The presence of these charges in the flame explains why excited non-conductors, like glass or ebonite, are so readily and completely discharged by holding them in the hot gases above it; the flame does not act *merely* like a point, as is often stated, but neutralises the charges by supplying others of an opposite kind.

Experiments[1] to show the Action of Lightning Conductors.—It is convenient to have two circular metal plates—about 12 or 15 inches in diameter—with rounded edges. One of the plates is laid upon the table and the other suspended above it, at any convenient height, by silk ribbons. Three small wooden feet carrying narrow brass tubes are also required (small cork borers pushed completely through a cork answer well); when these stand on the lower plate, the brass tubes make contact with it, and the tubes form holders in which brass wires, carrying brass balls of different sizes and slightly bent, may be adjusted to various heights without slipping (Fig. 58).

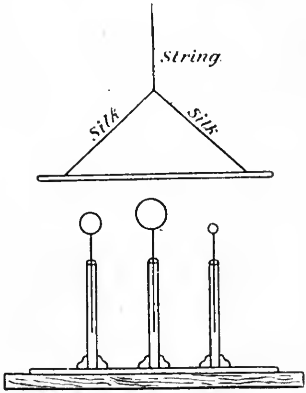


FIG. 58.

Exp. 63. Connect the upper plate to the *negative* terminal of a machine, and the lower plate to the *positive* terminal. Place the three stands in position. Work the machine and adjust the height of the balls by slipping the wire up or down the tube. The lower plate represents the earth; the three stands, buildings; and the upper plate, a thunder cloud. Notice that all *may* be struck, but that the spark shows a decided preference for the smallest ball, striking that alone even when it is much lower than the others. With a little care, the balls may be adjusted until the spark strikes any one of them indifferently, the largest ball being then nearest the upper plate, and the smallest furthest away.[2]

The important point to notice in this experiment is *not* the preference shown by the spark for the smallest ball, but the fact thereby indicated that the spark has the power of *selecting its path.*

Exp. 64. Now remove one of the balls, leaving the end of the wire free. On working the machine, observe that sparking is *stopped*, the point affording complete protection to the other conductors *even when it is considerably below their level.*

[1] These experiments were devised by Sir Oliver Lodge.

[2] Incidentally, this illustrates why the terminals of a Wimshurst machine are usually fitted with balls of unequal size, experiment showing that a longer spark can be obtained when the small ball is positive—the opposite arrangement giving shorter sparks and much brush discharge. For this reason, the student was advised to make the upper plate negative.

This illustrates the proper function of a lightning conductor. Its main purpose is not so much to be struck instead of the building itself, but rather to prevent any spark occurring at all—the silent and usually invisible brush discharges from its point quietly neutralising the atmospheric charges.

The student will notice that if all spark discharges were of the kind just investigated, the complete protection of buildings would be a very simple matter. Any projecting point, which need not necessarily be the highest part of the building, would safeguard quite a large area, and the question of a good "earth" to the conductor would not be material. Facts, however, prove that it is extremely difficult to protect buildings completely, and that the provision of a good earth-connection is of the first importance unless the lightning-rod is to be a source of danger instead of a safeguard. At this stage, it is impossible to explain the matter adequately, and, therefore, it is preferable to show experimentally that all spark discharges are not of the same kind.

Exp. 65. Disconnect the plates from the main terminals. Join them up to the outside coatings of the Leyden jars, removing the usual metallic connection between the latter. Adjust the main terminals until sparks are obtained between them. Probably it will also be necessary to diminish the gap between the balls and the upper plate. Observe that, at every spark between the main terminals, another spark occurs between a ball and the upper plate, but that *no preference is now shown* for any ball, the spark always striking the highest, whatever its size. Moreover, the point has completely lost its power of suppressing sparking; if it happens to be the highest it becomes struck by the spark, but there is no brush discharge, and, if it is the merest shade lower than any other conductor, it becomes absolutely useless. [This shows the practical necessity of making a lightning conductor the highest point of the building, and also of providing adequate cross-section to carry off a powerful discharge without fusion or without the risk of its leaving the rod.]

These experimental results will be understood better after reading the section on oscillatory discharges in Chapter XXXV. In the meantime, it may be remarked that the great difference revealed in the properties of the two kinds of sparks are natural consequences of the time factor involved. In the first case, the P.D. between plate and ball increased gradually during the time preceding the discharge, *i.e.* the electric field formed gradually and had ample time to choose its path. In the second case, we are dealing with a sudden impetuous rush from the outer coatings; it is like the giving way of a reservoir wall, or the discharge of a gun—no preliminary time is given for a choice of path, and the spark strikes whichever object is nearest, irrespective of its size or pointedness.

Exp. 66, *to show the instantaneous nature of the spark.* Work the machine until sparks are given. Notice that the rotating plates of the machine appear to be stationary.

Exp. 67, *to show the existence of two kinds of electrification.* Put a paper tassel, attached to a wire, first on one terminal and then on the other, In *each* case, the paper strips diverge widely by repulsion, and are attracted by the hand and by neighbouring objects. Hence both are electrified and apparently are alike. Now put tassels on both terminals; the result is a conspicuous and decided attraction between the two, the paper strips clinging together and forming a bridge across.

Exp. 68, *to illustrate the working of a Leyden jar.* Attach pith-balls, suspended by light threads about two or three inches long, one to the outside coating of the jar and one to the brass rod between the ball at the top and the lid. (*a*) Insulate the jar, place it so that its knob is nearly touching one terminal, and work the machine. Notice that only occasional feeble sparks pass to the knob—the better the insulation, the fewer there will be—and that both pith-balls instantly diverge to the fullest extent. This shows that when the outer coating is insulated, the jar does not act as a condenser, and that a very small charge raises the potential of the inner coating to an equality with that of the machine. Sparking then ceases, except to make up for leakage. Notice that, upon removing the jar and discharging it, the effect is very small. (*b*) Repeat these operations with the outer coating earthed, and notice that sparks pass freely to the knob for a few moments, after which the passage of sparks practically ceases. The pith-ball attached to the rod is meanwhile slowly deflected to its full extent, reaching its final position when the sparking nearly ceases. Observe, also, that the pith-ball on the outer coating is not deflected at all. Upon removing the jar and discharging it, a brilliant spark is obtained. This shows that a much larger charge is needed to raise the potential of the inner coating to an equality with that of the machine.

Exp. 69, *to compare approximately the capacities of two Leyden jars of different sizes.*[1] Place one of the jars with its knob *in contact* with the prime conductor of an ordinary frictional machine (an inductive machine is not suitable). Provide two insulated brass balls, placed about $\frac{1}{8}$ inch apart, and connect one to the prime conductor and the other to the outer coating of the jar, which may stand on the table as usual. On working the machine, the jar will charge up until the P.D. between the brass balls reaches a certain value, then it will be discharged by a spark, and the process will go on continuously. Turn the machine steadily and count the number of revolutions between one spark and the next. Let there be n revolutions. Repeat the experiment with the second jar, taking care not to disturb the balls, and suppose n_1 revolutions are now required. Then, assuming that the quantity of charge supplied to the jars is the same per revolution when rotation is steady, we have

$$Q = VC \text{ for the first jar, and}$$

$$Q_1 = VC_1 \text{ for the second jar,}$$

V being the same for both, as it is determined by the distance between the balls.

$$\therefore \frac{Q}{Q_1} = \frac{C}{C_1}$$

$$\text{but } \frac{Q}{Q_1} = \frac{n}{n_1}$$

$$\therefore \frac{C}{C_1} = \frac{n}{n_1}$$

[1] This is an instructive experiment, as showing what can be done with simple apparatus. It is quite capable of giving fair results, although it must not be regarded as a serious method of measurement.

CHAPTER VII

ELECTROMETERS

Electrometers are instruments in which the attractions or repulsions of electric charges are made use of in order to measure *differences of potential*, and may, therefore, be regarded as extensions of the principle embodied in the gold-leaf electroscope.[1]

The two most important and widely known types are those originally devised by Lord Kelvin, known as (1) the quadrant, and (2) the attracted disc electrometer.

The Quadrant Electrometer.—This consists of four metal quadrants, usually hollow, and forming, when placed together, a shallow circular box divided into quarters. In the earliest forms, these were insulated on separate glass pillars, and within them a light aluminium needle[2] (Fig. 59A) was suspended by two silk fibres—this is known as bifilar suspension, its purpose being *to secure a definite zero position.* The needle-system carries a small mirror, M (Fig. 59C), the deflections being read by means of a lamp and scale in the ordinary way.[3] The alternate quadrants are connected permanently by wires, as shown in Fig. 59B, and carry the terminals TT. Let us suppose that a small P.D. is produced between TT, say by connecting them to the terminals of a voltaic cell. Evidently the quadrants become charged as shown in the figure, but the forces upon the needle being equal and opposite, no deflection is produced. If, however, the needle itself is charged, a deflecting couple will be produced, which in magnitude must depend both upon the P.D. between the quadrants and upon the potential of the needle, and if the latter be high, very small differences of potential may be detected. If, however, any relative value is to be attached to the readings, it is obvious that the potential of the needle must not alter during the observations, and this introduces a difficulty, because the small charge required to raise the needle to a given potential would leak away very quickly

[1] Various new and elaborate forms of electroscope have recently been developed. It is now a refined and delicate instrument, capable of being used for very exact measurements. In some cases a single strip of gold leaf is used, only 1 millimetre wide and about 25 millimetres long, a microscope being used to read its movements.

[2] The word *needle* is conventionally used to denote the moving part of almost all instruments, and does not imply any particular construction. In this case, the needle is a piece of metal shaped like the figure eight.

[3] This is explained on p. 147.

in spite of the silk suspension. In theolder forms, the difficulty was overcome by attaching a platinum wire, P (Fig. 59c), underneath the needle, N, its lower end dipping into strong sulphuric acid, which formed the inner coating of a glass Leyden jar, the outer being made of tinfoil, and earth-connected. This enormously increased the capacity of the needle-system, and after charging the jar with a few sparks from an electrophorus (or in any other convenient way) its potential would, under favourable conditions, remain fairly constant for some time. The more elaborate forms of the instrument also contained devices for indicating any change in the potential of the needle, and for keeping it at a steady value. In any case, the instrument was difficult to use.

It is, however, unnecessary to discuss it in detail, because a greatly improved form has recently been devised by Dr. F. Dolezalek, which

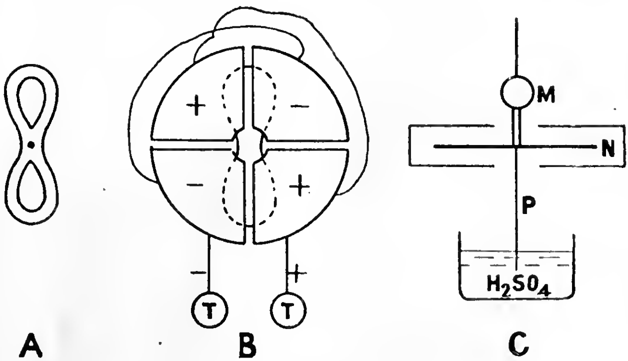


Fig. 59.

is shown in Fig. 60 (inserted by permission of the Cambridge Scientific Instrument Company).

The needle and quadrants are of small dimensions, thus reducing the capacity. This is advantageous, because in measurements of capacity, that of the instrument itself is always involved (see p. 90). The quadrants are mounted upon ambroid pillars (made from amber), which afford more perfect insulation than glass. The needle is made of paper thinly covered with silver, and is suspended by a quartz fibre, which provides excellent insulation combined with delicacy of suspension, and the extreme lightness of the moving system gives great sensitiveness. The needle and mirror are rigidly connected, but the quartz fibre is provided with a hook at each end, and is detachable, the hooks engaging in eyes in such a way that no slip or back-lash can occur. Several such fibres are supplied with an instrument in order to obtain a wide range of sensitiveness. No

Leyden jar is required, because the sensitiveness obtained by using a quartz fibre is such that much lower needle potentials can be used. It is usual to charge it to about 50 or 100 volts[1]—a pressure easily obtained from a battery or a lighting circuit. The insulation is so good that it is generally sufficient merely to connect the needle with the charging source for an instant, and for this purpose there is provided the switch K, which is an insulated brass rod carrying a

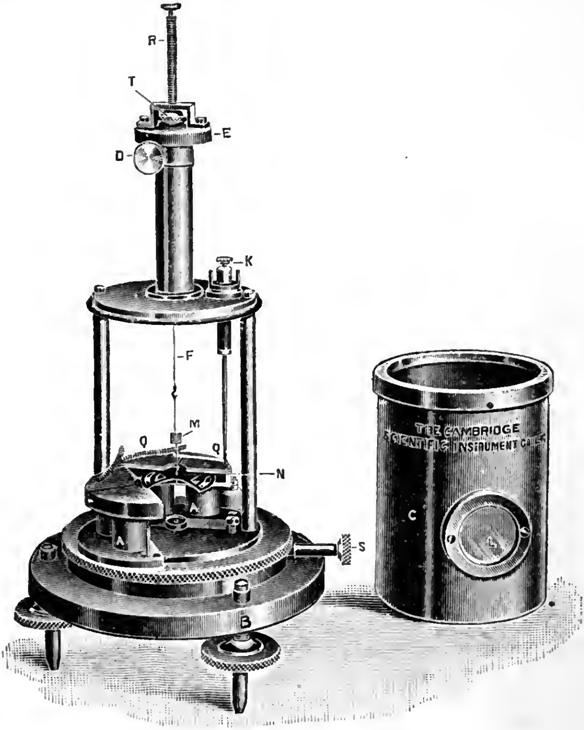


FIG. 60.

light strip of phosphor bronze at the lower end. By turning the insulated terminal at the top, this strip can be brought into contact with the needle and then withdrawn. In some cases, it may be desired to keep the needle permanently connected to the charging source, and then the quartz fibre is made to conduct by moistening

[1] In all forms of instrument, the quadrants should be earth-connected while the needle is being charged.

it with a weak solution of some hygroscopic salt, such as calcium chloride; the upper end, which is carried by an ebonite torsion head, can then be connected to the charging source.

The fibre itself is about $\frac{1}{4000}$ inch in diameter and $2\frac{1}{2}$ inches long, and a difference of potential of $\frac{1}{10}$ volt between the terminals gives a large deflection. In the figure, two adjacent quadrants are shown swung open to allow of easy access to the needle.

Theory of the Quadrant Electrometer.—It is only possible to find an approximate relation between the P.D. to be measured and the deflection produced. Hence, the quadrant electrometer is not well adapted to measure P.D.'s *absolutely*. When properly designed, however, its readings are proportional to the P.D. producing them over a wide range, and it, therefore, may be regarded as an extremely sensitive instrument suitable for "null" methods,[1] and for the exact comparison of differences of potential. When actual values are required it is best to calibrate it by noting the deflection produced by some known P.D., such as that given by a standard cell.

The usual theory is as follows, and, although it is not free from objection, it serves to indicate the nature of the relations between the various quantities concerned.

Let V = potential of needle, *i.e.* P.D. between needle and earth, and let V_1, V_2 = potentials of pairs of quadrants respectively with respect to that of earth. Assume that these values remain constant, and that $V > V_1 > V_2$.

The needle may be regarded as one plate of a condenser, a pair of connected quadrants forming the other plate, hence, there are two such condensers, whose capacities, by symmetry, will be equal when the needle is at rest in the zero position.

When the needle is deflected through an angle θ, the capacity of one of these condensers will be increased and that of the other diminished, and *it will be assumed* (as evidently approximately correct) that the amount of change is the same for both and is proportional to θ.

These alterations in capacity will also alter the energy (for energy $= \frac{1}{2}V^2C$).

Hence, if m be some constant, the energy of one condenser is decreased by $\frac{1}{2}(V - V_1)^2 m\theta$, and the energy of the other is increased by $\frac{1}{2}(V - V_2)^2 m\theta$ (for, as $V > V_1 > V_2$, the needle moves towards the quadrant at potential V_2).

$\therefore$ Nett gain of energy $= \frac{1}{2}m\theta\{(V - V_2)^2 - (V - V_1)^2\}$, and this is expended in doing work against the couple due to the torsion of the suspension.

$$\text{but work} = \text{couple} \times \text{angle}$$
$$= \tfrac{1}{2}m\{(V - V_2)^2 - (V - V_1)^2\} \times \theta,$$

i.e. the expression $\frac{1}{2}m\{(V - V_2)^2 - (V - V_1)^2\}$ is the moment of the couple acting upon the needle when the deflection is θ. Now, by the theory of torsion for small deflections, the angle is proportional to the couple producing it, and omitting $\frac{1}{2}m$ as no longer required, we may write

$$\theta \propto \{(V - V_2)^2 - (V - V_1)^2\}$$
$$\textit{i.e.} \quad \theta \propto \{2V(V_1 - V_2) - (V_1 + V_2)(V_1 - V_2)\}$$
$$\text{or} \quad \theta \propto (V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right).$$

In the ordinary use of the instrument, V is *very large* compared with V_1 or V_2, hence the term $\frac{V_1 + V_2}{2}$ is negligible, and, therefore, *when V is constant*, θ is

[1] See p. 305.

proportional to the P.D. between the quadrants, which is the P.D. to be measured. Evidently the *sensitiveness* depends on the magnitude of V.

There is another method of using the instrument. Instead of charging the needle from an independent source, it is put in conducting communication with one pair of quadrants. This makes $V = V_1$, and the expression for θ becomes

$$\theta \propto (V_1 - V_2)\left(V_1 - \frac{V_1 + V_2}{2}\right)$$

$$\textit{i.e.} \quad \theta \propto (V_1 - V_2)\left(\frac{V_1 - V_2}{2}\right)$$

$$\text{or} \quad \theta \propto (V_1 - V_2)^2.$$

The instrument is now much less sensitive, but in this state it will give steady readings with *alternating differences of potential*, for a little consideration will show that the direction of the deflection is unchanged, when the sign of all the charges is changed. Moreover, as the couple is at any instant proportional to the *square* of the P.D. to be measured, the readings will be proportional to its "virtual" or "square root of the mean square" value (see p. 429).

Experiments with the Electrometer.—In practice, the quadrant electrometer is useful as an instrument of research for many purposes, which cannot be adequately dealt with at this stage. The following experiments are suggested simply to illustrate general principles.

Exp. 70, *to measure the capacity of any conductor in static units, by comparison with the capacity of a sphere (Faraday's method, referred to on p. 67).* Connect the conductor to one terminal of the instrument by a long fine wire, the other terminal being "earthed." Charge it until a convenient deflection is obtained—suppose this is d divisions. Now, connect to the system an insulated uncharged sphere, also by means of a long fine wire. (Long wires are required because the various conductors must be kept at considerable distances from one another to avoid mutual influence.) The deflection will fall to some smaller value—suppose d_1 divisions.

If C be the capacity to be measured, C_1 the capacity of the sphere, q the charge given to the first conductor, and V the potential to which it is raised by that charge, then

$$V = \frac{q}{C} \propto d. \qquad (1)$$

In the second case, q was not altered in amount, but was simply shared between the conductors, their common potential being (say) V_1

$$\therefore \quad V_1 = \frac{q}{C + C_1} \propto d_1 \qquad (2)$$

$$\therefore \quad \frac{C + C_1}{C} = \frac{d}{d_1}$$

$$\text{or} \quad \frac{C}{C_1} = \frac{d_1}{d - d_1} \quad (\text{where } C_1 = \text{radius of sphere}).$$

It is important to notice that the value obtained for C includes the *capacity of the electrometer itself.* Hence the advantage obtained by keeping that capacity small, as mentioned in the description of the Dolezalek instrument. In exact work, its capacity would be measured separately and allowed for. It is obvious from the above argument that this can be done easily by connecting the instrument to a charged sphere, noting the deflection, and then connecting it to another insulated uncharged sphere, and again noting the deflection. The use of a sphere, however, as a standard requires caution, since its capacity is only equal to its radius in centimetres *when it is at a distance from other bodies*, and in an ordinary room this value may be seriously altered, because the walls and surrounding objects act towards it as the outer coating of a condenser.

Measurement of Specific Inductive Capacity (or of Dielectric Constant).—Here again it is intended only to indicate the principle involved in the methods actually used—the complicated arrangements required in practice to obviate many sources of error making a full description unadvisable at present.

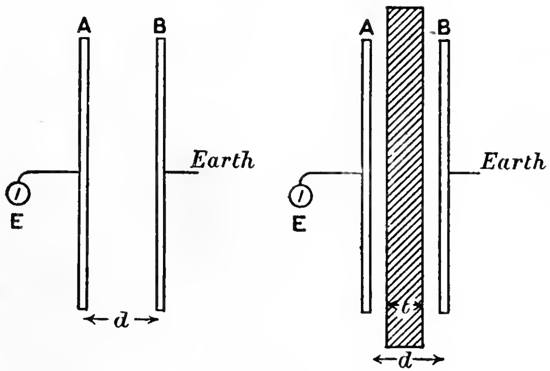


FIG. 61.

Exp. 71. Arrange a parallel plate condenser as in Experiment 42, using an electrometer in preference to the electroscope as being more suitable for exact readings. Charge the plate A (Fig. 61) until a convenient deflection is obtained—the other terminal of electrometer E being earthed as usual. Let d be the distance between the plates. Now insert, between the plates, but without contact, a slab of the dielectric in question of thickness t—taking precautions to avoid inadvertently charging the dielectric (see foot-note, p. 62). We know from Experiment 52, that this will increase the capacity and decrease the P.D. between the plates, thereby causing the deflection to be decreased. Now increase the distance between the plates, by moving one of them, say B, parallel to itself. Then, because the capacity is diminished, the deflection is increased. Let d_1 be the distance between the plates at which the deflection is the same as before. Then the capacity must also be the same as at first.

$$\text{In case (i.)} \quad C=\frac{A}{4\pi d}$$

In case (ii.) the actual air space is $d_1 - t$, and the dielectric itself is equivalent to an air space of $\frac{t}{K}$ (p. 62).

Therefore,
$$C_1 = \frac{A}{4\pi\left(d_1 - t + \dfrac{t}{K}\right)}$$

but we have made $C_1 = C$,
$$\therefore\ d_1 - t + \frac{t}{K} = d$$
$$\text{whence } K = \frac{t}{t - (d_1 - d)}$$

Now, as $d_1 - d$ is the distance the plate B was moved, it can be measured with great exactness.

Dielectric Constant of Liquids.—The method most generally used depends upon making two measurements of the capacity of any conveniently shaped condenser, first with air as the dielectric, and then with the liquid as the dielectric. Although such measurements can be carried out in various ways, the best methods involve the use of a galvanometer as described in Chapters XX. and XXIV.

Attraction between Parallel Plates.—Before describing Lord Kelvin's attracted disc electrometer, it will be necessary to investigate mathematically the law of attraction between parallel plates.

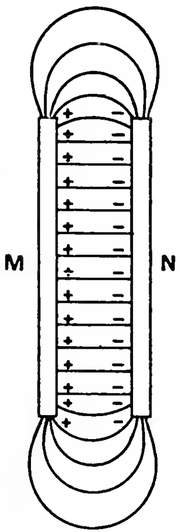


Fig. 62.

Let M and N (Fig. 62) be two oppositely charged plates of area A at distance, d, apart. We can easily determine the attraction between them, provided we simplify the problem by assuming—as in the previous theorem on p. 61—that the whole field is uniform, and confined to the space between the plates. Ordinarily the field will fringe out at the edges of the plates as shown, and some lines will diverge from the outer side of at least one of the plates—effects which may introduce serious errors—hence, we must be perfectly clear as to the conditions under which our argument is valid. It will be seen later how these conditions are satisfied in the actual instrument.

Let one plate, M, carry a charge $+Q$ wholly on one side, then by the assumptions made, the other, N, carries an equal charge $-Q$.

The field between the plates (see p. 61) is given by
$$\text{Field} = F = \frac{4\pi Q}{A}$$
$$\therefore\ \text{Force on unit charge} = U = \frac{F}{K} = \frac{4\pi Q}{AK} \qquad (1)$$

Now, if another charge q be placed anywhere in the space between the plates, *i.e.* in the electric field, it follows that the force on it would be qU dynes (it being understood that q does not, by its mere presence, alter the value of F).

This force will be the same at all points in the field—because the latter is by assumption uniform—however close q may be to the plates, but the force is modified if the charge is *on* one of the plates, for then it is situated on the boundary of the field. This case has been considered on p. 46, where it is shown that under such circumstances the force is $q \times \frac{1}{2}U$ dynes. A similar argument may be applied in the present instance, for if we regard each of the charges, $+Q$ and $-Q$, as possessing fields which extend horizontally in *both* directions, it will be found—by drawing a simple diagram—that in the space *outside* the plates the two fields are equal and opposite, thus neutralising each other; whereas *between* the plates they are in the same direction and strengthen each other, each charge contributing one-half of the actual field. Hence the attractive force on either plate will be $Q \times \frac{1}{2}U$ dynes, where $U = \frac{4\pi Q}{AK}$

$$\therefore \text{ the pull, or attraction} = Q \times \frac{1}{2} \cdot \frac{4\pi Q}{AK}$$

$$= \frac{2\pi Q^2}{AK} \text{ dynes} \qquad (2)$$

This important equation may be thrown into several different forms. Applying it, for instance, to the electrometer, we do not know Q, and we must, therefore, express Q in terms of V. This is done at once from the definition of V as force on unit charge × distance,

$$i.e. \; V = U.d.$$

$$= \frac{4\pi Q}{AK} \times d \qquad (3)$$

$$\text{whence } Q = \frac{V.A.K}{4\pi d}$$

Substituting this value in equation (2), we have

$$\text{attraction between the plates} = \frac{2\pi}{AK} \times \left(\frac{V.A.K.}{4\pi d}\right)^2$$

$$= \frac{V^2 AK}{8\pi d^2}$$

Let p = force of atttraction between plates (in dynes)

$$\text{then } V^2 = \frac{8\pi d^2}{AK} \times p$$

$$\text{or } V = d\sqrt{\frac{8\pi p}{AK}} \qquad (4)$$

This formula gives the P.D. between the plates, and is used in the attracted disc electrometer, an instrument in which the details are somewhat complicated. The principle, however, will be readily understood from the following explanation.

Lord Kelvin's Attracted Disc or Absolute Electrometer.—An attracted disc electrometer is one in which the attraction between two parallel discs at different potentials, and at a certain distance apart, is balanced by the weight of a given mass.

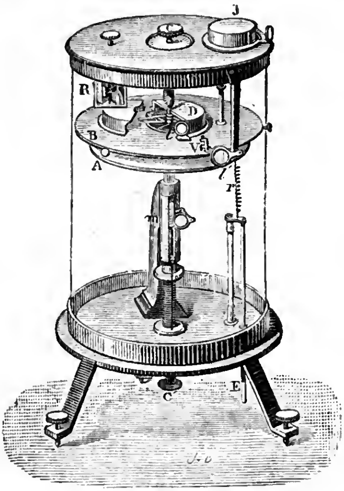


FIG. 63.

Sir William Snow-Harris constructed the first electrometer of this kind. It resembled a balance, having a flat disc suspended at one end of the beam, and an ordinary scale-pan at the other. This disc was suspended above a similar insulated disc, which was connected with the charged body to be tested. When attraction took place between the discs, weights were added to the scale-pan until equilibrium was restored. This electrometer is defective in many particulars; the chief one arises from the irregular distribution of the electricity on the plate—the surface density being much greater at the edges than on the flat surface.

In Lord Kelvin's form the principle only was retained, the details of construction being entirely different, and the result was an instrument capable of directly measuring difference of potential in static units.

Its general appearance is shown in Fig. 63, taken by permission from *Electricity and Magnetism* (Carey Foster and Porter), the details of which (as far as is necessary for our purpose) will be understood from Fig. 64. The attracted disc AB—made of aluminium for lightness—is suspended from the inner side of a circular metal box by means of three delicate springs (only one, S, is shown), similar in shape to coach springs, which cross each other at the middle points, where they are attached to box and plate respectively. This disc nearly fills the circular aperture in the "guard ring," GR, and is normally held by the springs slightly

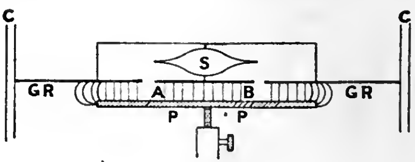


FIG. 64.

above that level. The guard ring is supported by the glass case, CC, and in Fig. 63 the box is seen opened out, showing the spring system and the disc AB of Fig. 64. Below is the attracting plate, PP, which is insulated, and can be raised or lowered by a micrometer screw.

The guard plate, disc and box form one conducting system, and if we connect this system to one, and the plate PP to the other, of the two points between which a P.D. is to be measured, an electric field will be formed between them. We then see

(1) that, because the attracted disc forms part of a hollow conductor, *the field is entirely confined to one side of it;* and

(2) that, because of the presence of the guard ring, all fringing of the field is confined to the latter, and the pull on the disc AB, which is the quantity to be measured, is due to a practically uniform field.

Hence the conditions previously mentioned are fully satisfied.

A delicate gauge is provided to show when the disc AB is exactly in the plane of the guard ring, and the force required for this purpose is found, to begin with, by using known weights. (In the original instrument this was 0·6 gram.) When the apparatus is connected up as mentioned above, the height of the plate is adjusted until the gauge shows that the disc is in the plane of the guard ring; then, we have

$$V = d\sqrt{\frac{8\pi mg}{AK}}$$

where d is the distance between the plates; m, the mass in grams found in the preliminary experiment with known weights; $g = 981$; A = area of attracted surfaces; and $K = 1$ (for air).

Hence the quantity under the root sign becomes a constant for the given apparatus, the only uncertainty being the precise value of A. This has been shown to be approximately the mean of the areas of the aperture and of the plate AB.

K is, of course, always unity in practice, but its presence in the equation tells us that if the discs were immersed in some dielectric liquid (in which case K would be greater than unity), the equilibrium distance, d, would be increased for a given P.D. in the ratio $\sqrt{K} : 1$.

It is, however, by no means easy to measure the distance d accurately, on account of the difficulty of making the plates parallel and perfectly plane, and hence the following method is actually used in taking measurements.

By some auxiliary means a constant P.D. is maintained between the disc and the earth, the lower plate P being also connected to earth. Then the latter plate is adjusted until the disc is in the plane of the guard ring, and the reading of the micrometer taken. If the corresponding distance be d (which is unknown), and if the auxiliary P.D. be V, then

$$V = d\sqrt{\frac{8\pi mg}{AK}}$$

The plate P is then insulated from earth, and connected to the body whose potential is to be measured (or to one of the points between which a P.D. is to be measured, the other being connected to earth), and again moved up or down until the disc is in adjustment with the guard ring—a second reading of the micrometer being taken. If d_1 be the new distance between the plates, and if V_1 be the potential, which has to be measured, then the P.D. between the plates is $V-V_1$ (assuming V to be the greater), and we have

$$V-V_1=d_1\sqrt{\frac{8\pi mg}{AK}}$$

Hence, by subtraction

$$V_1=(d-d_1)\sqrt{\frac{8\pi mg}{AK}}$$

Now $(d-d_1)$ is a distance, which can be measured with great accuracy, as it is the difference in the micrometer readings. Obviously it must be expressed in centimetres.

Lord Kelvin used the term *heterostatic* to express the method of using an electrometer when an auxiliary P.D. is made use of (as in the above case and in the case of the quadrant electrometer), and he applied the term *idiostatic* to the method in which the P.D. to be measured is alone made use of (as in the second method we have given in using the quadrant electrometer, and in the direct method of using the above instrument).

Lord Kelvin also devised a smaller portable form, known as the *portable electrometer*, which is essentially the same in principle, but which, on account of its smaller size, involves certain difficulties in satisfying the theoretical conditions. It is usual to calibrate it by using known potentials.

Further Consideration of the Attraction between Parallel Plates.—Apart from its application to an electrometer, the expression for the attraction between two parallel plates will repay further examination.

(A) In equation (2), p. 93, we have attraction $=p=\frac{2\pi Q^2}{AK}$ dynes. As this does not involve the distance between the plates, we learn that *when Q is constant, the attraction is independent of the distance.* This may seem strange at first sight, but the fact becomes obvious if we remember that our conditions require the field to be uniform and parallel at *all distances*, and that the "attraction" is really the pull due to the tension of the lines of force which terminate on a given plate. If the number and distribution of these lines are unaltered, so is the pull, and the distance has nothing whatever to do with it. This assumes that the medium is also unaltered; if it is changed, we see that the pull is inversely proportional to K, although the field is the same as before; *i.e.* we must suppose that the *tension* of the lines of force is lessened as K increases.

(B) Again, equation (2) is often expressed in terms of surface density.

$$\text{Now, surface density}=\rho=\frac{Q}{A},$$

whence by substitution in (2) on p. 93

$$p=\frac{2\pi}{AK}\cdot\rho^2A^2=\frac{2\pi\rho^2A}{K} \tag{5}$$

Hence in air, the pull of a uniform field is $2\pi\rho^2$ dynes per square centimetre, as already found on p. 46. This is true generally, *e.g.* for the pull of the field on a sphere charged to surface density ρ.

(C) It is instructive to obtain an expression for the pull of the field in terms of the field strength itself. For this we have

$$p=\frac{2\pi Q^2}{AK}$$

$$\text{Now Field}=F=\frac{4\pi Q}{A}$$

whence, eliminating Q, we obtain

$$p=\frac{F^2A}{8\pi K}\text{ (dynes)} \tag{6}$$

from which we deduce the very important fact that the pull of the field varies directly as the *square* of its strength. It is in this respect chiefly that the analogy between a line of force and a stretched string breaks down. In the latter case, the pull on a body would depend upon the number of strings alone, and not at all upon their concentration, *i.e.* upon F. In the former case, the pull depends largely upon the concentration. For instance, if a certain number of elastic strings pulled on one square centimetre, doubling their number on that area would simply double the pull; but if we do the same thing with lines of force, the pull is four times as great. (The student will find later that this peculiarity is also characteristic of magnetic lines of force, although they are essentially different in nature.)

(D) We may also write F = KU in equation (6), and thus obtain an expression for p in terms of U. This gives

$$p=\frac{U^2AK}{8\pi} \tag{7}$$

Hence, from equations (6) and (7),

if F is kept constant (which means Q is constant), p varies inversely as K,
and if U is kept constant (which means V is constant), p varies directly as K.

(E) Another instructive expression is obtained for p in terms of Q and V. We have, for this purpose,

$$p=\frac{2\pi Q^2}{AK}$$

$$\text{and } V=\frac{4\pi Qd}{AK}$$

whence, eliminating AK, we obtain

$$\tfrac{1}{2}QV=p\times d. \tag{8}$$

Now, $\frac{1}{2}QV$ is the energy of the system, and we, therefore, learn from this equation that it resides entirely in the field (as previously indicated on p. 54). For suppose that the plates have a constant charge of +Q and −Q respectively, then—with the limitations already stated—we know that p is the same at all distances. Now let the plates be placed infinitely near together, then d and V are both nearly zero, and so is the energy. If the plates are now separated against the constant pull, p, work is done of amount $p\times d$, and this is the energy expended in establishing the field, and also the potential energy of the field when formed.

Energy of Field per Unit Volume.—From equation (6) the value of p is $\frac{F^2}{8\pi K}$ dynes per square centimetre, and if this force is overcome through a distance of 1 centimetre, the result is the formation of a cubic centimetre of field by an expenditure of energy of amount $\frac{F^2}{8\pi K} \times 1$ ergs.

Hence, we arrive at the important result :—

Energy of an electric field per unit volume

$$= \frac{F^2}{8\pi K} \text{ or } \frac{U^2 K}{8\pi} \text{ ergs.} \qquad (9)$$

Example.—If the force of attraction, F, between two large parallel plates, charged with equal and opposite charges $\pm Q$ is independent of the distance between them, find the energy of the charge, and the difference of potential between them in terms of F and Q, when they are at a distance of 1 centimetre apart. (B. of E., 1903.)

The full solution of this question would merely be a repetition of some of the preceding results.

It follows from equation (8) that

$$\tfrac{1}{2}QV = F \times 1.$$

$$\therefore \text{ energy of charge} = F \times 1 \text{ ergs}$$

$$\text{also, P.D. between plates} = V = \frac{2F}{Q}.$$

Stress in a Dielectric.—Let us suppose that the equal and opposite charges upon the two plates under consideration (or upon any other pair of conductors) are steadily increased. Then the mechanical stress on the dielectric, which is always produced by the presence of an electric field, is also increased, and eventually it will give way and be pierced by a disruptive discharge or spark.[1] The limiting stress varies very much with the nature of the dielectric, and is much greater for materials like glass or ebonite than for a gas. In the latter case, it also varies with the nature of the gas, and with the pressure it supports. Consider, as an illustration, the case of air at ordinary pressure. The relation between sparking distance and potential difference has been measured for brass balls of different sizes by various experimenters. Although the spark length increases with the P.D. between the terminals, the exact value depends very largely upon their size and shape, and is so much affected by slight changes in the conditions, that only a rough general agreement can be obtained. It is, however, a well-marked fact that the P.D. required to break down a very thin layer of air, is relatively much greater than that required for a thicker layer. For instance, under certain experimental conditions, it has been found that an air film 2 millimetres thick required

[1] The student may find it difficult to realise how such a stress can exist in a mobile body like a gas. It may be remarked that the stress occurs between different parts of the atoms or molecules, and that it is compatible with freedom of mass motion as a whole.

a P.D. at the rate of 190 static units per centimetre to pierce it, whilst a layer 26 millimetres thick required only 90 static units per centimetre. The full discussion of this peculiarity is not advisable at present, but it has been suggested that it may be due to the fact that we arbitrarily measure the spark length as the shortest distance between the terminals, whereas in short air gaps, the spark conspicuously avoids the shortest paths; an effect perhaps best seen under reduced pressure.

For our present purpose, it may be taken that the average breaking stress for air under ordinary pressures is a P.D. of about 130 static units per centimetre of length.

For a uniform field, such as that between parallel plates, it follows from the definitions that the force on unit charge is then 130 dynes,

$$\text{for } V = U.d, \text{ } i.e. \text{ } U = \frac{V}{d} = \frac{130}{1}$$

This gives some idea of the greatest possible value of U under the given conditions.

Now, we have already shown that the pull of an electric field per unit area is $\frac{U^2K}{8\pi}$ dynes, therefore, in this case, the pull is $\frac{(130)^2}{8\pi}$ or 672 dynes approximately, which is the breaking-down stress of air at ordinary pressures.

Again, we have shown that the stress or pull is also given by $\frac{2\pi\rho^2}{K}$ dynes per unit area, and if we write $\frac{2\pi\rho^2}{1} = 672$, then $\rho = 10$ units approximately.

Hence, the greatest possible charge which, under the circumstances, can be given to a conductor is about 10 units of quantity per square centimetre. When the pressure of a gas is reduced, the breaking stress becomes greatly reduced, and reaches a minimum value at about ·5 millimetre pressure. Beyond this, it increases again, and finally the gas becomes non-conducting. The study of the discharge under these conditions is discussed in Chapter XXIX.

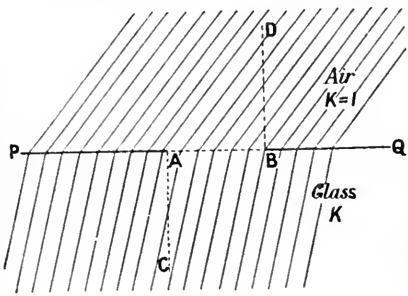


FIG. 65.

Refraction of Lines of Electric Force.—When lines of force pass obliquely across the bounding surface between two dielectrics, for which K has different values, say from air into glass or paraffin-wax, their direction changes abruptly at the surface, *i.e.* they experience a kind of refraction.

To understand this we must examine the conditions which hold good at the boundary of the two media. Let PQ, Fig. 65, indicate the boundary between (say) air and glass. We will suppose that the field inside the glass is uniform, but not normal to the bounding surface. Let AB be 1 centimetre in length and regarded as indicating 1 square centimetre marked out on the surface. Also, let AC and BD indicate square centimetres marked out at right angles to the surface in glass and in air respectively. Then, as explained on p. 174, the number of lines of force passing through AB will be the measure of the normal component of the field, and the number passing through AC and BD will be the measure of the tangential component in glass and in air respectively. It will be seen that, if the lines of force pass through the surface *without* change of direction, the tangential components will be the same in the two media, but not otherwise. On the other hand, the normal component is necessarily the same in the two media, whether there is bending or not.

Consider the electric force (U). Remembering that F = KU, we see that, if there is no change of direction at the surface, *i.e.* if F is the same on both sides, then U must have different values on the two sides. Now, the normal component of U *may* have different values, but we can show that the tangential components of U *must* be the same close to the surface on each side, for otherwise it would be possible to get work done without expending energy, *i.e.* a kind of perpetual motion could be obtained.

Let AB, Fig. 66, be any two points on the surface, and let U_1, U_2 be the tangential components of the force in air and glass respectively. Then if $U_1 > U_2$, a unit charge placed at A, just outside the surface, might be driven from A to B by the force, thus doing work. When it arrived at B, a very slight (and in the limit, infinitesimal) force would move it across the surface, and we could bring it back to A_1 *against* the weaker force U_2, thus doing work upon it. Arriving at A_1, an infinitesimal force would move it again through the surface, and it would reach its original position with a nett gain of energy during the cycle, for *less* work would have to be done in bringing it back against the weaker force U_2. Such a gain of energy without equivalent expenditure of work is contrary to all experience, and hence we infer that the tangential components of the force must be the same very close to the surface on each side of the boundary. Hence, the lines of force must be bent or refracted at the surface by an amount sufficient to make the tangential components of the force equal to each other.

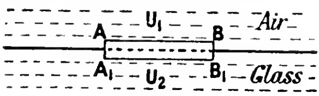


FIG. 66.

Example. — Let a line of electric force, Fig. 67, pass from a dielectric, whose constant is K_1, into another whose constant is K_2. Draw NN_1 perpendicular to the surface at the point of incidence P, and suppose that the tangent to the curved line of force at P makes with NP an angle θ_1 in the first medium, and an angle θ_2 in the second medium. We can state the two conditions which must be satisfied as follows:—

(1) The normal components of the *field* must be the same in each medium.

(2) The tangential components of the *force* must be the same in each medium.

(Of course, it follows that the tangential components of the field and the normal components of the force are necessarily *unequal*.)

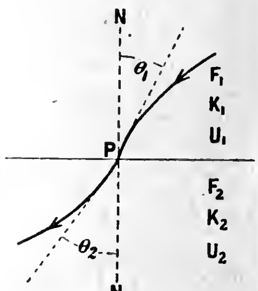


FIG. 67.

Let F_1U_1, F_2U_2 refer to the two media respectively. Then the first condition means that $F_1 \cos \theta_1 = F_2 \cos \theta_2$ (1)

and the second condition requires

$$U_1 \sin \theta_1 = U_2 \sin \theta_2 \quad (2)$$

$$\text{but} \quad F_1 = K_1 U_1, \text{ and } F_2 = K_2 U_2$$

$$\therefore \quad K_1 U_1 \cos \theta_1 = K_2 U_2 \cos \theta_2 \quad (3)$$

$$\therefore \frac{(2)}{(3)} \qquad \frac{1}{K_1} \tan \theta_1 = \frac{1}{K_2} \tan \theta_2$$

$$\text{or} \quad \frac{K_1}{K_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

From which it appears that when lines of electric force enter a medium of greater dielectric constant, they are bent away from the normal. If the second medium is a conductor, $K_2 = \infty$, in which case $\tan \theta_1 = 0$, *i.e.* the lines of force are incident normally on a conductor, as already inferred from simple reasoning.

Exercise VI

1. A is a gold-leaf electroscope, B a quadrant electrometer. For measuring small potential differences B is found to be more sensitive than A: does it necessarily follow that it will be more sensitive for the measurement of small charges? Give reasons for your answer. (B. of E., 1905.)

2. An insulated sphere having a diameter of 20 centimetres is charged. It is then connected with an electrometer by a fine wire, the deflection being 50 divisions. An insulated and uncharged sphere of 16 centimetres diameter is then joined to the first by a long wire, and the electrometer deflection falls to 32. Calculate the capacity of the electrometer. (B. of E., 1906.)

3. Describe and explain the construction and action of the essential parts of Lord Kelvin's quadrant electrometer. What is the instrument designed to measure?

4. In some forms of electrometer there is a movable disc, surrounded by a wide flat ring in the same plane as the disc. Explain the use of this ring.

5. Describe and explain the use of any form of "attracted-disc electrometer." Show what it measures, and explain the action of the "guard-ring."

6. An insulated plate 10 centimetres in diameter is charged with electricity and supported horizontally at a distance of 1 millimetre below a similar plate suspended from a balance and connected to earth. If the attraction is balanced by the weight of 1 decigram, find the charge on the plate ($g = 980$ C.G.S.). (B. of E., 1902.)

7. Define a unit of electricity and the strength of an electric field. Two very large metal plates are parallel to one another, and at a distance of 2 centimetres apart; one plate is charged with positive and the other with negative electricity; and the surface density of the charge on each plate is 20. Calculate (*a*) the strength of the field between the plates, (*b*) the potential difference between them. (Science and Agriculture, Inter., 1905.)

8. A condenser is made of two parallel metal plates. These are maintained at constant potentials. What effect will be produced on the attraction between the plates when a slab of heavy glass, specific inductive capacity 10, and thickness one-half of their distance apart, is inserted between them? The faces of the slab are parallel to the plates. (Lond. Univ. Inter. B.Sc., Hon., 1895.)

CHAPTER VIII

ATMOSPHERIC ELECTRICITY

The early observers soon perceived that thunder and lightning were similar in their nature to the crackling and light of the electric discharge. Franklin proved an exact similarity between the two discharges in (1) giving light, (2) noise, (3) conduction by metals and moisture, (4) fusing metals, (5) rending imperfect conductors, (6) killing animals, and (7) odour. He succeeded in drawing electricity from the clouds by means of a kite having a pointed wire attached to it. The kite was held by ordinary packthread, having a key at the end to which a silk cord was fastened. Having tied the silk to a tree, he held his hand to the key; but at first he was unable to obtain any result. A storm of rain, however, came on, which, wetting the string, made it a good conductor, and he then obtained sparks in sufficient quantity to charge a Leyden jar.

Since Franklin's famous experiment, numerous observations have proved that the atmosphere is *constantly* in a state of electrification, even during fine weather.

In such observations, it is usual to measure the difference of potential between a point in the air and the earth; or, preferably, to obtain a series of such values for points at different heights and vertically above each other.

If, for instance, the base of an electroscope is earth-connected as usual, and the cap is connected by a conductor to a point (say) two yards above the earth's surface, then, if the end of the conductor can be made to acquire the potential of the air at that point, a deflection will be produced when any P.D. exists between the point and the earth; and as the leaves and conductor form a connected system necessarily at the same potential, the magnitude of the deflection will be a measure of the P.D. in question. The practical difficulty arises from the fact that air is a non-conductor, for the end of the conductor can only change in potential by giving up or by receiving a charge, and we have seen that such an interchange does not readily occur between a conductor and a non-conductor. The earliest observers, some 150 years ago, used an electroscope to which a long upright pointed rod was attached (Fig. 68), but this application of the properties of pointed conductors is not satisfactory, for the equalisation of potentials at the tip takes place slowly, and there is never any certainty that it is complete.

Volta attached a burning match or torch to the top of the rod, and many later observers have used similar methods. This was a great improvement, but it is somewhat inconvenient, and, moreover, there is a certain small P.D. produced due to the combustion itself.

Lord Kelvin's water-dropping arrangement is very satisfactory, and is now almost universally used. This is shown in Fig. 69 (which is taken from Carey Foster and Porter's *Electricity and Magnetism*). A vessel containing water is well insulated by supporting it on a glass rod rising from the centre of a vessel containing strong sulphuric acid (to keep the rod dry). To this vessel is attached a long tube extending through an aperture in in the wall of the room into the open air, and from the end of this tube the water slowly drops away. The water-vessel and rod may be regarded as being under induction from the air surrounding the jet; if the air is positively charged, the jet will become negative and the vessel inside positive, although of course both

FIG. 68.

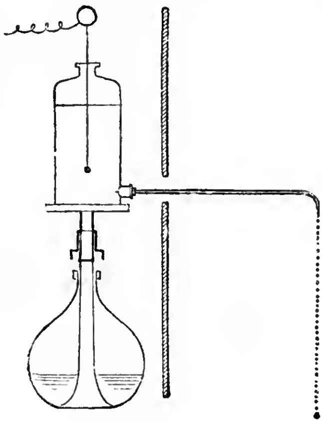


FIG. 69.

are at the same potential, but the fact that the jet is negatively charged shows that this potential is less than that of the adjacent air. As the water drips away, each drop carries with it a small negative charge, and in this way, in a very short time, the potentials are equalised, so that an electroscope[1] whose cap is connected to the water-vessel will correctly indicate the P.D. between the air around the jet and the earth.

[1] For the purpose of exact measurement an electrometer is used.

Very simple apparatus may be used to demonstrate the existence of large differences of potential between the earth and points above it.

Exp. 72. Select a tinned iron can of moderate size. Close to the bottom make a small hole through which may be inserted the end of a narrow brass tube about 6 feet in length. Let this project inside the can, and solder it to the bottom. Also solder round the hole. Fill up with water, and partially close the end of the tube with a hammer until the water slowly trickles from it. Take this to the highest room in a building, and place it on a cake of paraffin-wax so that the tube projects as far as possible through a window (or if practicable, take it outside and work in the open air at the top of the building). Connect the can to the cap of an electroscope, and then fill up with water. As a rule, with an instrument of average sensitiveness a large deflection will be obtained very quickly. Test the sign. In all probability it will be positive.

Such observations show that in fine weather the atmosphere is, as a general rule, positive to the earth, *i.e.* the earth is negatively charged, the lines of electric force extending vertically upwards and terminating somewhere in positive charges. Where they terminate is a question still under investigation. The potential increases rapidly with vertical height for a considerable distance; at still higher elevation with less rapidity. The rate of increase of potential with height is an important quantity known as the **potential gradient.** It is a very variable quantity, the average value being something like 200 volts per metre of vertical height above open ground. It may, however, greatly exceed this value, and there is good reason for believing that the average is different in various parts of the earth.

During wet weather, the potential gradient is often reversed in sign, the earth being positive to the air, but, taking the whole year into account, such reversals are for relatively short periods.

Surface Density of the Earth.—Assuming that the electrification of the air is entirely due to a uniform negative charge possessed by the earth, we can estimate its surface density as follows:—

If ρ is the surface density, the field close to the earth's surface is given by

$$F = 4\pi\rho \quad \text{(p. 46)}.$$

Also, if V is the P.D. between the earth and a point at a height of h centimetres above it (the curvature of the earth being neglected), we have

$$V = Uh, \text{ also } F = KU,$$

and as K = 1 for air

$$V = 4\pi\rho h$$

$$\therefore \rho = \frac{V}{4\pi h}$$

If we take V as 300 volts per metre (for convenience),

then V = 1 static unit

and $\rho = \dfrac{1}{4\pi \times 100} = \dfrac{1}{1257}$ static units of charge per square centimetre.

This calculation assumes a uniform field; if it is not uniform, we have

$$\frac{dV}{dh} = 4\pi\rho, \text{ where } \frac{dV}{dh} \text{ is the potential gradient.}$$

It is, however, very improbable that the electric field in the atmosphere is entirely due to a charge situated upon the earth. The evidence suggests very strongly that the air itself is often locally charged, and that as masses of air sweep past the place of observation they carry their charges with them.

Sources of Atmospheric Electrification.—The actual sources of atmospheric electrification are still unknown, and the question is rendered more difficult by the fact that the earth is always rapidly losing its charge.

Comparatively recent research has established the fact that the air is not a perfect insulator, for charged bodies, even when thoroughly insulated, always slowly lose their charge. For a long time this was attributed to the inevitable leakage over the supports due to imperfect insulation, but it is now definitely known to be due to a small but true conductivity of the air itself. Recently, C. T. R. Wilson, using an instrument devised by himself, measured the rate of loss of charge in fine weather, and found that the average value was 8 per cent. per minute in the open air. This means that the total charge on the earth's surface is lost (and renewed) every 12 or 13 minutes.

As it has been known for some time that rain-drops are frequently (but not always) electrically charged—the sign of the charge being supposed to be usually negative—Wilson suggested that districts experiencing fine weather are losing negative electrification, and at the same time other districts are gaining negative from falling rain, equilibrium being restored by earth currents below, and above by currents at a considerable height in the atmosphere. This suggestion has so far been neither confirmed nor disproved, but some additional evidence has been obtained at Simla in India by Dr. G. C. Simpson. He invented and used an apparatus which gave a practically continuous record of the amount of the charge during rain, and its sign. He obtained the unexpected result that positive electrification was in excess. During 37 per cent. of the total time of rain, no charge was registered; and when the rain was charged, the observations showed that about three-quarters of the total amount recorded was positive.

Dr. Chree has, however, pointed out that there may be some local influence at work, and that observations in other parts of the world are required before any general conclusions can be drawn. It appears that at Simla rain chiefly falls during thunderstorms, and that the excess of positive electrification was especially pronounced during such storms.

How the rain becomes charged is another question, which still awaits a definite answer. In this connection, a research made by Lenard, about twenty years ago, may be mentioned. He found that, when water fell upon a solid and broke into spray (*e.g.* upon the rocks and stones at the base of the waterfall), the drops became positive, while the air around acquired a negative charge. He also showed that when a drop of water (or of mercury) splashed on a metal plate, the same effect was produced. A slight alteration in the purity of the water, however, modified the result obtained; exceptionally pure water showed the effect very strongly, while it was almost imperceptible in the case of water slightly less pure. On the other hand, a weak salt solution gave an opposite effect, the air becoming positive instead of negative.

Sir J. J. Thomson found that minute traces of certain substances in the water produced surprisingly large results, but that strong solutions were practically inert, giving little or no electrical effect by splashing.

Lenard came to the conclusion that no electrical separation occurred when water broke up into drops without splashing against some obstacle, a conclusion which appears to need modification in the light of some recent experiments made by Dr. Simpson, who broke up falling water into drops by means of a vertical air jet, and found no perceptible action in the case of Simla tap water, although there was a strong effect when distilled water was used, the water becoming positive.

Now, rain-drops may be broken up in the atmosphere by air currents, and hence it is possible that their charge may be accounted for in some such way.

It is not easy to explain the frequent presence of a negative charge, although Dr. Simpson suggests that it may be directly derived from air which has been previously negatively charged by breaking drops.

Elster and Geitel have suggested that the electrification of the air may be due to the action of ultra-violet rays from the sun upon the upper strata of the atmosphere (see p. 613).

Such an action may be of two kinds: (1) the rays may dissipate negative charges from ice crystals or dust in the atmosphere; in fact, *dry* ice is known to act in this way; (2) the rays may simply *ionise* the air. Probably both effects are produced, but it is difficult to estimate their real importance.

The slight conductivity of the air already mentioned indicates that it is ionised to a very small extent, and the presence of these ions may greatly influence the formation of water particles from vapour.[1]

The facts of surface tension show that there is a great difficulty in starting condensation, and very small drops, even if formed, would tend to evaporate rapidly. As a matter of fact, condensation appears always to commence around some nucleus, such as a dust particle, which provides a finite radius to begin with. Now, the effect of surface tension appears to be purely mechanical; like a tightly stretched skin, it increases the pressure inside the drop, and it has been found by C. T. R. Wilson that this pressure may be compensated by the opposite outward pull due to an electric charge, or rather to the lines of force which extend outwards from it. As a result, ions serve as nuclei on which moisture may condense, the negative ions being far more effective than positive ones. Hence, Sir J. J. Thomson has suggested that such condensation may produce a separation of electric charges in ionised air, the negative ions being selected as nuclei in preference to positive, and brought down as negatively charged rain.

In whatever way the water particles acquire their charge, a cloud may be regarded as a movable conductor, and, if it is charged, it may approach by electrostatic attraction other clouds or the earth. If the charge is initially on the individual particles, and then becomes gradually transferred to the cloud as a whole, *i.e.* to its external surface, the potential may rise enormously, for the external surface is very much less than the total surface of the whole of the particles, and the final result may be a spark between two clouds, or between a cloud and the earth.

Similarly, it is easy to see that the potential must rise if small charged drops coalesce to form larger ones.

For suppose that n globules, each of radius r, in falling unite to form one globule, then the radius (r_1) of this globule can be obtained by the following method:—

$$\text{The mass of a sphere} = \text{volume} \times \text{density}$$

$$= \frac{4\pi r^3}{3} \times d$$

Now, the mass of the sphere of radius r_1 is n times that of a sphere of radius r,

$$\text{whence } \frac{4\pi r_1^3 d}{3} = n\frac{4\pi r^3 d}{3}$$

$$\therefore\ r_1 = r\sqrt[3]{n}$$

[1] We may remark that the ionisation is probably due to the generally diffused presence of exceedingly small amounts of radio-active matter. This matter as it breaks up, liberates α and β rays, which are the direct cause of the ionisation. See Chapter XXIX.

If, for example, $n=8$, the radius of the larger globule will be twice that of each of the eight, but the charge is eight times as great. Now

$$V=\frac{Q}{r}$$

and, therefore, its potential is four times greater than that of the constituent drops.

Thus the potential of a cloud may increase enormously by the coalescence of the minute particles, but when dealing with such calculations it must be remembered that they ignore the possible influence of neighbouring particles on the result.

Lightning.—These discharges are flashes of lightning, of which three kinds have been distinguished: (1) *forked lightning*, which is no doubt due to the character of the air through which the discharge occurs, certain portions offering greater resistance than others; in fact, its path is the one of least resistance. (2) *Sheet lightning*, which is probably due to the illumination of the cloud where the flash occurs, or is merely the reflection on the clouds of a distant discharge. It is called *heat* or *summer lightning*, when the whole horizon is illuminated by flashes so far distant that the thunder is not heard. (3) Another form—known as *globular* or *ball lightning*—has not yet been explained. It is said to consist of balls of fire, which travel slowly, or even remain stationary, and then explode with sudden violence. Fortunately, its occurrence is very infrequent.

Thunder is the report which follows the discharge. It is probably due to the fact that the lightning heats the air in its path, producing sudden expansion and compression, which is followed by an extremely rapid rush of air into the rarefied space. A thunder *clap* is produced when the path of the discharge is short and straight. The *rattle* is produced when the path is long and zigzag. *Rumbling*, or *rolling*, is produced by the echoes among the clouds.

As the velocity of light is about 186,000 miles per second, we may consider it as practically instantaneous; and as that of sound, at the ordinary atmospheric temperature, is 1120 feet per second, the distance of a discharge can be ascertained by multiplying 1120 by the number of seconds which elapse between the lightning and the first sound of the thunder.

Lightning Conductors consist of three essential parts: (1) a *rod*, usually of copper or of galvanized iron, elevated above the highest portion of a building and terminated at the top in a fine point; (2) a *conductor*, connecting this rod with the ground, often made of a stout copper ribbon; (3) the *earth connection*, which is of very great importance. The lower end of the conductor should terminate in several branches passing into a well or at any rate into moist earth. Of course, the continuity of the conductor is of the utmost importance.

The protective action of a lightning conductor is very simple; for suppose that a charged cloud passes over a building provided with a conductor; induction is set up, but the induced electrification accumulates on, and is then discharged from, the raised point, and so tends to neutralise the charge of the cloud. By this means a disruptive discharge is frequently prevented. Such protective action has been illustrated in Experiments 63–65, pp. 83, 84. But lightning discharges are often oscillatory and of very high frequency.[1] The *impedance* of the conductor may then become enormous, and the discharge strikes off the conductor with disastrous effect upon neighbouring bodies. This "choking" effect, due to impedance, is usefully applied in order to protect electric cars supplied from overhead wires. A small coil consisting of a few turns of thick copper wire (often termed a "kicking" coil) is placed in the circuit between the trolley wire and the motors, and an alternative path provided through a spark gap to earth. The coil offers no appreciable resistance to the direct current which drives the motors, whereas the spark gap offers an infinite resistance. On the other hand, to oscillatory lightning discharges of high frequency, the impedance of the coil acts very much like an infinite resistance, while, as the potential is high, the discharge readily sparks across the gap.

It is by no means a simple problem to protect a building from risk of damage by lightning.

The late Professor Clerk-Maxwell suggested that a building should be covered with a network of metallic wires, so that the building forms, as it were, the interior of the conductor.

Sir Oliver Lodge condemns the use of a rod passing above the highest point of a building, and suggests that the conductor should consist of a number of lengths of common telegraph-wire connected with large masses of metal, *e.g.* leaden roofs. The connection must be thorough, and made at many points. Barbed wire should be run round the eaves and ridges of the roof so that many points are exposed. Balconies and other accessible places should not be connected.

The Aurora is a luminous phenomenon occurring chiefly in high latitudes and depending upon the electrical condition of the atmosphere. If it occurs in northern latitudes it is known as *aurora borealis*, or *northern lights*, while in southern latitudes it is called *aurora australis*.

In the arctic regions the aurora occurs almost nightly, and it occasionally extends over very large areas. The light assumes various forms and colours; *e.g.* it sometimes appears merely as pale and flickering streaks, occasionally tinged with various colours, passing from the horizon towards the north magnetic pole.

Usually the light is silvery or greenish, less frequently a red glow is seen. The spectrum of this light *invariably* shows a yellowish green line, which is highly characteristic of a true aurora. Sometimes fainter blue lines are seen, and occasionally red ones. For a long time the origin of the chief auroral line was a profound mystery, for it did not correspond to any known element, but

[1] These terms are explained in Chapters XXVI. and XXXV.

it is now fairly certain that it is due to krypton, one of the inert gases in the atmosphere, discovered by Sir Wm. Ramsay. It is difficult to say why this gas should appear so prominently. At the earth's surface it is not present in relatively great quantity, ordinary air containing about one part in a million by volume; less than the amount of either hydrogen, helium, or neon normally present, and very much less than the amount of argon (about 9000 parts in a million). We may, however, point out that the percentage composition of the atmosphere at extremely high altitudes is certain to differ widely from that obtaining at the sea-level.

Possibly one of the red lines occasionally seen is due to neon, but this is by no means certain.

Arrhenius in 1900 worked out a very important theory of the aurora, in which he attributed the phenomenon to the presence of cathode rays, *i.e.* electrons, shot off from the sun, and which under the influence of the magnetic field of the earth (by an effect identical in nature with that described on p. 486), tend to rotate round the lines of force and to drift towards the poles. There is very strong evidence that some such action originating in the sun is involved. Auroræ are always most plentiful at the time of sun-spot maxima, and are invariably accompanied by "magnetic storms" (p. 185).

As far as the visible phenomena are concerned, they are, no doubt, analogous in many ways to vacuum discharges, occurring in the rarefied, and therefore semi-conducting upper strata of the atmosphere. At the same time, observations are on record, which seem to indicate that occasionally such visible effects occur quite close to the earth's surface.

Note.—The student must not expect to find any *direct* connection between the phenomena dealt with in **Electrostatics,** and those to be discussed in the next section. **Magnetism** is, for all practical purposes, a new subject. It is, therefore, desirable to point out that the *main* object of the first two sections of the book is to develop the ideas embodied in the terms "lines of electric force," and "lines of magnetic force," respectively, *while such lines are in a state of rest.* The nature of the relation which exists between them will become apparent, when in the third and most important section—**Voltaic,** or **Current Electricity**—we apply such ideas to phenomena depending upon the coexistence of *both* kinds of field, *while in a state of relative motion.*

MAGNETISM

CHAPTER IX

MAGNETIC ATTRACTION AND REPULSION

Lodestones or Natural Magnets.—**Exp. 73.** Plunge a piece of lodestone into iron filings; on withdrawal notice that tufts of filings cling to certain parts. In Fig. 70 the piece has been shaped with a hammer so that this attractive power is most apparent near the ends.

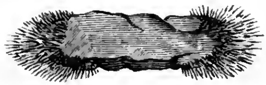


FIG. 70.

Exp. 74. Suspend the piece of lodestone so that it can turn freely. Either of the following methods of suspension may be adopted. Fasten a thread of raw silk to a suitable support, and (*a*) tie the other end to a piece of wire bent as shown in Fig. 71, or (*b*) pass it through the two free ends of a doubled strip of paper. We may call (*a*) a wire stirrup, and (*b*) a paper stirrup. Observe that the lodestone sets itself in a definite direction, pointing nearly north and south. If disturbed from this position it oscillates for a time, and then comes to rest in exactly the same position as before, the same end always pointing towards the north.

FIG. 71.

This hard, dark-coloured, stone-like body is widely distributed in nature, being met with in great abundance in Norway and Sweden, and in some parts of America. From the circumstance that it was originally found in *Magnesia*, in Asia Minor, it was probably called *magnēs* by the Greeks, whence we have derived our words *magnet*, *magnetism*, &c. Although this substance does not always possess the properties of attracting iron and of setting itself in a north and south direction, it constitutes one of our most important and valuable iron ores, known as *magnetite*, or *magnetic oxide of iron.* Its chemical formula is Fe_3O_4. It is called a *natural magnet* because it is found in a natural state, and *lodestone* (A.S. loedan, to lead) because it has the remarkable property referred to in Experiment 74, which caused it to be subsequently used in navigation.

Artificial Magnets.—**Exp. 75.** Draw a piece of lodestone fifteen or twenty times over an ordinary steel knitting-needle, or a piece of watch-spring, taking care to move it always in the same direction, not to and fro.[1] (*a*) Plunge it into iron filings, and observe that, on withdrawal, tufts similar to those obtained in Experiment 73, cling to the ends; (*b*) suspend it in a paper stirrup, and observe that it sets itself in a north and south direction.

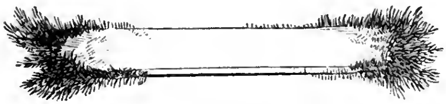


FIG. 72.

We learn from these experiments that a new property, which manifests itself in several ways, has been imparted to the steel needle by rubbing it with a lodestone. Such a piece of steel is an *artificial magnet.* The process by which this property is acquired is called *magnetisation*, and the steel is said to be *magnetised.*

Attraction of Iron by Magnets.—**Exp. 76.** Obtain a number of small, soft iron nails, as nearly as possible of the same size and weight. (1) Near the end, *a*, Fig. 73, of a strong magnet, support the greatest possible number of the nails.

(2) Hang the nails at points *b* and *d*, nearer the middle of the magnet. Observe that, as we approach the middle, a smaller number can be supported, while (3) at the middle, *e*, even a small particle of iron cannot be supported.

Test the other half of the magnet in a similar manner, and observe that equal weights are supported at equal distances from each end.

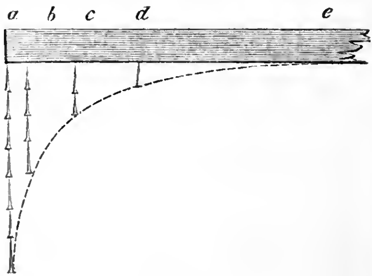


FIG. 73.

The curve drawn through the free ends of each series is a rough measure of the attractive force at different points along the magnet.

We therefore learn from this experiment that—

(*a*) The maximum attractive force of a magnet is situated in somewhat indefinite areas near the ends. In certain cases, however, especially when the length of a magnet is great compared with its thickness, it is permissible to regard these areas as points. Such parts are called the *poles* of the magnet. Later, the student will realise that a magnet pole is a region from which "lines of magnetic force" pass into the surrounding medium.

(*b*) As we approach the middle of a magnet, the attractive power becomes smaller until (*c*) all round the magnet, midway between the poles, it ceases altogether. This is called the *neutral line.*

The line joining the poles is called the *magnetic axis.*

Poles of a Magnet.—We have seen that, in our latitude, one pole of a magnet always points northwards, and the other southwards. From this property the poles are distinguished one from the other by

[1] This method is of no practical value. The ordinary methods of making magnets will be described later.

calling that which is directed towards the north, the *north* pole, while the opposite one is called the *south* pole.

Magnetisation by Single Touch.—**Exp. 77.** Place a strip of steel on a table. Bring one pole of a magnet in contact with one end of the strip (Fig. 74). Move the magnet, parallel to its first position, to the other end, then lift it and replace it in its first position. After rubbing one side ten or twelve times, turn the strip over, and treat the other side similarly.

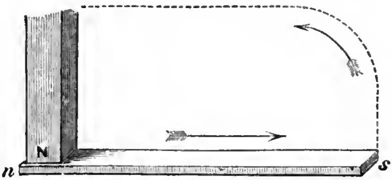


FIG. 74.

The polarity of the end of the bar, where the magnet leaves it, is always of *opposite* name to the magnetising pole. Thus, if a N pole be used, the end where the magnet leaves the bar is S. and that where it is first placed, N.

Magnetisation by Separate or Divided Touch.—**Exp. 78.** Place the bar to be magnetised horizontally, and then place the opposite poles of two bar magnets at the middle of the bar as shown in Fig. 75. Draw them simultaneously from the middle to the ends. Lift them and place them again at the middle. Repeat this operation ten or twelve times. Turn the bar over and rub the other side in a similar manner.

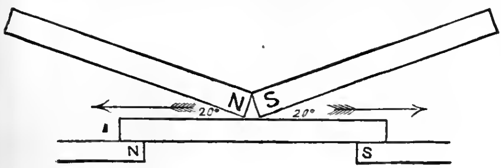


FIG. 75.

This process is rendered easier and more effectual if the bar to be magnetised is supported on the opposite poles of two bar magnets, so that the poles of the lower magnets are similar to those of the magnetising magnets immediately above them (Fig. 75).

These methods were important before the discovery of electro-magnetism. They are now only occasionally used, as steel bars may be much more readily and strongly magnetised by electrical methods.

To make a Magnetic Needle.—(*a*) Cut a piece of clock-spring, with a pair of shears used for cutting metal, into either of the shapes shown in Fig. 76. Magnetise it by the method of single touch. Balance it accurately by placing it across a knife-edge, and then scratch a line to mark the position of the knife.

(*b*) Make a glass cap in the following manner—

Take a piece of glass tubing ($\frac{3}{16}$-inch bore is the most useful), and holding it in a Bunsen's or spirit-lamp flame, turn it continually until it is quite soft, and then pull the ends apart so that it has the appearance shown in Fig. 77, *a*. Break the thin thread, and hold one piece in the flame until the end is rounded off (Fig. 77, *b*). It is often necessary to remove the bead which forms on the end with another piece of hot glass-tubing. When the glass is cold, make a mark (represented by the dotted line in the diagram) with a sharp triangular file. The rounded portion, which forms the cap, can then be easily separated from the rest of the tube by gentle pressure.

(*c*) Soften the central part of the strip of magnetised steel, by holding it in a flame until it is red-hot, and then *gradually* removing it from the flame so that it cools *slowly*. Drill a hole through the needle at the middle of the line about which it balanced, taking care that it is slightly *smaller* than the diameter of the cap. After boring, the strip must be *hardened* by again making it red-hot, and then suddenly plunging it in cold water. Now place the needle upon a small sheet of red-hot iron, when it will first turn yellow, and then gradually blue. When this occurs, slide it off the iron into cold water. This is called *tempering*.

(*d*) Fasten the cap, with a trace of glue, in the hole, so that it is perpendicular to the needle, and then put it aside to dry.

(*e*) Make a support by gluing a cork to the centre of a board (6 in. × 3 in.

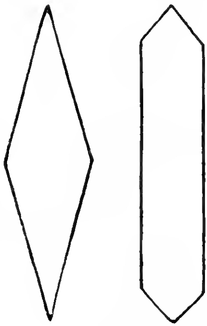


FIG. 76.

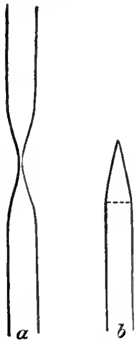


FIG. 77.

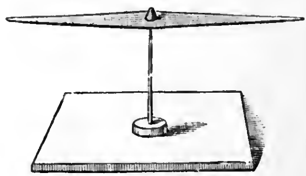


FIG. 78.

× ½ in.), and then passing the eye of a fine sewing-needle into the cork. Take care that the needle is quite vertical (Fig. 78).

(*f*) Bore a hole, sufficiently large to admit the glass cap, in the base of the support. Place the needle on the board, with the cap in the hole, and re-magnetise it. This is best done by *separate touch*.

Exp. 79. Place the needle on its support, and notice that it sets itself in a north and south direction.[1]

Action of Magnetic Poles on each Other.—Exp. 80. Suspend a magnet in a wire stirrup. Mark the end which points northwards with a piece of gummed paper. (1) Bring the marked end of the magnet near the N pole of the needle. Observe that repulsion takes place. Therefore, two N poles repel one another. (2) Bring the marked end near the S pole of the needle. Attraction ensues. Therefore, a N and a S pole attract. (3) Repeat these experiments with the unmarked end.

When performing this experiment, the magnet pole must be gradually brought near the needle from a distance, for when a fairly strong magnet is

[1] In these experiments care must be taken that the needle is removed from the influence of other magnets and of pieces of iron.

brought quickly up to one end of the needle, attraction may take place although the poles are alike, and it will then be found that the polarity of the needle has been reversed.

These results enable us to state the first law of magnetism—*like poles repel one another, unlike poles attract.*

Exp 81, *to find if a given piece of iron or steel is magnetised.* Bring the specimen to be tested towards one end of a compass needle, and observe whether attraction or repulsion is produced. If repulsion, then the specimen is certainly magnetised; if attraction, try the effect of the other end on the *same* end of the needle. If the effect is still attraction, the specimen is not magnetised, for *either* end of the compass needle will be attracted by *unmagnetised* iron.

In performing this experiment, care must be taken to observe the effects at the greatest possible distance; for if a weakly magnetised piece of iron is gradually brought near the compass needle, it will often be found that repulsion occurs when the distance between the two like poles is considerable, and that this changes to attraction when the distance is small. Hence, a rapid approach may cause the initial repulsion to be overlooked. In this case, the effect is the converse of that described above—the pole of the compass needle reversing the polarity of the iron.

Position of Poles.—**Exp 82.** Place a magnet on a sheet of paper, and trace its outline with a pencil. Place a short compass needle near one end of the magnet, and mark with dots the positions of the ends of the needle. Repeat this for several positions of the compass-needle around the end of the magnet. Remove the magnet and draw lines through each pair of points, producing them until they intersect. Observe that the intersections do not occur in a single point, but that they cover a small area, whose approximate centre may be regarded as the position of the pole. This point may be a centimetre or more from the end of the magnet.

The positions of the poles of three magnets were found by this method with the following results:—

Length.	Breadth.	Distance of Pole from End.
31·3 centimetres.	3 centimetres.	1·7 centimetre.
15·85 ,,	1·8 ,,	1·15 ,,
10·7 ,,	2 ,,	0·5 ,,

These results are important, because in numerical calculations the axis or "length" of a magnet is *the distance between its poles* and not its total length. This definition holds good with magnets of any shape, *e.g.* in the horseshoe form the distance between the poles may be quite small, although the total length is considerable.

The Earth a Magnet.—The direction which a horizontally suspended magnet takes, is due to the fact that the earth itself is a huge magnet, having its magnetic poles comparatively near the geographical poles. From the law just enunciated, "like poles repel, unlike attract," we can easily perceive that, as we have defined the pole, which points north, the N pole, the earth's pole, situated near the geographical north pole, is a S pole.

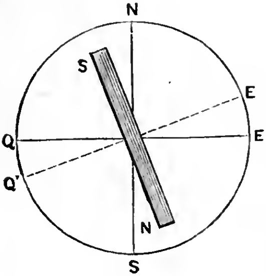


FIG. 79

An approximate representation of the magnetic condition of the earth can be made by placing a bar magnet within a wooden globe, so that the centre of the magnet coincides with the centre of the globe, its S pole being about 16° to the west of the point which represents the geographical north pole.[1] This will be understood by reference to Fig. 79, in which EQ represents the geographical equator, and EQ′ the magnetic equator.

The Earth's Action on a Magnet is merely Directive. —**Exp. 83.** Pass each end of a magnetised needle through a small piece of cork, and float on water. Observe that the N pole turns towards the north magnetic pole of the earth, but that there is no movement towards the side of the vessel.

The size of the earth is enormously great as compared with that of an artificial magnet, so that the attractive force exerted by the earth's north magnetic pole upon the magnet's N pole, is equal and opposite to the repulsive force exerted on the S pole; and the attractive force exerted by the south magnetic pole of the earth upon the S pole of the magnet is equal and opposite to the repulsive force exerted upon the N pole. The total effect of these forces upon the poles of the magnet is therefore equivalent to a *couple*, one force acting towards the north magnetic pole of the earth, and the other towards the south magnetic pole. This will be more easily understood when the properties of lines of force have been explained (see Chapter XI.).

Magnetic Meridian.—The magnetic meridian of any place is sometimes defined as the plane drawn through the zenith (the point in the heavens immediately overhead), and the magnetic north and south points of the horizon; it may be more accurately and simply defined as the vertical plane in which lies the axis of a freely suspended magnetic needle at rest.[2]

Exp. 84. Suspend a magnetised knitting-needle by means of a fibre of raw silk, and allow it to rest just above a table. Mark the position of the two ends of the needle. Remove it, and draw a line joining these two points. This line is the intersection of two planes, viz. the magnetic meridian and the surface of the table.

No Isolated Poles.—**Exp. 85.** Harden, but do not temper, a piece of watch-spring. Magnetise the hardened steel by single touch, and then show

[1] Approximately true in 1912.

[2] The geographical meridian of any place is the plane passing through the zenith, and through the geographical north and south poles.

that one end contains a N pole, and the other a S pole. Mark the former with gummed paper. Break the newly made magnet into two pieces, and prove that each piece is a perfect magnet. In fact, we shall find that (1) we obtain complete and perfect magnets if the magnetised strip of steel be broken into any number of pieces (Fig. 80), and (2) it is impossible to obtain a magnet with one pole only.

This fact, taken in conjunction with others which cannot be discussed at present, points to the conclusion that each ultimate particle of iron is naturally a perfect magnet of exceedingly small dimensions. Under ordinary circumstances, we may suppose that these atomic magnets arrange themselves under the influence of their own mutual forces in such a way that their resultant action is zero as far as external bodies are concerned.[1] We may further suppose

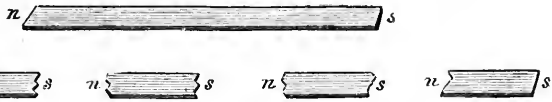


FIG. 80.

that the process of magnetisation is equivalent to compelling some of them to take up an ordered direction, with like poles pointing in the same direction. We need not think that *all* the particles take up such a configuration; if only a small percentage do so, the iron will behave as a feeble magnet. As a matter of fact, when a bar of steel is treated as in Experiment 77, only the surface layers are usually magnetised, as may be demonstrated by dissolving them off in acid. Neither are we obliged to think of the particles as arranged exactly end to end. If that were the case, only the *ends* of a bar would exhibit magnetic properties, the *sides* throughout being neutral. It must be remembered that in addition to the endwise attractions of "unlike" poles along a chain of particles, there must exist a lateral repulsion between adjacent chains due to "like" poles, which, towards the ends, will cause the chains to widen out, some ending on the sides, as shown diagrammatically in Fig. 81. It is easy to see that such lateral repulsion assists the formation of shorter and more nearly closed chains of particles (never of longer and more open chains), the final result of gradual rearrangements being the *equivalent* to a tendency of the ends A and B, Fig. 81, to approach each other,

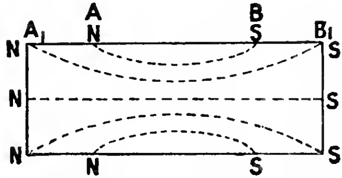


FIG. 81.

[1] In elementary text-books, the atomic magnets in unmagnetised iron are often said to be mixed up in general confusion. Such a statement completely overlooks the existence of mutual forces between them, and, as Ewing has shown, the particles really arrange themselves in little groups, more or less ring-shaped, which are stable under ordinary conditions.

and similarly A_1 and B_1, and so on. In pure iron this takes place with great readiness, the particles settling down into the stable configurations already mentioned.

FIG. 82.

In steel magnets, this action is resisted by something akin to friction, due to the presence of carbon, &c., and hence we see that a magnet with free poles always has a natural tendency to demagnetise itself. This action is often called "the demagnetising effect of the poles." When there are no free poles, *e.g.* when the particles can form closed chains entirely in iron, it practically disappears. It is for this reason that bar magnets, when not in use, are arranged as shown in Fig. 82, opposite poles being joined by short bars of soft iron, A and B, known as *keepers.*

Our argument necessarily implies that the ultimate particles of a solid piece of iron possess some freedom of rotation—a fact strongly supported by other branches of science.

Exp. 86. Partially fill a small test-tube with *steel* filings. (Steel is required, because we have found that once magnetised it retains its magnetism. It will be shown later that this is not the case with soft iron). Holding the tube horizontally, magnetise it by single touch. Observe that the filings set themselves end to end, having their longest directions parallel to the length of the tube (Fig. 83). (*a*) Without disturbing the arrangement, bring the tube near

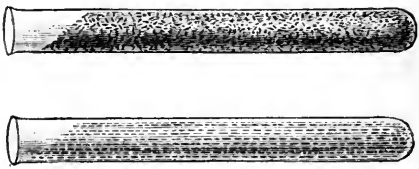


FIG. 83.

a horizontally suspended magnetic needle, and notice that one end of the tube attracts one pole of the needle and repels the other. We, therefore, conclude that the filings form a magnet. (*b*) Now disturb the arrangement by shaking the tube, and test again. Observe that no *repulsion* occurs, *i.e.* the filings *as a whole* have lost their magnetism, although each individual filing may be as strongly magnetised as before.

Why each particle of iron acts as a magnet, whereas the particles of other elementary bodies, as a rule, do not (except with great feebleness in a few cases), is a question of some difficulty.

It will be shown later that magnetism can be produced by an electric current, and Ampère suggested that each atom of iron may have such a current circulating round it. This hypothesis, which affords an adequate explanation, and which, in a modified form, is supported by later discoveries, will be referred to subsequently.

Magnetic Battery.—If a number of magnets, either bar or horse shoe, be used, having their similar poles adjacent, they form what is known as a *magnetic battery*.

Fig. 84 represents such a battery, in which there are twelve magnets—

FIG. 84.

arranged in three sets, each set consisting of four magnets. Their similar poles are bound together by pieces of soft iron, A and B.

In this way, a magnet is produced of much greater power than could be obtained by using the same mass of steel in the solid form. For, as already stated on p. 117, it is difficult to magnetise a single thick bar equally and strongly throughout the whole of its thickness, whereas thin strips can be readily magnetised and then built up into a compound bar. The principle of this method of construction is often used in instruments where magnets are required, *e.g.* in Ewing's hysteresis tester (p. 415), and in Lord Kelvin's compass (p. 191), &c.

Consequent Poles.—Owing to irregular and imperfect magnetisation, a magnet sometimes contains more than two poles. In such a case, the bar really consists of several magnets placed end to

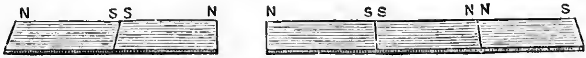


FIG. 85.

end, having their similar poles together at *intermediate* points, as shown in Fig. 85. The extra poles are called **intermediate poles, consecutive poles,** or **consequent poles.**

The presence of consequent poles in a magnet is generally due to accidental causes; they may, however, be produced at will by several methods, of which the following is one :—

Exp. 87. Place *like* poles of two bar magnets at the middle of a strip of steel, draw them simultaneously to the ends, lift, and place them again at the middle. Repeat this operation a few times, and then show, by placing the strip in iron filings, that there are three poles—one near each end, and the other of double strength at the middle.

Magnetic Substances.—Mutual magnetic attraction does not take place between magnets and *all* bodies. Those substances which have the property of attracting and of being attracted by a magnet, are called **magnetic bodies.** Besides iron and steel, the following bodies are recognised as magnetic—cobalt, nickel, chromium, manganese, and cerium. Of the latter, cobalt and nickel are the best, but even they are distinctly inferior to iron or steel in this respect. Many other bodies, *e.g.* paper, porcelain, oxygen, and certain salts of iron, are feebly attracted by very powerful magnets. This subject is further discussed in Chapter XXV.

CHAPTER X

INDUCTION

Induced Magnetism.—**Exp. 88.** Place a piece of soft iron either in *contact with* or *near* one pole of a magnet. Bring iron filings to the lower end of the iron, and observe that they cling in a tuft (Fig. 86). Remove the magnet, the filings immediately fall.

FIG. 86.

The magnetism thus communicated to the iron is called *induced magnetism;* the magnet communicating it is called *the inducing magnet;* the action is known as *magnetic induction.*

Nature of Induced Polarity.—**Exp. 89.** Cover a horizontally suspended magnetic needle with a beaker.

(*a*) Place a magnet, N (Fig. 87), in such a position that its N pole does not appreciably repel the N pole of the needle.

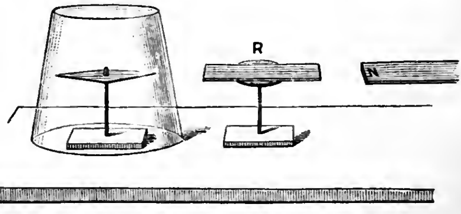


FIG. 87.

(*b*) Bring a soft iron rod, R, between the magnet and the needle. Observe that the N pole of the needle is immediately repelled.

We infer, therefore, from this experiment, that the end of the rod near the needle acquires N polarity under the influence of the magnet, the other end becoming, of course, S—i.e. *a magnetic pole induces opposite polarity in the end of an iron rod near to it, and similar polarity in the end remote from it.*

Exp. 90. Repeat the last experiment with two or three smaller iron rods between the needle and the magnet, and observe the repulsion.

We, therefore, learn that the inductive influence takes place through a series of iron rods.

Exp. 91. Obtain a number of small, wrought (soft) iron nails. Support one on the end of a strong magnet. Place another on the free end of this, and so on. This forms what is commonly called *magnetic chain* (Fig. 88).

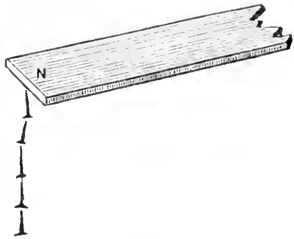


FIG. 88.

As will be understood from the figure, the N pole of the magnet induces S magnetism in the point of the first nail, and N in the head; this again induces S in the point of the next, and so on through the whole series.

Exp. 92. Show, by using a magnetic needle, that the polarity of the free end of the series is similar to that of the inducing pole.

Coercive Force and Retentivity.—There is a marked difference between steel and iron, with regard to (1) the difficulty of magnetisation, and (2) the retention of magnetisation. This difference may be easily shown by the following experiments :—

Exp. 93. Form a magnetic chain with pieces of well wrought (*i.e.* very soft) iron. Remove the magnet from the uppermost piece, and observe that the others fall away. Test one of the pieces by bringing it near both poles of a magnetic needle. There is no repulsion, and, therefore, the pieces of iron were *temporarily* magnetised.[1]

Exp. 94. Now form a chain with pieces of steel (*e.g.* steel pens). When the magnet is removed from the uppermost piece, the others do not drop off, *i.e.* steel is said to retain its magnetism *permanently*.

Exp. 95. Break a steel knitting-needle (four or five inches long) at the middle. Raise both pieces to a white heat. Plunge one in cold water to harden it, and allow the other to cool slowly in order to keep it soft. Dip both pieces in iron filings, and then bring a magnet in contact with the other ends. Notice that on withdrawal, the mass of filings attached to the hard iron is smaller than that attached to the soft iron. Remove the magnet, and observe that most of the filings adhere to the hard piece, but that they drop off the soft piece.

The student will notice that, under similar conditions, soft iron becomes more strongly magnetised than steel, but that it loses its magnetism more readily when disturbed or subjected to rough usage. It is usual to denote these properties by the terms *Coercive Force and Retentivity,* although they will not convey any real meaning to the student until he has read Chapter XXV.; in the meantime, it will be sufficient to define coercive force as a measure of the power of the material to resist changes of magnetisation; *e.g.* steel has a much greater coercive force than soft iron. Retentivity is a measure of the power of retaining magnetism, when the material is quite undisturbed and free from vibration or shock, and it may be greater for soft iron than for steel. It must also be borne in mind that any

[1] Generally there will be a slight action, as the iron would retain *some* magnetism, however feeble.

piece of iron, once magnetised, cannot be *completely* brought back to its original condition unless it is heated above redness. Otherwise it always retains a small amount of what is termed "residual magnetism."

Induction precedes Attraction.—We are now in a position to explain more clearly why mutual attraction takes place between a magnet and a magnetic substance. In Fig. 89 a small piece of soft iron is suspended by a thread. When the N pole of a magnet approaches the iron, induction is set up, the near side of the iron acquiring S polarity and the remote side, N. Attraction, therefore, takes place between the two opposite polarities and repulsion between similar polarities. The distance between the two opposite poles is, however, less than that between the two similar poles, and hence the attractive force overcomes the repulsive force.

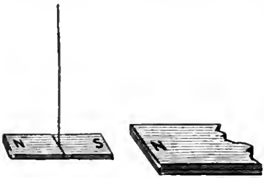


FIG. 89.

Influence of Medium.—**Exp. 96.** Magnetise a knitting-needle (two or three inches long), and suspend it horizontally by a fibre of raw silk. If moved from its position of rest, it will make a certain number of oscillations in a fixed time—say one minute. If, however, the N pole of a magnet be brought towards the S pole of the needle after the latter has been moved from rest, it will make a greater number of oscillations than before. Count the number of oscillations made in one minute, when a large sheet of glass, cardboard, a wooden board, or even the body is interposed between the needle and the magnet. The distance between the two magnets being constant in each case, observe that the number of oscillations are equal.

Exp. 97. Now interpose a large sheet of soft iron. Observe that the number of oscillations made by the needle in the same time is less than that in the last experiment. If the iron were quite soft and very thick, the number would approximate to that obtained when the needle oscillated under the earth's influence alone.

It appears, therefore, that magnetic force acts across all media, except iron and the other magnetic substances, and that they (or rather the ether which surrounds the molecules of the medium), directly transmit the force from one point to another.

The influence of the medium can be better shown by means of a reflecting magnetometer (see Experiment 110, p. 149).

Magnetisation by an Electric Current.—This method, although exceedingly important, will be merely mentioned here, as a knowledge of voltaic electricity is necessary before it can be rightly understood.

Exp. 98. Wind a spiral coil of copper wire round a bar of soft iron. The turns of wire must neither touch each other nor the bar. A good plan is to paste a layer of brown paper round the bar, and then, when it is dry, to wind cotton-covered wire upon it.

Connect both ends of the spiral to the terminals of a voltaic battery. Bring a piece of iron or steel to the bar, and observe that it is attracted and supported.

Magnetisation by Electro-magnets.—In the last experiment we made and used an **electro-magnet**, which merely consists of a core of soft iron, often of horse-shoe shape, round which a coil of insulated copper wire is wound. As we have learnt, the core is magnetised during the passage of an electric current round the coil. This produces very powerful magnets, and on this account they are frequently used to magnetise bars of steel.

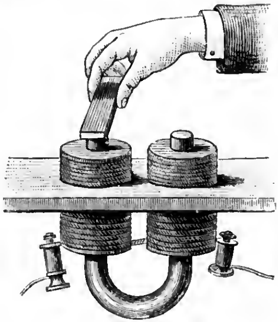


Fig. 90.

Exp. 99. Hold an electro-magnet upright, or, as is the common practice with magnet-makers, fix it in a board (Fig. 90). During the passage of the current, (1) move a steel bar from end to end across one pole of the electro-magnet; (2) move it in the opposite direction across the other pole. The reason of these movements will be understood from previous explanations.

Loss of Magnetisation.—Magnetisation may be destroyed or weakened under the following circumstances:—

(1) By arranging magnets, when not in use, with their similar poles adjacent. The tendency of each pole is to induce opposite polarity in the other, which, of course, weakens or destroys the original polarity (see p. 118).

(2) By the earth's induction. The tendency of the earth is to induce N magnetism in the lower end of the vertical or nearly vertical bar, so that if a magnet is placed with its S pole downwards, its polarity is weakened.

(3) By rough usage, either wilful or accidental. Such treatment, no doubt, disturbs the molecular arrangement described on p. 117.

(4) By making a magnet red-hot (see p. 423). If it be only slightly heated, it is weakened, but most of its original strength is regained on cooling.

Exp. 100, *to illustrate* (4). Suspend a long thin iron wire—about 15 inches of 20 gauge—by a bifilar suspension, *i.e.* by two cotton threads, so that it may swing end on (Fig 91). Support a magnet NS, on a suitable stand, B, and adjust the distance so that the wire, when it is swung forward, just adheres to the magnet. Place a small sheet of mica (which may be kept in place by a weight, W), on the end of the magnet, so that actual contact between the magnet and the wire is prevented. Now, place a Bunsen flame between the magnet and the wire, and notice that the latter oscillates. This is due to the fact that the end becomes non-magnetic in the flame, and the wire therefore falls away, although it cools sufficiently to become magnetic before the return swing, and is again attracted. It does not merely *lose* its magnetism; the iron becomes actually a non-magnetic substance above a certain temperature. It becomes a magnetic substance again when it cools, but is completely demagnetised.

Effects of Magnetisation.—(1) When an iron or steel bar is magnetised it becomes very slightly longer. This increment is very small, for even when a bar is magnetised to its maximum, it merely amounts to $\frac{1}{720,000}$ of its original length.

This observation is due to Joule, but later experiments of Bidwell have shown

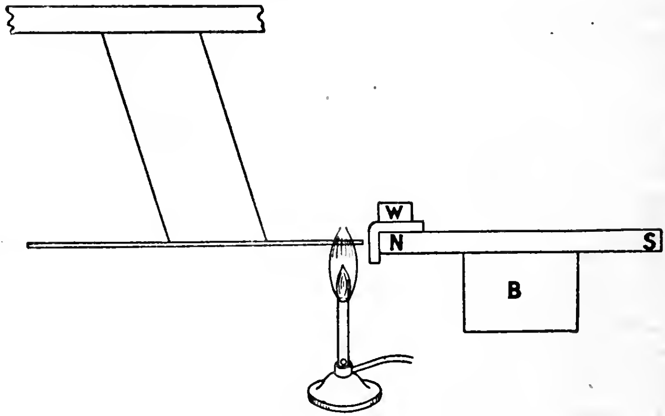


FIG. 91.

that the effect is somewhat complicated. Iron *at first* elongates, and then, with increasing magnetisation, contracts again until it is shorter than its original length. Cobalt first shortens and then lengthens, whereas nickel always shortens and never lengthens.

(2) Heat is produced when a bar is rapidly magnetised and demagnetised, apparently pointing to the conclusion that friction is set up between the molecules of the bar during magnetisation. Another explanation is given on p. 416.

(3) During magnetisation, a twisted iron bar tends to untwist itself.

CHAPTER XI

MAGNETIC FIELDS

Magnetic Field and Lines of Force.—The space surrounding a magnet through which its influence extends is called the *magnetic field* of that magnet. At every point in the field the magnetic force has a definite strength depending upon the distance from the poles; and it has a well-defined direction at every point, as indicated by what is called the *line of force* passing through the point.[1]

Exp. 101. Place a sheet of cardboard on a magnet. Sprinkle iron filings from a muslin bag over the cardboard. Gently tap the cardboard as the filings fall, and observe their arrangement along certain curves (Fig. 92.)

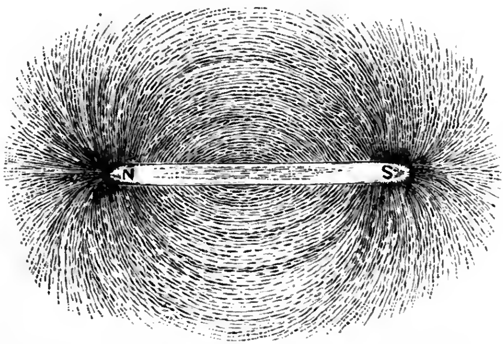


FIG. 92.—Arrangement about a Single Bar Magnet.

Each particle of iron assumes a definite direction, due to the action of both poles of the magnet. Tapping the cardboard merely facilitates the arrangement of the particles. The curves thus obtained may be taken to represent what are known as *magnetic lines of force*.

Exp. 102. Obtain the curves with the magnets arranged in various positions, *e.g.* as shown in Figs. 93 to 98.

These lines of force can be obtained in an interesting manner by

[1] The actual direction of the magnetic force at any point is a tangent to the *line of force* at that point.

means of a compass-needle pivoted in a small circular case (about $\frac{3}{5}$ of an inch in diameter).

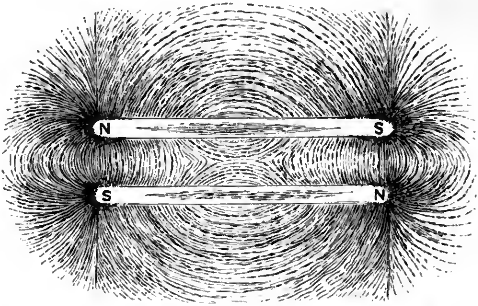


FIG. 93.—Arrangement about two Parallel Bar Magnets with their *Dissimilar Poles* adjacent.

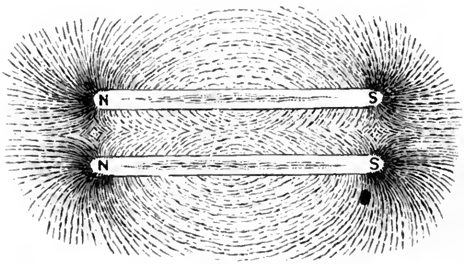


FIG. 94.—Arrangement about two Parallel Bar Magnets with their *Similar Poles* adjacent.

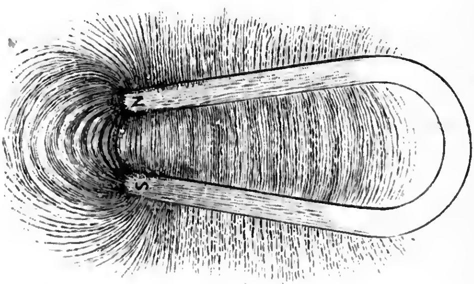


FIG. 95.—Arrangement about a Horseshoe Magnet.

Exp. 103. Place a bar magnet on a large sheet of drawing-paper fastened on a board. Rule lines round the magnet to show its position. Place a small compass-needle near one corner—say, the S pole of the magnet—and make pencil

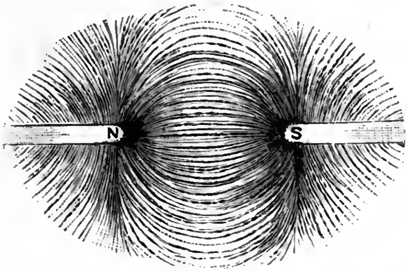


FIG. 96.—Arrangement about the *two Dissimilar Poles* of two Bar Magnets.

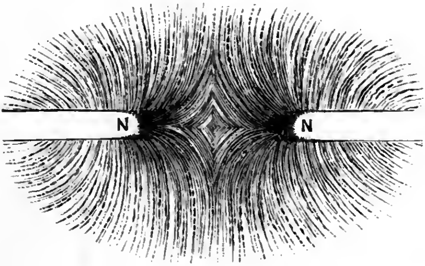


FIG. 97.—Arrangement about the *two Similar Poles* of two Bar Magnets.

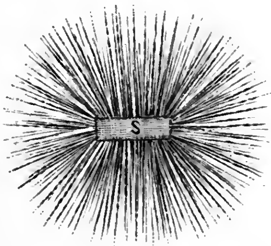


FIG. 98.—Arrangement about *One Pole* of a Bar Magnet.

dots exactly opposite the poles of the needle. Move it so that its N pole is on the S pole dot, and make another dot opposite its S pole. Continue in this way until a series of dots is obtained, and then join them by a freehand curve. This curve represents a line of force. In a similar manner other curves can be obtained until the whole field is mapped out.

Neutral Points.—If the student has a sufficiently large sheet of paper, he will find that, when the needle is removed from the neighbourhood of the magnet, the lines traced out by the compass needle are parallel. Such are the lines of force due to the earth. He will also notice that, in certain parts of the field, the lines of force due to the earth and those due to the magnet form irregular lozenge-shaped figures (Figs. 99–101), in which the compass-needle sets itself

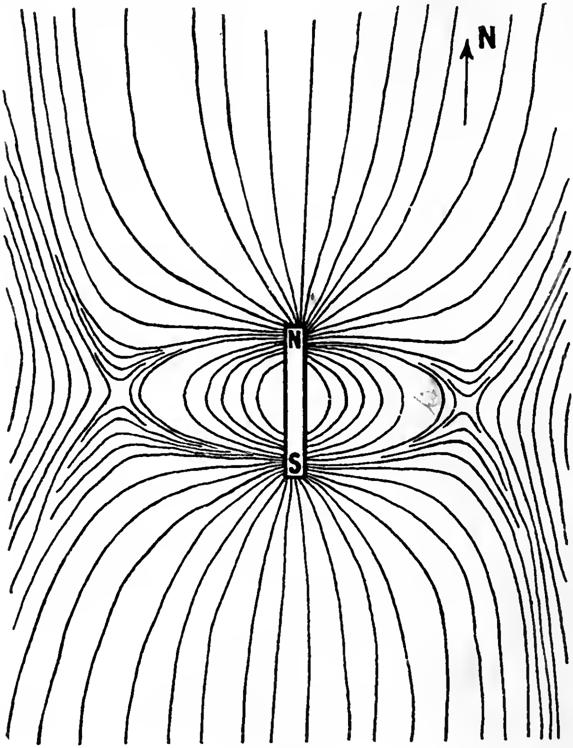


Fig. 99.

indifferently in any direction. These points are called *neutral points* or *points of zero force*, because the force due to the earth and that due to the magnet exactly balance each other. For an explanation, see pp. 132, 133.

Lines of Force through Magnetic Substances.—When the lines of force pass through a magnetic substance (p. 119), they crowd into the substance, *e.g.* if a soft iron ring be placed near one pole of a bar magnet, the lines of force arrange themselves as shown in Fig. 102. These lines were obtained in a manner similar to that described in Experiment 103. It will be observed that there are no lines of force inside the ring, *i.e.* a thick piece of iron acts as a magnetic screen. If, therefore, a compass-needle be placed inside a space enclosed by sufficiently thick iron, it is practically uninfluenced by the magnetic field in which it is placed.

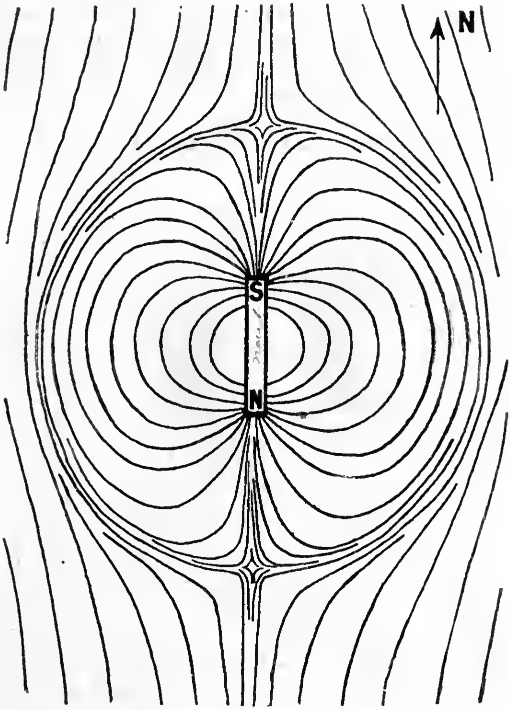


Fig. 100.

Properties of Lines of Force.—There are two fundamentally distinct methods of dealing with magnetic problems. The first, and oldest, was naturally suggested by the phenomena of attraction and repulsion exhibited by magnetic poles; it concentrates the student's attention more particularly on the poles themselves, which for purposes of mathematical treatment, although often in defiance of actual facts, have to be regarded as points; it seeks to determine the laws according to which such poles act on each other and to deduce their consequences. It does these things quite correctly, but it leaves untouched the vital question: How can bodies act upon each other without any tangible connection? The method is sound in principle, and is, for some purposes, the more useful, but it is only conveniently applicable to a limited class of problems.

The second method was initiated by Faraday, who clearly saw that the most important machinery of magnetism, from a practical point of view, lay in the space outside the magnet itself, and that the poles were more or less incidental circumstances. By introducing the idea of "lines of magnetic force" he showed how the properties of that space could be represented in a useful form. Clerk-Maxwell and others developed the method, which has become the basis of nearly all calculations in electrical engineering.

The patterns framed around magnets by iron filings have in them-

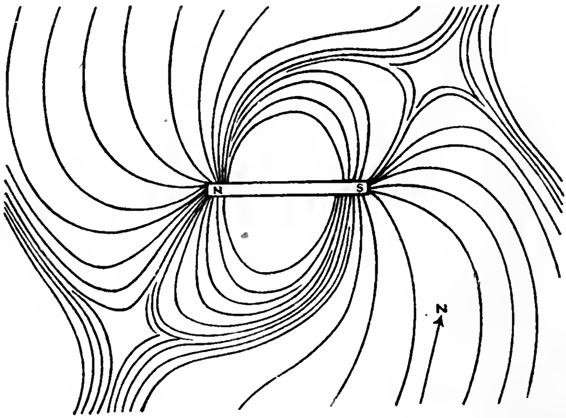


FIG. 101.

selves no special significance, and are easily explainable; the importance of the idea lies in perceiving that the facts of magnetism can be accurately represented by assuming that a magnet carries with it a number of "lines of force," which possess certain definite properties—the iron-filing figures merely enabling us to visualise their distribution (over one plane) in simple cases.

Their most important characteristics may be summarised as follows:—

1. The poles of a magnet are those regions where the lines of force emerge from the iron. (From this point of view, a pole is the analogue of a static charge, but whereas the lines of electric force *terminate* in a charge, we must suppose that the lines of magnetic force extend *through* the iron.)

2. Lines of force never cut one another; for if they did, the small magnet, used in tracing the lines, would tend to set itself in two directions at once at the point of intersection, which is impossible. Two lines from independent sources merge together to form a resultant line, which has *one* definite direction.

3. Lines of magnetic force always form closed curves.

4. Lines of magnetic force tend to contract in length, and to repel each other laterally.

5. A piece of iron, when placed in a magnetic field, becomes magnetised, and tends (*a*) to set its longer axis parallel to the lines of force, (*b*) to move into the strongest part of the field.

6. As shown in Fig. 102, iron (or other magnetic substance) causes

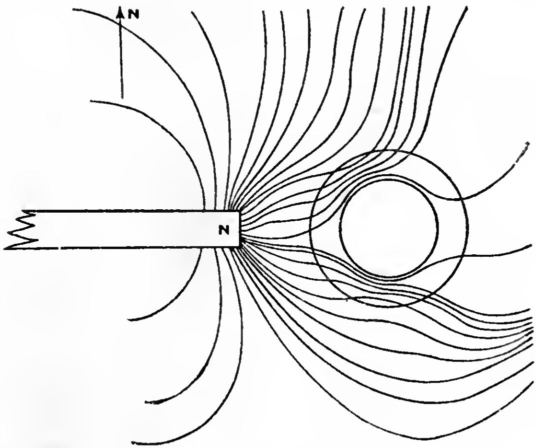


FIG. 102.

the lines of force to deviate from their normal path through air (or other non-magnetic substance). We may express this in a simple way by saying that the lines of force tend to pass through iron, wherever possible.

As we shall frequently discuss the properties of lines of force without reference to the poles from whence they emerge, it is necessary to adopt some convention to indicate their direction. The accepted custom is to mark them with arrows *pointing in the direction in which a single free N pole would tend to move.* It must be understood that these arrows do not suggest any motion of the line, and in fact do not have any physical meaning whatever. They are simply convenient aids to thought.

As an illustration, consider Fig. 99, p. 128. It represents certain facts, but no explanation has been given why the lines of force should have that particular configuration. Suppose, however, that we represent the field of the earth by a series of uniformly spaced straight lines. Then assuming the magnetic meridian to run vertically across the page, and remembering that a free N pole would tend to move northwards, we see that the direction of the earth's field must be marked with arrows pointing north, as in Fig. 103. On this set of lines is superposed another due to a bar magnet placed in the meridian with its N pole towards the north. If we draw a few of

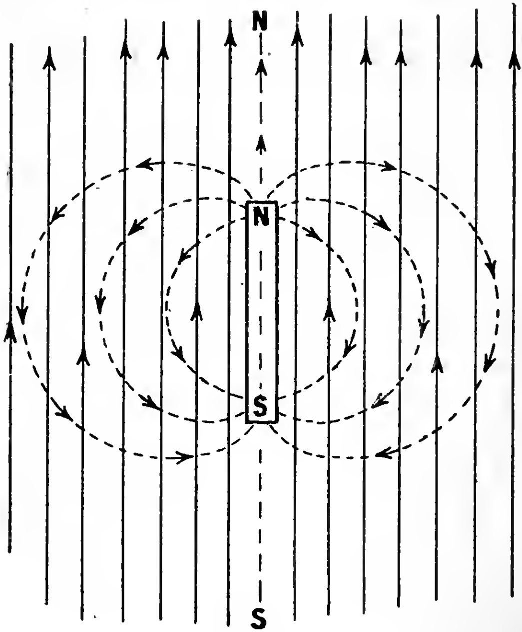


FIG. 103.

its lines and mark them according to rule (remembering that a free N pole would tend to move from a N pole towards a S pole), we get the result shown. From this we see that *both* sets of lines of force are in the same direction at all points on the axis of the magnet produced, so that along this line the field is stronger than before, whereas at all points on a line drawn through the centre of the magnet and at right angles to its axis, the two fields are in *opposite* directions, and therefore weaken each other. But whilst the strength of the earth's field is uniform, that of the magnet is variable, being stronger near the magnet than that of the earth, and rapidly becoming weaker as the distance from the magnet increases. Hence there will be some point at which the two fields are equal and opposite, and the resultant zero. These points are indicated by the small enclosed areas in Fig. 99.

Evidently, if the magnet were to be turned round, the direction of its lines would be reversed, which would make the two fields agree in direction at right angles to the axis, and interfere along the axis. This case is shown in Fig. 100, the neutral points being now on the axis of the magnet.

The student should draw a similar diagram to represent the state of affairs shown in Fig. 101, and he will then find the cause of the peculiar distribution obtained with that arrangement.

Strength of a Magnetic Field.—It is convenient to represent the strength of a field by the number of lines of force passing through a given area, *i.e.* the closer the lines are together, the stronger is the field. This method is defined quantitatively on p. 136.

It must, however, be borne in mind that this method of expressing numerically the strength of a magnetic field does not imply that the magnetism only exists along certain lines and that in the space between the lines there is no magnetism. The essential unseen forces are continuous throughout the field, and the lines are only diagrammatical devices for indicating their direction and magnitude. These remarks also apply to *electric* fields.

For instance, the strength of the earth's magnetic field at London is ·47 lines per square centimetre, which simply means that to represent it in terms of our units, we must draw, in the plane of the meridian, a set of parallel equidistant lines inclined at an angle of (about) 67° to the horizontal, and so spaced that there is one line to about 2 square centimetres of surface. *Everywhere* in this field, a unit magnetic pole would be subjected to a force of ·47 dyne tending to urge it in a direction parallel to the lines. If its constraints allow it to move only horizontally, it behaves as if acted upon by a horizontal force of ·18 dyne, and if only vertically, as if acted upon by a vertical force of ·44 dyne.

These numbers are known as the "horizontal" and "vertical" components of the magnetic force (or field); see p. 175.

Another conception, that of "tubes of force," may be used in order to fill up the total space (see p. 28), and this alternative method is usually discussed in text-books. We think, however, that it is less useful to beginners, and, therefore, we have not introduced it.

Exercise VII

1. Explain what is meant by a *line of magnetic force*.

Describe some experimental method of tracing such a line, and explain the principles upon which the method depends. (Camb. Local, Senior, 1900.)

2. A bar magnet is placed in the meridian with its north pole pointing north. Show that there are in general two points, on the line perpendicular

to the meridian passing through the centre of the magnet, at which the field due to the magnet is equal and opposite to the horizontal field of the earth. (Lond. Univ. Matric., 1907.)

3. A bar magnet is placed on a table. Describe how you would proceed to determine the directions of the lines of magnetic force in the plane of the table due to the magnet and to the earth's magnetism. (B. of E., 1901.)

4. A short bar magnet is placed at the centre of a circle three feet in diameter, its axis being in the magnetic meridian. Trace the changes in the direction in which a compass-needle points as it is carried round the circumference of the circle. (B. of E., 1902.)

5. A magnet is placed horizontally in the magnetic meridian due south of a compass-needle. How will the action on the latter be affected if (1) a plate of soft iron is interposed between the two, (2) a rod of soft iron is placed along the line which joins the centres? Give reasons. (B. of E., 1891.)

CHAPTER XII

MAGNETIC MEASUREMENTS

Any measurements of the forces between magnetic poles are necessarily complicated by the fact that we cannot isolate two poles for the purpose. Two magnets, each having two poles, must be used, and the actual effect depends upon all four poles. The simplest way to overcome this difficulty is to use long magnets, so that the two disturbing poles are removed some distance away, thus making their influence sufficiently small to be neglected in a rough experiment.

Coulomb adopted this plan in the earliest quantitative measurements of magnetic forces. He used the torsion balance method (already explained on p. 23 in connection with his work on electric forces. A long thin magnet was suspended horizontally from the torsion head by means of a fine wire, and another similar magnet replaced *i*, Fig. 22).

Without discussing the details of such experiments, it will be sufficient to say that Coulomb found that the attraction or repulsion between two small magnet poles varied directly as the product of their strengths, when the distance between them was constant; and inversely as the square of the distance between them, when the pole strengths were constant. Hence, if m and m_1 are the pole strengths, and d the distance between them, Coulomb's law may be written in the form

$$\text{Attraction or repulsion} \propto \frac{mm_1}{d^2}$$

It will be at once noticed that this expression resembles that already given on p. 24 for electric charges, and that in this case, limitations similar to those mentioned on that page are involved, *e.g.* the law applies only when the poles can be treated as points in comparison with the distance between them.

From this result, it follows that *unit magnetic pole* may be defined in the same manner as unit charge (although it must not be supposed that this is due to any similarity in their nature). Hence, a pole is of unit strength, when it attracts or repels an equal pole, placed at a distance from it of 1 centimetre in air, with a force of 1 dyne (both poles being mere points).

In accordance with this definition we may write

$$\text{Attraction (or repulsion)} = \frac{m \times m_1}{d^2} \text{ dynes}$$

When discussing the corresponding expression for electric charges, it was stated that the attraction or repulsion depended also upon the nature of the surrounding medium, whereas all ordinary substances (except iron and a few other markedly magnetic bodies) behave practically alike with regard to lines of magnetic force, and the attractions or repulsions are, therefore, independent of the nature of the surrounding medium. But when later, it becomes necessary to study more fully the magnetic properties of iron, it will be found that we have to distinguish carefully between the ideas conveyed by the terms *magnetising or magnetic force* and *magnetic field*, just as we had to distinguish between the ideas *electric force* and *electric field*. Definitions similar to those given in statical electricity will apply to magnetism, e.g. *magnetising or magnetic force* will be numerically the mechanical force in dynes on a unit pole placed in a magnetic field; and the *magnetic field* itself will be measured by the number of lines of force per square centimetre. The relation between these two quantities is written—

Strength of Magnetic Field $= \mu \times$ **Magnetising Force,**

where μ, called the "permeability" of the medium, is practically unity for all substances, except markedly magnetic ones. By common agreement, it has become usual to put B for *strength of field*, and H for *magnetising force*, so that we have

$$B = \mu H$$

Evidently, in air, B and H are numerically identical. The importance of the preceding statements will not be realised until the properties of iron are discussed (see Chapter XXV.).

Further, the reasoning already given for electric charges on p. 26 may be applied to magnetic poles by merely writing "m" in place of "q." We then find that the complete form of the law discovered by Coulomb is

$$\text{Attraction (or repulsion)} = \frac{n \times mm_1}{4\pi\mu d^2}$$

and, therefore, our definition of magnetic pole implies that unit pole has 4π lines of force emerging from it, and that μ is unity for air.

Similarly it may be deduced that the field strength at distance d from a point pole of strength m units is $\frac{m}{d^2}$ lines per square centimetre, and that the force on unit pole is $\frac{m}{\mu d^2}$ dynes. Hence, it follows that the force in dynes on unit pole is, in all practical cases, numerically identical with field strength. For this reason, confusion often arises as to the terms themselves. For instance, B and H are both vector quantities, and in non-magnetisable substances they always coincide in direction (although that is not always the case inside magnetic substances). But this numerical identity does not mean that B and H are identical in nature, and where necessary we shall point out to which of the two ideas reference is made.

The letter H is also used to denote the horizontal component of the earth's magnetic force at a given place, *i.e.* the horizontal component of the force in dynes on a unit pole at that place (see Chapter XIII.), and in the following pages we shall frequently use it in this sense, as there is no real danger of confusion arising in consequence of such usage. Hence, μH is, strictly speaking, the horizontal component of the earth's field.

Some writers use the term "Gauss" to indicate strength of field, *e.g.* a field of 50 lines per square centimetre is termed a field of strength 50 gausses. This term, although often used, has not received unanimous international sanction, and we do not think that it possesses any advantage over the word "line," which is universally adopted in practical calculations.

The Inverse-square Law.—Coulomb's method requires considerable experimental skill in order to obtain even roughly consistent results, and it is, therefore, unsuitable for ordinary purposes. By the nature of the case, *direct* experimental proof is difficult, because it is impossible to satisfy strictly the required conditions.

It is shown later that the best proof, and one capable of great accuracy, is an indirect one.

A Magnetic Balance.—Any experimental method, which is capable of yielding approximately correct results, is, however, instructive, and Mr. W. Hibbert has devised a form of magnetic balance,[1] which is very useful for this purpose (as well as for many others).

We have used a much less perfect form of balance, which involves the same principle and which is easily constructed as follows: Two long magnets are required, each $\frac{1}{4}$ inch in diameter and about 1 yard in length. One of them is fastened with copper wire to the edge of a metre scale (the wire passing through small holes drilled in it). The scale is then pivoted on a wooden stand by means of a pin passing through a hole at the middle—the arrangement being a modification of the one commonly employed for demonstrating the properties of levers. Two stops, A and B, Fig. 104, are fixed in some convenient position in order to limit the play of the lever, and to facilitate the adjustment; a small weight, W (from 2 to 5 grams), is suspended by a fine thread on one arm; and one end of the second long magnet is brought underneath the end of the first, as at S.

[1] See *Magnetism*, by W. Hibbert, published by Messrs. Longmans.

Exp. 104. Place the second magnet either endwise on, or at right angles to the first, as may be the more convenient, but take care to ensure that the points, which may be regarded as the effective poles, are in the same vertical line. This can be done by adjusting the position of the second magnet until the best effect is obtained.

The points in question will probably be an inch or more from the ends.

When this is done, note the position of the sliding weight giving balance for as many distances between the two poles as is practicable. (It is not desirable to attempt adjusting these distances to any *definite* value—it is far better to take them as they come and to measure the distances as exactly as possible after getting balance.)

Now, if p is the force between the magnet poles; l_1 the distance

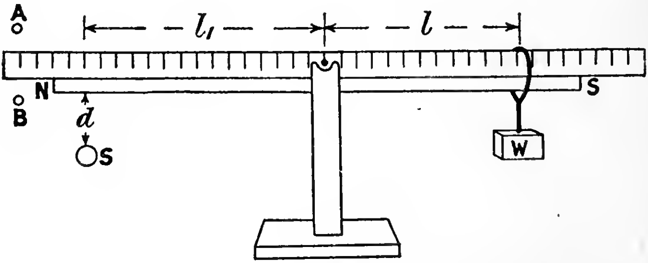


FIG 104.

of the pole from the fulcrum; W, mass of weight in grams; and l, its distance from the fulcrum; then, taking moments about the fulcrum,

$$p \times l_1 = W \times 981 \times l$$

$$\text{or} \quad p = \frac{W \times 981 \times l}{l_1} \text{ dynes.}$$

Hence, the force between the magnet poles is directly measured for some distance d between them, the other two poles being so far away as to exert no serious disturbing action.

Again, we notice that W and l_1 are constant, and, therefore, $p \propto l$. Hence, if $p \propto \frac{1}{d^2}$, we have $l \propto \frac{1}{d^2}$, or $ld^2 = $ a constant. A rough test might be applied by multiplying together these values, but for the purpose of determining the law connecting the force with the distance, it is more convenient to proceed as follows:—

Assume $l \propto \frac{1}{d^n}$, where n is unknown,

$$\text{then} \quad ld^n = \text{constant.}$$

Taking logs, we have

$$\log l + n \log d = \text{another constant.}$$

If the values of log l be taken as ordinates, and those of log d as abscissæ, and plotted on squared paper, the result will be a straight line as shown in Fig. 105, and we know by elementary co-ordinate geometry that, in the above equation, n is the *tangent of the angle of slope*—*i.e.* $\frac{Ao}{oB}=n$.

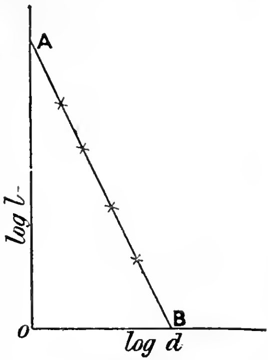


FIG. 105.

Hence, if the experimental results give an approximately straight line, we may infer that the force varies inversely as some power of the distance, and by measuring Ao and oB, we can determine the probable value of that power.

The student will readily obtain such a line, but it is very doubtful if he will find $\frac{Ao}{oB}=2$. It is much more probable that he will find that the ratio lies between 1 and 2. This arises from the fact that the poles are not points, and, therefore, the inverse-square law does not hold good over the short distances used in the experiment.

Robison Magnets.—When the ends of a long thin magnet are fitted with spherical steel balls, the poles are much more definitely localised, and may be taken without much error as being at the centres of the balls. If such ball-ended magnets are used, a much closer approximation to the inverse-square law will be obtained. This form of magnet was devised many years ago by Robison, but its peculiar properties have only recently been brought into notice by Dr. G. F. C. Searle of Cambridge.

Other experiments bearing on the inverse-square law are given on pp. 150 and 158.

Determination of the Field Strength at a given Point near a Bar Magnet, taking the Effect of both Poles into Account.—Consider any point o (Fig. 106), at distances d and d_1 from the two poles of a magnet respectively. Then, if m be the strength of each pole, the field at o due to the N pole will be $\frac{m}{d^2}$, and will act along No, and the field due to the S pole will be $\frac{m}{d_1{}^2}$, and will act along oS (these directions being taken in accordance with our convention, *i.e.* the direction in which a free N pole would tend to move). The actual field at o will be the

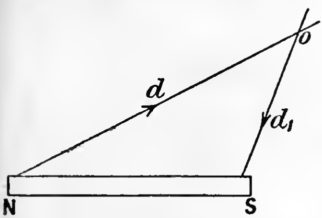


FIG. 106.

resultant of these two components, but it is not necessary here to show how an expression for it can be obtained. When, however, the point o is on the axis of the magnet, or when it is on the line at right angles to the magnet and passing through its centre, it is easy to determine the resultant field, and it will be sufficient to consider these two cases.

CASE I.—*When the point is on the axis of the magnet produced.*

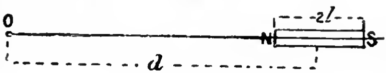


FIG. 107.

Let o (Fig. 107) be the point; NS the magnet, of which the length is $2l$ centimetres, and the strength of each pole m units; d, the distance between the point o and the *middle* of the magnet;

$$\text{then } oN = d - l$$

$$\text{and } oS = d + l$$

$$\text{Now the field at } o \text{ due to N} = \frac{m}{(d-l)^2}$$

$$\text{and } \quad ,, \quad ,, \quad ,, \quad S = \frac{m}{(d+l)^2}$$

$$\therefore \quad \text{the resultant field} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

$$= \frac{m(d+l)^2 - m(d-l)^2}{(d^2-l^2)^2}$$

$$= \frac{m \, . \, 4dl}{(d^2-l^2)^2}$$

We must at this point introduce a new and important definition—

The moment of a magnet *is the product of the strength of one of its poles, and the distance between them*—*i.e.* the moment (M) of the magnet $= m \times 2l$ ($2l$ being the length of the magnet).

$$\therefore \text{ the resultant field at } o = \frac{2M \, . \, d}{(d^2-l^2)^2}$$

Now, if half the strength (l) of the magnet be very small compared with d, l^2 may be neglected without making any appreciable difference, so that we, then, have

$$\text{the resultant field at } o = \frac{2Md}{d^4} = \frac{2M}{d^3}$$

Hence, the field strength at any point on the axis, whose distance is great compared with the length of the magnet, varies directly as its moment (which is a perfectly definite quantity) and inversely as the *cube* of its distance from the centre of the magnet.

If the point is fairly near the magnet, the unsimplified form of the expression must be used.

CASE II.—*When the point is on the straight line at right angles to the magnet and passing through its centre.*

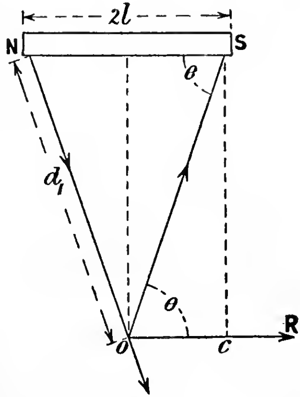


FIG. 108.

In this case, the poles are equidistant from the point o (Fig. 108). Let d_1 be this distance, so that $d_1 = o\mathrm{N} = o\mathrm{S}$. Each pole produces at the point o, a field of magnitude $\frac{m}{d_1^2}$, acting respectively along No produced and oS.

As the two fields are equal, their resultant must act along the line bisecting the angle between them, *i.e.* along oR.

Draw Sc perpendicular to oR, then by the properties of the triangle of vectors, the component along oR due to the S pole is found by writing

$$\frac{\text{Component along } o\mathrm{R}}{\text{Field along } o\mathrm{S}} = \frac{o\mathrm{C}}{o\mathrm{S}} = \cos\theta$$

$$\therefore \text{ Component due to S pole along } o\mathrm{R} = \frac{m}{d_1^2}\cos\theta$$

To this must be added the component due to the N pole acting along No produced, which evidently has the same magnitude and direction.

$$\therefore \text{ Resultant field at } o = \frac{2m}{d_1^2}\cos\theta$$

$$\text{Again, the figure shows that } \cos\theta = \frac{l}{d_1}$$

$$\therefore \text{ Field at } o = \frac{2m}{d_1^2} \times \frac{l}{d_1} = \frac{2ml}{d_1^3}$$

but the moment (M) of the magnet is $2ml$

$$\therefore \text{ Field at } o = \frac{\mathrm{M}}{d_1^3}$$

Hence, the field at o varies inversely as the *cube of the slant distance* from either pole.

If d be the direct distance from o to the centre of the magnet, then, when d is great compared with l, we may write $d_1 = d$ for a first approximation. For points too close to the magnet to satisfy this condition, we must write $d_1 = \sqrt{d^2 + l^2} = (d^2 + l^2)^{\frac{1}{2}}$.

$$\therefore \text{ Field at } o = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$$

Experimental methods of verifying these results are given later (see pp. 149 and 159).

Deflection of a Compass-Needle by a Magnetic Field.—A freely suspended, or pivoted, magnet, *e.g.* a compass-needle, when left to itself, comes to rest in the magnetic meridian under the influence of the horizontal component of the earth's magnetic field (see p. 175). Let H denote the strength of this field.[1] If another entirely distinct magnetic field inclined to that of the earth be produced by any means, *e.g.* by a magnet or by an electric current, then the two fields merge into a resultant field inclined to the direction of both.

Consider the simple case in which the second field is at right angles to the direction of H.

Let F be the strength of this second field, assumed to be uniform throughout the space occupied by the compass-needle. Then as F and H are both vectors, their resultant is given in magnitude and direction by oR in Fig. 109, and the needle will be deflected through an angle θ to point along oR. It is at once evident from the figure that

$$F = H \tan \theta$$

and that the angle of deflection is independent of the strength of the needle. This gives a method of measuring F when H is known.

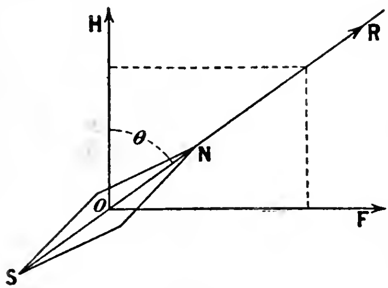


Fig. 109.

We have regarded F and H as *field* strengths, but because $\mu = 1$ for air, this expression holds good, and is the same numerically, if we consider F and H to be the *forces* acting on unit pole.

Application to Previous Theorems.—I. Let a magnet of moment M be placed *at right angles to the magnetic meridian.* Consider any point o on its axis produced, and at distance d from its centre (d being great in comparison with the length of the magnet). Then the field at o is at right angles to the horizontal

[1] Strictly, the field is μH.

component of the earth's field. At this point, let a *short* compass-needle be placed (if the needle is a *long* one, it will extend into regions off the axis to which our results do not apply). The needle will be deflected from the meridian through some angle θ, such that

$F = H \tan \theta$, where F is the field due to the magnet.

Now it has been shown that $F = \frac{2M}{d^3}$

$$\therefore \frac{2M}{d^3} = H \tan \theta$$

This is known as the **end-on position**, or the **A position of Gauss.**

II. Let the point o be taken on the line at right angles to the magnet and passing through its centre at a relatively great distance d from its centre.

The field due to the magnet at this point is also at right angles to the earth's field, and we have, by a repetition of the above argument,

$$\frac{M}{d^3} = H \tan \theta$$

This is known as the **broadside-on position**, or the **B position of Gauss.**

As before, in these expressions, the numerical values are the same whether we regard H as *field* or *force*.[1]

In order to make use of these results, some form of small compass-needle, arranged to facilitate exact measurements of θ, is required.

Such an instrument is known as a magnetometer. A simple form may be made as follows :—

To make a Magnetometer.—(1) The box, A (Fig. 110), is made by gluing together four strips of wood—$4\frac{1}{4}$ inches long, and $1\frac{1}{2}$ inches high. The sides are then glued to the bottom, which consists of a square piece of looking-glass. Small cubes of wood should be glued in the top corners of the box, so that a square of window-glass may rest upon them to form the cover of the box.

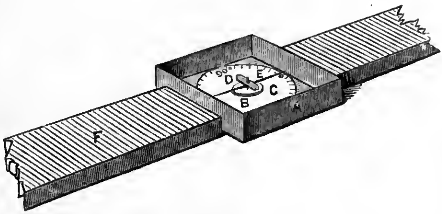


FIG. 110.

(2) Glue a small flat cork, B, at the centre of the looking-glass, and insert a fine needle, point upwards, into it. The needle must be fixed with great accuracy at the centre of the box.

---

[1] It is instructive to notice that if we write $\frac{M}{d^3} = \mu H \tan \theta$, then each of the terms, $\frac{M}{d^3}$, μH, means "field"; and if we write $\frac{M}{\mu d^3} = H \tan \theta$, each means "force.

(3) Cut a piece of watch-spring to form a small needle (about 1·5 centimetres long) of the shape shown in Fig. 111. Drill a small hole through the centre of the needle. Make and fix a small glass cap in the hole, so that it is perpendicular to the needle, as explained on p. 114. Glue a fine pointer at right angles to the needle —this may be made of any light rigid body; those used by the writer are very fine glass fibres, made by heating glass rod and then drawing it out.

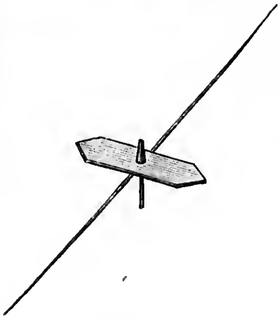


FIG. 111.

(4) Make a graduated scale as follows: Construct a circle of 2-inch radius on paper, and divide the circumference into 1-degree spaces. Remove the central part of the paper, so that a ring, a $\frac{1}{4}$ inch wide, is left. Glue this carefully to the bottom of the box. (A printed scale is readily obtainable and should be used if possible.)

(5) Take a piece of wood, 4 feet long, $2\frac{1}{2}$ inches wide, and about $\frac{3}{4}$ inch thick, and cut a square groove at the middle to hold the box, A (Fig 110). The outstanding portions, F, may be called the *arms*.

(6) Gum a strip of paper on each arm, and then graduate them in centimetres, making the zero under the centre of the box, and graduating outwards.

The great disadvantage of this type of instrument is due to the friction at the pivot, which makes it relatively insensitive. It is not difficult, however, to modify the design in such a way that the needle is suspended by a silk fibre. In fact, a galvanometer in which such a needle is used, *e.g.* a tangent galvanometer, or the simple form described on p. 282, makes an excellent magnetometer for many purposes of demonstration.

Exp. 105, *to magnetise two pieces of steel to the same pole strength.* Cut two pieces of clock-spring, each piece being, say, $8\frac{1}{2}$ centimetres long. Thoroughly harden them, and, placing them side to side, magnetise them together. If this is done carefully the pole strength of the two magnets will be equal. To test them, arrange the magnetometer so that, when the pointers are at zero, the arms are in the magnetic meridian. Place one of the magnetised pieces across one arm, so that its centre is on the middle line. Read the angles at both ends of the pointer—suppose they are $10\frac{1}{2}°$ and 11°. Reverse the poles of the magnet, and repeat these observations—suppose the angles are 11° and $11\frac{1}{2}°$. Take the mean of the four readings—in this case 11°—which gives the true deflection.

Repeat these operations with the other magnet. If the mean of the four readings is the same as before, the magnets are of equal pole strength.

If they are found to vary, magnetise the weaker one until the deflections are equal.

Exp. 106, *to prove that the force exerted by a bar magnet does not depend merely upon its pole strength, but also upon its length,* i.e. *the force is proportional to the magnetic moment of the magnet.*

(1) In the last experiment, we found that the mean of the four deflections for each magnet was 11°.

(2) Now place the two magnets end to end, with their opposite poles together —of course, the distance being the same as in (1). Again take the mean of the four readings. This, in an actual experiment with the two pieces of magnetised steel, was 21°.

Both the deflections in (1) and (2) are produced by magnets of the same pole strength; the greater deflection in (2) must therefore be produced by the greater length of the magnet.

Exp. 107, *to find the moment of a magnet by the A position of Gauss.* Arrange the magnetometer so that the arms are at right angles to the meridian, and the pointer at zero.

(*a*) Place a short magnet on the arm which lies towards the east, and observe the exact distance (which must be great compared with half the length of the magnet) between the middle of the magnet and the point of suspension of the needle.

(1) Let the N pole lie towards the needle, and read the deflections at both ends.

(2) Reverse the magnet so that the S pole lies towards the needle, and again read the deflections.

(*b*) Now place the magnet, at the same distance as before, on the arm which lies towards the west. Repeat (1) and (2).

(*c*) Take the mean of the eight readings. This gives the true deflection.

(*d*) Repeat the eight observations at a different distance, and then apply the formula given on p. 143.

$$M = \frac{H \cdot d^3 \tan \delta}{2}$$

In a particular experiment the following results were given with a magnet 15 centimetres long:—

Distance between Centres.	Position of Magnet.	Deflections.		Mean Deflection.	Natural Tangent of Mean Deflection.	Value of M.
38 centimetres	E 1	$24\frac{1}{2}$	25	23·5	·4348	2147
	E 2	23	$22\frac{1}{2}$			
	W 1	$21\frac{1}{2}$	22			
	W 2	$24\frac{1}{2}$	25			
35 centimetres	E 1	30	30	29·375	·5628	2171
	E 2	31	$30\frac{1}{2}$			
	W 1	$26\frac{1}{2}$	27			
	W 2	30	30			

Experiment 107 must be regarded as giving only an approximate value. In the first place, d is only about five times greater than l, and the use of the simplified formula introduces an error of about 10 per cent. In the second place, it assumes that the value of H is known. It will be shown later how this may be measured, but if, in the meantime, the accepted value for *the open air* be used, a serious error may be introduced, for sometimes the value is widely different inside a building from that outside, and indeed it often varies at different points in the same room. This is due to the presence of iron used in the construction of the building.

It is, however, easy to *compare the moments* of two magnets without knowing the value of H. Either the A or the B position may be used, although the former is mentioned in the next experiment.

Exp. 108. Arrange the magnetometer for the A position of Gauss.

(*a*) Take the mean of the eight readings mentioned in Experiment 107, with one magnet, which we will call A. Suppose that it is 20° 30′.

(*b*) Take the mean of the eight readings with the other magnet, which we will call B, with its centre in *exactly* the same position. Suppose that it is 8° 15′.

$$\text{Then } \frac{\text{Moment of A}}{\text{Moment of B}} = \frac{\tan 20° 30'}{\tan 8° 15'} = \frac{\cdot 374}{\cdot 145}$$

$$= \frac{2 \cdot 6}{1} \text{ nearly.}$$

Another method of comparison may also be indicated :—

Exp. 109. Place the two magnets, whose moments are to be compared, on opposite sides of the magnetometer—in either A or B position—and adjust their distances until the deflection is zero.

Evidently the fields due to the two magnets are equal and opposite, and if d and d_1 be the distances corresponding to the moments M and M_1 respectively, we have

$$\text{for A position } \frac{2M}{d^3} = \frac{2M_1}{d_1^3}$$

$$\text{or, for B position } \frac{M}{d^3} = \frac{M_1}{d_1^3}$$

which, for both positions, becomes

$$\frac{M}{M_1} = \frac{d^3}{d_1^3}$$

(It will be understood that, as the distances have to be cubed, a small experimental error will cause a relatively large error in the final result.) On p. 165 another important method of comparing the moments of two magnets is explained by means of a worked example.

Reflecting Magnetometer.—An enormous gain in sensitiveness and accuracy is obtained by applying the reflecting principle to magnetometers, and such instruments are always used in actual practice. Probably the most useful form for teaching purposes is one similar to that shown in Fig. 112 (for which we are indebted to Messrs. J. J. Griffin & Co.). A small concave mirror of about 1 metre radius of curvature and $\frac{3}{8}$-inch diameter is suspended by a silk fibre in a strip of wood *ab*, slotted out to receive it, the height being adjusted by a brass screw *d*, to which the upper end of the fibre is attached. The cavity in which the mirror

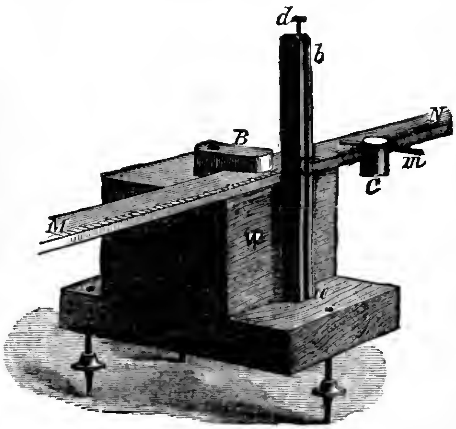


FIG. 112.

swings is closed by a sheet of glass, and is quite small—only slightly wider than the mirror itself, and not deep enough to allow the latter to turn completely round. In this way (*a*) the fibre is kept from twisting, and (*b*) the oscillations of the mirror are partly checked or "damped" by air friction, so that it comes to rest more quickly.

It is usual, as shown in Fig. 113, to attach to the back of the mirror one or more very small and light magnets made from watch-spring. This construction works very well, but such short strips tend to lose their magnetism readily, for reasons explained on p. 117. Longer strips could be magnetised more strongly, and would retain their magnetism better, but it is especially desirable to have the poles close together, so that the arrangement may be regarded as a point. Both conditions are satisfied by using the horse-shoe type, a small and nearly circular magnet being made from a fine knitting-needle, and magnetised by passing a current through a temporary winding of copper wire. This is attached to the back of the mirror, as shown in Fig. 114.[1]

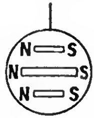


FIG. 113.

The whole arrangement is fixed to a solid wooden block, W (Fig. 112), provided with levelling screws, and carrying at the back a small clamp, B, suitable for holding an ordinary metre scale, MN. In many experiments, short magnets made from knitting-needles are used, which are held in position by a wooden slider, C.

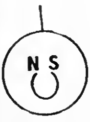


FIG. 114.

Evidently, the instrument is best adapted for working with the B position, because in the A position the deflecting magnets would sometimes have to be placed between the mirror and the scale, which would be inconvenient.

A temporary wooden support of any desired length may, however, be arranged at the back of the instrument in order to demonstrate the properties of the A position; indeed, such supports are often useful in either position when heavy magnets are employed.

A simple form of lamp and scale to use with this instrument is shown in Fig. 115, for which (and for Fig. 116) we are indebted to Mr. R. W. Paul.

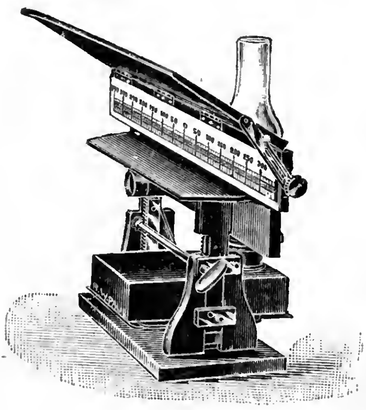


FIG. 115.

The light from an oil lamp, having its wick endwise on, passes through a convex lens of about 4 inches focal length (in front of

[1] This type of magnet appears to have been first suggested by Mr. W. Hibbert.

which is a vertical wire), then falls upon the magnetometer mirror, and is reflected back to the scale. In use, the stand is first placed in such a position that the distance between scale and mirror is equal to the radius of curvature of the latter (usually about a metre). Then, after raising or lowering the lamp to a suitable height, its horizontal distance from the lens is adjusted until a sharp inverted image of the flame falls upon the mirror. This gives the maximum illumination. A real image of the surface of the lens, and of the wire crossing it, is formed by the mirror on the scale, the result being a circular disc of light crossed by a fine vertical dark line, from which the readings are taken. If this image does not fall on the scale at the first trial, search must be made for it with a piece of white paper, and then the height of the stand or lamp must be altered to correspond.

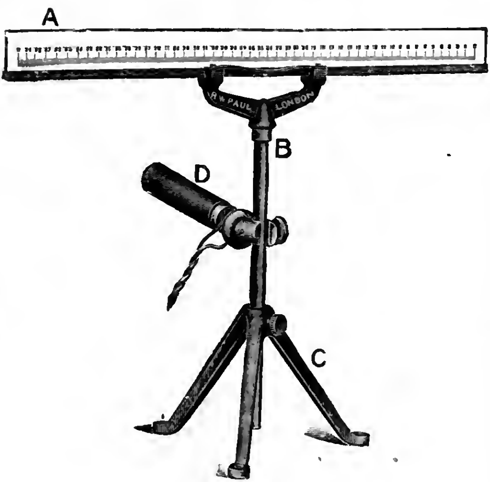


Fig 116.

A more recent type of lamp and scale is shown in Fig. 116. In this pattern the source of light is a small electric lamp contained in the tube D, which is fitted with a convex lens at the end. The scale itself is transparent, and is read from behind by transmitted light, the spot of light being visible without requiring a darkened room.

When the mirror moves through an angle θ, the spot of light moves through an angle 2θ; hence, if s (Fig. 117) is the deflection as read on the scale, and l the distance of the scale from the mirror, we have $\frac{s}{l} = \tan 2\theta$.

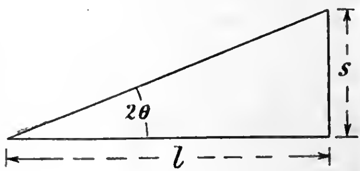


Fig. 117.

In ordinary cases, s is so small compared to l, that we may write $\tan\theta = \frac{1}{2}\cdot\frac{s}{l}$, without sensible error, and for comparative purposes we may therefore regard the scale reading as proportional to the tangent of the angle of deflection. As a further consequence of the smallness of θ, in many cases it will be sufficiently accurate to regard θ as identical with its tangent.

Experiments with Reflecting Magnetometer.—Exp. 110, *to show the influence of medium* (see p. 122). Arrange a reflecting magnetometer so that the mirror can swing in the magnetic field of the earth. Raise the mirror by slightly turning the screw, and level until it moves freely. Adjust the scale and lamp until the spot of light and the dark central line are well defined. Arrange a suitable support, on which a bar magnet can be placed in a horizontal position about level with the mirror, and preferably due north or south of it. Place the magnet at right angles (*i.e.* Gauss's B position), and move it gradually nearer until a convenient deflection is obtained. Between the magnet and the magnetometer hold various articles, *e.g.* a drawing board, a plate of glass, a brick, or the like. Observe that no alteration in the deflection can be seen. Now, between them, place a sheet of iron, and observe that the deflection is decreased, but that it returns to its old value when the iron is removed. This suggests that the iron puts some impediment in the path of the lines of force, but the effect really depends upon the fact that the lines pass more readily through iron than through air (see Fig. 102). The iron facilitates the return of the lines to the opposite pole and cuts off part of them from the space beyond.

Exp. 111, *to compare the moments of two magnets.* For this purpose two small knitting-needle magnets may be used. Let the moments be M and M_1 respectively. Place one of them in the holder C, taking care that the centre is in a line with the mirror, and adjust it until a convenient deflection is obtained. Read this deflection. Turn the needle round, and read the new deflection on the other side of the scale. Place the needle at exactly the same distance on the other side of the instrument, and obtain two more readings. Let s be the mean value of the four readings.

Repeat the observations with the second magnet, and let s_1 be the mean of the four readings.

Now, for the B position,

$$\frac{M}{d^3} = H \tan \theta$$

$$\text{or} \quad M = d^3 H \tan \theta$$

$$\text{and} \quad M_1 = d^3 H \tan \theta_1$$

$$\therefore \quad \frac{M}{M_1} = \frac{\tan \theta}{\tan \theta_1}$$

But, as already stated, in this case $\dfrac{\tan \theta}{\tan \theta_1} = \dfrac{s}{s_1}$

$$\therefore \quad \frac{M}{M_1} = \frac{s}{s_1}$$

The moments of large magnets may be similarly compared with great accuracy, using suitable supports in place of the metre scale.

Exp. 112, *to verify the inverse-cube law.* For this purpose it is desirable to work with a greater range of distance than the metre scale permits, and it is better to arrange a temporary support for the magnet to be used. It will be sufficient to obtain readings on one side only.

Begin by placing the magnet (using B or A position) as far away as possible, consistent with obtaining a readable deflection. Turn the magnet round and read the deflection again. Take the mean. Repeat observations at various distances, tabulating the values of the deflections and of the distances.

It will be found that as long as the distance is great compared with the length of the magnet, *halving* the distance gives *eight* times the deflection.

The law can, however, be more clearly brought out by employing the method already described in connection with the inverse-square law.

For, if $F \propto \frac{1}{d^3}$, where F is the field strength at distance d,

$$\text{then } Fd^3 = \text{a constant.}$$

$$\text{But } F \propto s \quad \therefore \quad s.d^3 = \text{a constant,}$$

$$\therefore \quad \log s + 3 \log d = \text{another constant.}$$

Hence, plot a graph with the values of $\log s$ for ordinates, and of $\log d$ for abscissæ. This will be a straight line if F varies as some power of the distance, and the tangent of the angle of slope will be that power.

It will be found that the ratio $\frac{Ao}{oB}$ (Fig. 105) is 3, and there will be no difficulty in obtaining very consistent results, thus showing that the inverse-cube law expresses the facts with great accuracy.

The Inverse-square Law for Point Poles.—We have already stated that direct experimental proof is not capable of any great accuracy. The results obtained on pp. 140 and 141, however, lead to a much more exact, but indirect, method of verification.

Instead of assuming its truth, as we did before, let us write $F = \frac{m}{d^n}$ for the law of action, where n is unknown, and again find expressions for the field strength at any point O on the axis, and also on the line at right angles to the magnet.

In the first case, we have

$$\text{Field} = \frac{m}{(d-l)^n} - \frac{m}{(d+l)^n} = m\{(d-l)^{-n} - (d+l)^{-n}\}$$

Expanding binomially, this becomes

$$\text{Field} = m\{(d^{-n} + nd^{-(n+1)}l + \dots) - (d^{-n} - nd^{-(n+1)}l + \dots)\}$$

which, neglecting terms involving l^2 and higher powers, becomes

$$\text{Field} = \frac{2mnl}{d^{n+1}} = \frac{n\text{M}}{d^{n+1}} \qquad (1)$$

In the second case, we have

$$\text{Field} = \frac{2m}{(d^2+l^2)^{\frac{n}{2}}} \times \cos\theta = \frac{2m}{(d^2+l^2)^{\frac{n}{2}}} \times \frac{l}{(d^2+l^2)^{\frac{1}{2}}}$$

$$= \frac{2ml}{(d^2+l^2)^{\frac{n+1}{2}}}$$

which, when l is small compared with d, becomes

$$\text{Field} = \frac{\text{M}}{d^{n+1}} \qquad (2)$$

The results (1) and (2) show that if the law of action for a single pole varies inversely as the nth power of the distance, then the field at a relatively great distance from the magnet is n times stronger at a point on the axis than at a point at the same distance, but on the line passing through the centre and at right angles to the magnet. Hence, to determine the value of n, we have only to compare the field strengths at two such points, equidistant from the centre of the magnet, and this can be done with great accuracy in several ways. For instance, a reflecting magnetometer may be arranged so that a bar magnet can be placed in either A or B position with reference to it, and the deflection noted for equal distances. As a result, n will be found equal to 2, within a range of experimental error which will be much less than for the method previously given. This is the best method of verifying the inverse-square law.

Measurement of the Temperature Coefficient of a Magnet.—When a magnet is slightly heated, its moment varies with the temperature—decreasing uniformly as the temperature is raised, and increasing as the temperature is lowered. We shall now give a method of finding the relation which exists between the temperature and the moment, for a moderate range of temperature.

Exp. 113. It is convenient to use a steel bar magnet about 6 inches long—any shape, however, may be used, provided that some method is devised by which it can be heated to various temperatures up to (say) 100° C. without disturbing its position. Place it in a simple triangular trough made of zinc, or of any other non-magnetic substance, and supported on brass legs. Arrange the trough so that the magnet is in either the A or the B position—usually the latter is the more convenient—with respect to the reflecting magnetometer. Fill the trough with water and adjust the distance (which should be fairly great) until a convenient deflection is obtained. Then heat with a Bunsen flame. Notice that, as the temperature is raised, the deflection slightly decreases. Take a number of readings both of temperature and of deflection between 0° C. and 100° C. Remove the burner, and take another series of readings as the magnet cools. Notice that the deflection increases again; although it is quite possible that its original value will not be reached.

Experience has shown that in order to secure consistent results in these measurements, the magnet must be carried through several cycles of alternate heating and cooling, a process which appears to bring it gradually into a definite physical state.

It will also be found that the amount of variation with temperature changes considerably with different magnets; for those of poor quality it is sometimes quite large, but for those of good quality it is small.

Plot graphs for both rise and fall of temperature, taking deflections as ordinates and temperatures as abscissæ. The graphs will show that the change is linear, and hence the results can be expressed by an equation of the form

$$M_t = M_o(1 - at),$$

where M_t is the moment at any temperature t° C.; M_o, the moment at 0° C.; and a, the temperature coefficient of the magnet, *i.e.* the change of moment per degree centigrade. From the graph, obtain the deflections corresponding to 0° C. and 100° C. respectively. We know that these are proportional to M_o and M_{100}. Insert them in the above equation and calculate the values of a (which from the form of the equation, depends only on the ratio of M_t to M_o).

Moment of Couple acting upon a deflected Compass-needle.—Let AB, Fig. 118, indicate the direction of the magnetic meridian, and let the needle be deflected by some means through an angle θ.

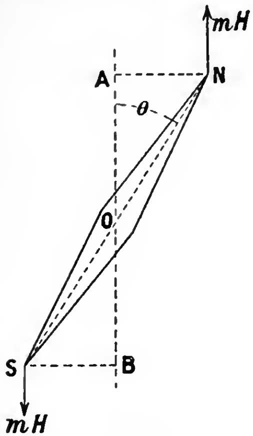


FIG. 118.

The force acting on each pole

$= $ strength of pole $\times$ horizontal component of the earth's magnetic force

$= m \times H$ dynes

Hence the forces acting upon the needle, tending to bring it back into the meridian, constitute a couple, whose moment is

$$mH \times (AN + BS) = mH \times 2AN$$

Now $AN = ON \sin \theta$

$\therefore$ moment of couple $= mH \times 2ON \, . \sin \theta$

but 2ON is the length of the magnetic needle,

$$\therefore \text{ moment of couple} = mH \times l \sin \theta$$
$$= MH \sin \theta$$

This result is of great importance.

Evidently, the moment of the couple increases from zero when $\theta = 0°$ to a maximum value of MH when $\theta = 90°$.

If the deflection is due to the influence of a field, F, at right angles to the meridian, and uniform in strength over the space traversed by the needle, then the deflecting forces on each pole will be $m \times \frac{F}{\mu}$ (where $\mu = 1$), and these will constitute another couple (Fig. 119) whose moment is

$$mF \times AB = mF \times 2AO$$
$$= mF \times 2ON \cos \theta$$
$$= MF \cos \theta$$

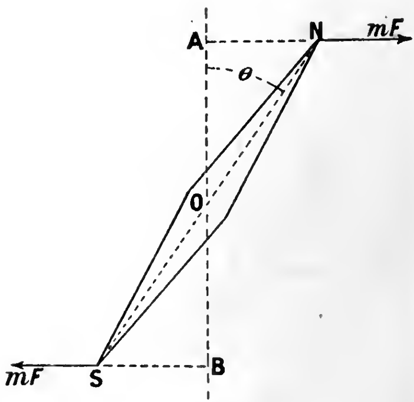


FIG. 119.

Evidently, the moment of this couple will have a maximum value of MF when $\theta = 0°$, and will decrease to zero when $\theta = 90°$.

If the magnet is in equilibrium under the influence of the two opposing couples, their moments must be equal, so that

$$MF \cos \theta = MH \sin \theta$$

$$\text{or } F = H \tan \theta$$

This agrees with the result obtained by simple reasoning on p. 142.

Oscillations of a Magnet in a Magnetic Field.—Before giving any experimental work on this subject, it will be advisable to construct a simple and useful form of vibration apparatus by means of which the oscillation of a magnet may be studied.

Obtain a circular glass dish, A (Fig. 120), about 6 inches in diameter and 3 inches deep. Cut a glass cover, slightly larger than the dish, having a hole (a quarter of an inch in diameter) drilled through the centre. Small pieces of wood should be glued

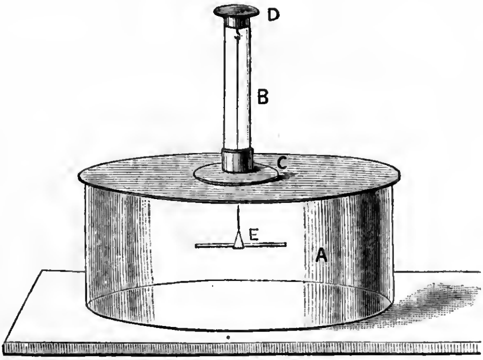


FIG. 120.

to the cover to keep it in position. Take about 5 inches of glass tubing (half an inch in diameter) and fasten it over the hole in the cover. This can readily be done by using the cover of an ordinary deflagrating spoon, which consists of a circular brass plate, C, about 3 inches in diameter, having a hole at the centre, and provided with a circular brass collar. Fit the glass tube into the collar by means of a bored cork, and fasten the brass plate to the glass cover. Make a wooden cap, D, for the top of the tube, and fix a brass hook into it. Make a stirrup of copper-foil or zinc-foil, E, and suspend it by a fibre—or a few fibres, if a heavy magnet is to be oscillated—of unspun silk to the hook in the cap. In most experiments only light knitting-needle magnets are used, and then the best stirrup is a double loop of silk.

When a magnet is balanced in the stirrup, it can be drawn out of the meridian by bringing another magnet carefully up to it. It will then oscillate about its position of rest, until finally it again becomes stationary.

Exp. 114, *to prove that, although the extent of the oscillation gradually diminishes, the time of performing each oscillation is the same.*

Suspend a magnet in a stirrup of a magnetometer and draw it aside through a small angle—say 4° or 5°. Determine the time of a definite number of oscillations—say 10 or 20. Draw it aside through a larger angle—say 8° to 10°—and again determine the time of the same number. Observe that the times are the same in both cases.

The actual time of vibration depends upon several conditions :—

1. Upon the mass and shape of the magnets.

Exp. 115. Magnetise a knitting-needle and suspend it in the vibration apparatus. Draw it aside, and, as before, determine the time of a number of complete oscillations.

Load each end of the needle with a piece of sheet lead, thus increasing the mass and changing the shape, and again determine the time of the same number of oscillations. Observe that the time in the latter case is greater than in the first case, *i.e.* the greater the mass, the longer the time of vibration.

2. Upon the moment of the controlling couple, which in its turn depends upon

(*a*) the moment of the magnet,

(*b*) the magnetic force due to the field, and hence upon the strength of the field

Exp. 116. Determine the time of a definite number of complete oscillations of a magnet. Now remagnetise the magnet as strongly as possible. This increases its moment without altering its mass or shape. Observe that the time of vibration of the remagnetised magnet is less than before.

Exp. 117. Determine the time of a definite number of oscillations (*a*) when a small heavy magnet oscillates under the earth's influence alone; (*b*) when the strength of the field of force is altered by bringing the S pole of a long magnet near the N pole of the oscillating magnet. Notice that in the second case the oscillations are much more rapid, *i.e.* the time of oscillation is diminished.

A magnet in oscillation is a particular instance of a "compound pendulum," and we can apply to it the general equation for the time of vibration of any compound pendulum executing what are known as "simple harmonic vibrations.'

If t be the time of one complete vibration, then

$$t = 2\pi \sqrt{\frac{\mathrm{I}}{\frac{\text{moment of couple}}{\text{angle of deflection}}}}.$$

where I is a quantity depending upon the shape and mass of the vibrating body, known as its *moment of inertia*, and the denominator is the ratio between the couple producing any steady deflection and the deflection produced. Experiment shows that this ratio is constant for different deflections, as long as they are not very great in amplitude.

In the particular case of a pivoted or suspended magnet vibrating under the influence of a magnetic field, it has just been shown that the moment of the couple required to deflect the magnet through an angle θ is MH sin θ,

$$\therefore t = 2\pi\sqrt{\frac{\mathrm{I}}{\frac{\mathrm{MH}\sin\theta}{\theta}}}$$

Now, *when θ is small,* the ratio $\frac{\sin\theta}{\theta}$ is practically unity, and

$$\therefore \text{ we have } t = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{MH}}}$$

It must be very carefully remembered that this expression only applies to oscillations of *small* amplitude. It may also be pointed out that H is not the field strength, but that it is the magnetic force *i.e.* the force in dynes on unit pole, although in all practical cases, they will have the same numerical value.

The term *moment of inertia* will be understood only by students who have some acquaintance with the theory of rotatory motion. It may be defined as follows :—

If the mass of every particle of a body be multiplied by the square of the distance from the axis of rotation, the sum of these products is the moment of inertia of the body about that axis,

although this definition is not likely to convey much information.

For the present purpose, it will be sufficient to know that the moment of inertia of bodies of simple geometrical shape can be calculated from their dimensions.

For instance, for a rectangular bar magnet of mass m, length a, width b, and thickness c, vibrating about an axis through its centre perpendicular to the surface containing a and b, we have

$$\mathrm{I} = m \,.\, \frac{a^2 + b^2}{12} \text{ (notice that it is independent of } c\text{)}$$

For a cylindrical bar magnet of mass m, length a, and radius r, vibrating about a central axis perpendicular to the axis of the cylinder,

$$\mathrm{I} = m\left(\frac{a^2}{12} + \frac{r^2}{4}\right)$$

Very frequently, bar magnets are used in the form of thin rods (*e.g.* knitting-needles), and then these two expressions may be simplified. For when b is small compared with a, or r is small compared with a, both forms reduce to

$$\mathrm{I} = \frac{ma^2}{12}$$

Bottle Form of Oscillation Apparatus.—It is often necessary to experiment upon fields due to magnets, &c., which are very far from uniform, and therefore, in order to compare the strengths at different points in a varying field, it is necessary that the oscillating magnet should be very short and that it should be placed with its centre exactly at the point in question. Again, there may not be sufficient room for the large vessel shown in Fig. 120, and hence it is convenient to use a needle about an inch long, suspended inside a bottle just wide enough to allow it to oscillate. Fig. 121 is a diagrammatic sketch of such an instrument, in which the magnet NS has soldered to it a brass rod, B, in order to increase its moment of inertia, and thus to make it vibrate slowly for convenience in counting the oscillations. A is a silk fibre attached to a brass rod, R, which passes through the cork of the bottle.

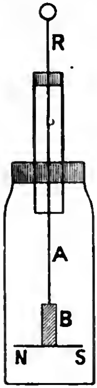


FIG. 121.

Comparison of Field Strengths by the Method of Oscillation.—If the time of oscillation of the *same* magnet be taken in fields of different strengths, M and I are constant, and the equation on p. 155 reduces to

$$t \propto \sqrt{\frac{1}{H}} \text{ or } H \propto \frac{1}{t^2}$$

where H is the force on unit pole.

This affords an easy and accurate method of comparing field strengths, and thereby of indirectly measuring various other quantities.

Exp. 118, *to compare the value of the horizontal component of the earth's field in the open air with its value at a point inside a building.* Use a knitting-needle magnet, 4 or 5 inches long, inside the apparatus shown in Fig. 120. Take its time of vibration in the open air, at a distance from surrounding buildings. To do this, disturb it slightly from its position of rest by bringing a piece of iron near the side of the glass cover, wait until the oscillations are of small amplitude (4° or 5°), then determine the time taken in executing, say, 20 complete vibrations. Repeat this several times, and, from the results, find the mean time of one vibration. Let this be t seconds.

Repeat the experiment at a given place inside the building, and let the mean time be t_1 seconds. The difference between t_1 and t will depend on circumstances, but in the majority of cases a small but distinct difference will be found.

Let H and H_1 be the values of the horizontal component in the open air and inside the building respectively, then

$$\frac{H}{H_1} = \frac{\frac{1}{t^2}}{\frac{1}{t_1^2}} = \frac{t_1^2}{t^2}$$

As the value of H is well known for different parts of the country, we can easily deduce the *actual* value inside the building.

This experiment also illustrates the method adopted in comparing the values of H at different parts of the earth. Evidently, it is only necessary to take the time of oscillation of the *same* magnet in different places, although precautions must be taken to ensure that its moment is not altered during transit from one place to the other.

It may be pointed out that if n and n_1 are the number of oscillations in one second, corresponding to times of vibration t and t_1 seconds respectively, then

$$n = \frac{1}{t} \text{ and } n_1 = \frac{1}{t_1}$$

and we may, therefore, write $\frac{H}{H_1} = \frac{n^2}{n_1^2}$, and this form is sometimes convenient. In practice, however, it is always the value of t that is found (by determining the time taken in executing a given number of vibrations), and not the value of n (for it is difficult to determine with accuracy the number of vibrations in a given time), and, hence, we shall usually express our results in terms of t.

Exp. 119, *to compare the moments of two magnets.* A reference to the equation on p. 155 will show that, if the two magnets have the same size and weight, *i.e.* if their moments of inertia are the same, it will be sufficient to take their respective times of vibration at the same place, for when I and H are constant, we have

$$t \propto \sqrt{\frac{1}{M}}, \text{ or } M \propto \frac{1}{t^2}$$

As a general rule, this condition will not be satisfied, and then it is necessary to proceed as follows:—

Two slots are cut in a small wooden block to receive the magnets, another similar block is laid upon them, and the two blocks are screwed together with brass screws. In this way, the magnets are held rigidly one above the other with their axes parallel to each other, but without contact. A hook is screwed into the block, and the whole is suspended by a suitable fibre inside a glass case (*e.g.* an empty balance case) to avoid draughts.

The time of vibration is then taken (1) with both N poles pointing the same way, (2) when one magnet is reversed.

Let M and M_1 be the moments of the magnets, and let t and t_1 be the times of vibration in (1) and (2) respectively.

The moment of inertia of the *whole* system is evidently the same in each case, and hence we have (assuming $M > M_1$)

$$t = 2\pi \sqrt{\frac{I}{(M + M_1)H}}$$

$$t_1 = 2\pi \sqrt{\frac{I}{(M - M_1)H}}$$

Therefore $$\frac{t}{t_1} = \sqrt{\frac{M - M_1}{M + M_1}}$$

$$\text{or} \quad \frac{M - M_1}{M + M_1} = \frac{t^2}{t_1^2}$$

$$\text{from which} \quad \frac{M}{M_1} = \frac{t_1^2 + t^2}{t_1^2 - t^2}$$

Example.—Two magnets are placed in an aluminium frame with their axes horizontal and parallel, one being vertically over the other. The frame is suspended and oscillates in the earth's horizontal field, making 20 and 5 vibrations per minute respectively when similar poles of the magnets are together or opposed. If the moment of the stronger magnet is 300, what is that of the other?

(B. of E., 1907).

Let M and M_1 be the moments of the stronger and weaker magnets respectively, and I the moment of inertia of the combination. Let t and t_1 be the times of vibration in the two cases respectively; then, by the result given above,

$$\frac{M}{M_1} = \frac{t_1^2 + t^2}{t_1^2 - t^2}$$

Now 20 vibrations in 1 minute = 1 vibration in 3 seconds, and
5 vibrations in 1 minute = 1 vibration in 12 seconds.

$$\therefore \quad \frac{300}{M_1} = \frac{12^2 + 3^2}{12^2 - 3^2} = \frac{153}{135}$$

$$\therefore \quad M_1 = \frac{300 \times 135}{153} = 265$$

Further Experiments on the Inverse-square and Inverse-cube Laws.—**Exp. 120**, *to determine the law of action of a single pole.* A third method of testing the inverse-square law may now be given.

Place on a table a very long, thin bar magnet in the magnetic meridian, with its N pole directed towards the north. Then at all points on its axis produced, its field is in the same direction as the earth's field, and the resultant field is the sum of the two components.

Determine as nearly as possible the point which may be taken as the position of a pole of the magnet (see p. 115).

Using the bottle form of vibration apparatus, place it about 3 inches from the pole on the axis of the magnet produced, and then determine its time of vibration. Move it about 3 inches further away, and again take the time of vibration.

Repeat the observations for a number of points on the produced axis, in each case measuring the distance from the pole.

Finally remove the long magnet, and take the time of vibration under the influence of the earth's field alone.

Let F = Field due to magnet at one of the points where the observations are made, E = Field due to earth, and R = Resultant field.

Then, in this case, $F + E = R$, or $F = R - E$.

But if t be the time of vibration at the point, and t_1 the time under the influence of the earth alone, then

$$R = C \times \frac{1}{t^2}, \text{ where C is some constant,}$$

$$\text{and } E = C \times \frac{1}{t_1^2}$$

$$\therefore F = C\left(\frac{1}{t^2} - \frac{1}{t_1^2}\right)$$

$$\text{or } F \propto \left(\frac{1}{t^2} - \frac{1}{t_1^2}\right)$$

Hence, calculate the value of $\frac{1}{t^2}$ in each case, subtract from the result the value $\frac{1}{t_1^2}$, which is the same in all positions, and thus prepare a table of numbers proportional to F corresponding to various distances (d) from the pole. Then, as shown on p. 138, if the field varies as d^n, we have

$$Fd^n = a \text{ constant,}$$

$$\text{and } \log F + n \log d = \text{another constant.}$$

Hence, as already explained, plot log F against log d, and determine n from the tangent of the angle of slope. Again, it is unlikely that n will be found equal to 2, but it will probably be nearer this value than it was in Experiment 104, simply because it has been possible to work with greater distances, for which the inverse-square law is approximately satisfied.

Exp. 121, *to verify the inverse-cube law.* Proceed exactly as in the last experiment, but use a *short* bar magnet instead of a long one, and measure the distances from its *centre.* In this case, it will be found that n equals 3 (within reasonable limits of experimental error) except for points too close to the magnet.

Absolute Measurement of M and H.—We have given methods of *comparing* (1) the moments of two magnets; and (2) the horizontal components of the earth's field at different places, and we must now give a method of obtaining M and H in *absolute* measure.

For this purpose, it is preferable to use a reflecting magnetometer. A small magnet, about 4 inches long, is made from a knitting-needle; it should be of uniform section, and the ends should be ground flat, and, in order to secure uniformity of magnetisation, it should be magnetised by placing it inside a solenoid carrying a current, and not by rubbing on another magnet. Either A or B position may be used, but, with the instrument described on p. 146, the latter position is the more convenient.

Exp. 122. (a) Put the small magnet in position at distance d from the mirror (about 40 centimetres will probably be suitable), and take the scale reading; then turn the magnet round, thus reversing the direction of deflection, and again take the reading. Now place the magnet at exactly the same distance on the other side of the magnetometer, and by repeating the operations, obtain two more readings. Take the mean of the four readings. Let this be n divisions. Measure as accurately as possible the distance from the mirror to the scale, and also express the mean deflection n in terms of the same unit of length. From this determine (as on p. 148) tan 2θ, remembering that the reflected ray turns through twice the angle of rotation of the mirror. But, for these small angles, it will be sufficiently exact to take tan θ as $\frac{1}{2}$ tan 2θ, so that the value of tan θ can be written down at once in terms of the measured distances.

We have now $\frac{M}{H}=d^3 \tan\theta$ (for B position)

where M is the moment of the knitting-needle magnet. It is better to work this out instead of leaving it in factors. Let this value be a, so that $\frac{M}{H}=a$.

(*b*) Now suspend the magnet by a fine silk fibre in the apparatus shown in Fig. 120. Make a double loop in the fibre, instead of using any form of stirrup, so as not to add appreciably to the mass. Using swings of small amplitude, determine the time of executing about 20 complete swings. Repeat carefully several times, and obtain the time, t, of one complete oscillation.

$$\text{Then, we have } t=2\pi\sqrt{\frac{I}{MH}}$$

$$\text{whence } MH=\frac{4\pi^2 I}{t^2}\ (=b, \text{ say})$$

For a round bar, whose radius is small compared with its length, the moment of inertia, I, is, $\frac{Ml^2}{12}$ (see p. 155).

Two equations are thus obtained—

$$\frac{M}{H}=a \text{ and } MH=b$$

Then, by multiplication, we have

$$M^2=ab \text{ or } M=\sqrt{ab}$$

and by division,

$$H^2=\frac{b}{a} \text{ or } H=\sqrt{\frac{b}{a}}$$

The method may, therefore, be used to determine either M or H, although, for the sake of simplicity, we have omitted numerous precautions and corrections, which are necessary for an accurate measurement.

Experiment 122 (*a*) can, of course, be carried out, although with much less accuracy, by means of the magnetometer described on p. 143, and we give below the results of a particular experiment by this method. A rectangular bar magnet—10·7 centimetres long, 2 centimetres wide, and 110·8 grams in weight—was used in the B position. When the distance was 25 centimetres, the following eight readings were taken :—

Position.	Deflection.	Mean	Natural tangent.
E 1	$21\frac{1}{2}°$, $22°$	21° 26¼′	·3926
E 2	$21°$, $22°$		
W 1	$21\frac{1}{2}°$, $20\frac{1}{2}°$		
W 2	$21\frac{1}{2}°$, $21\frac{1}{2}°$		

$$\text{Whence} \quad \frac{M}{H}=\frac{d^3 \times \tan\theta}{2}$$

$$=\frac{25^3 \times \cdot 3926}{2}$$

$$=3067 \text{ nearly.}$$

Using the same magnet in the oscillation apparatus (Fig. 120), 30 oscillations were made in 11 minutes 20 seconds.

$\therefore$ time of one oscillation was 22·6 seconds.

Calculating the moment of inertia, we have

$$I=110\cdot 8\left\{\frac{(107\cdot 2)^2+2^2}{12}\right\}=1094$$

Substituting in equation

$$MH=\frac{4\pi^2 I}{t^2}, \text{ we have}$$

$$MH=\frac{4\times(3\cdot 14)^2\times 1094}{(22\cdot 6)^2}$$

$$=84$$

$$\text{Whence} \quad MH\times\frac{M}{H}=M^2=84\times 3067$$

$$\therefore \quad M=508$$

$$\text{and} \quad MH\div\frac{M}{H}=H^2=\frac{84}{3067}$$

$$\therefore \quad H=\cdot 16 \text{ nearly.}$$

It may be pointed out that the term *magnetic moment* has a much more definite meaning than the term *magnetic pole.* Whatever may be the shape of the magnet, its *moment* can be measured as an exact numerical quantity, whereas a magnetic *pole* is little more than a convenient mathematical conception, and can be measured only approximately by indirect and doubtful processes.

Construction for Magnetic Curves.—The construction given on p. 41 for electric lines of force can be applied to the field produced by a bar magnet, on the assumption that its poles may be represented by points. As the two poles are necessarily equal in strength, the two sets of radiating lines must be equal in number; and if n lines are drawn from each point, they will represent poles of strength $\frac{n}{4\pi}$. It will be found, on carrying out the construction as already explained, that the distribution *outside* the magnet resembles the figure given on p. 125. (The distribution *inside* the magnet must be inferred from other considerations.)

Direction of the Field at a given point near a Magnet.—This can easily be determined by a simple construction, although the method is not suitable when the whole field has to be plotted. Let O (Fig. 122) be the given point, and let d and d_1 be its distances from the N and S poles respectively treating them as points). Writing F_n F_s for the fields at O due to the N and S poles respectively, we have

$$F_n = \frac{m}{d^2} \text{ acting along NO produced}$$

$$F_s = \frac{m}{d_1^{\,2}} \text{ acting along OS}$$

$$\therefore \quad \frac{F_n}{F_s} = \frac{d_1^{\,2}}{d^2}$$

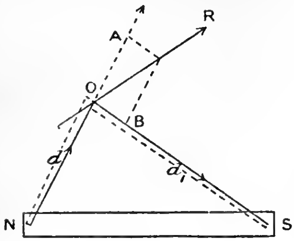


FIG. 122.

Hence, mark off along NO produced, a length, OA, to represent d_1^2, and along OS a length, OB, to the same scale to represent d^2; complete the parallelogram and draw the diagonal OR. This will give the *direction* of the field at O, *i.e.* OR is the tangent to the curved line of force at that point.

As the direction of the line indicates the direction of the force acting upon a magnetic pole, it follows that a *single* free pole placed anywhere in the field would tend to move one way or the other along the curved line until it reached the magnet. But such poles do not exist, and in considering the behaviour of a piece of iron or of a small magnet (say an iron filing) in a magnetic field, we must take into account the force on each pole.

Let A (Fig. 123) represent the iron filing in any non-uniform field, of which only one line is drawn for clearness. Then each pole will tend to move along the line of force passing through it, but in opposite directions, and it is easy to see that these two forces will have a resultant tending to move the filing laterally towards the magnet. As the filing is nearer to the N pole of the magnet, the force in that direction is the greater, and hence the resultant is inclined to the axis of the filing, which tends to move *across* the lines of force until it reaches the magnet. We may express these results by saying that whereas a single pole tends to move along the lines of force, an actual piece of iron tends to cut across them in the direction in which the field is increasing in strength most rapidly.

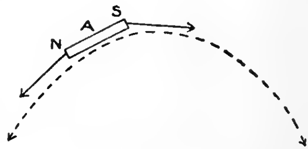


FIG. 123.

Examples.—1. Two exactly equal magnets are attached together at their mid points so that their axes are at right angles, and the combination is pivoted so that the axes of the magnets are horizontal and they can turn freely about a vertical axis. How will the system set itself under the influence of the horizontal component of the earth's field? If the moment of each magnet is M, and the moment of inertia about the axis round which it can turn is K, what will be the period of vibration of the system?

(B. of E., 1906.)

Taking the latter part of the question first (for convenience), its point depends on recognising that if *each* magnet has moment of inertia K and magnetic moment M, the values of these quantities in the combination are obtained by simple addition with regard to the moments of inertia, and by vector addition with regard to the magnetic moments; *i.e.* the moment of inertia of the combination is 2K, and the moment of the equivalent magnet is $\sqrt{M^2+M^2}=M.\sqrt{2}$

$$\text{Hence, } t=2\pi\sqrt{\frac{2K}{M.\sqrt{2}}}$$

Considering the first part of the question, it is evident that, as the moments are equal, the magnets will set themselves symmetrically with respect to the meridian.

It will be instructive to discuss the more general question—how will two magnets of moments M and M_1, and rigidly fastened together at their centres at an angle θ, set themselves when free to move in the earth's field in a horizontal plane?

The dotted line in Fig. 124 indicates the magnetic meridian. The moments of the opposing couples are MH sin α and M_1H sin β, and hence the condition of equilibrium is given by

$$MH\sin\alpha=M_1H\sin\beta$$

$$\text{or}\quad \frac{M}{M_1}=\frac{\sin\beta}{\sin\alpha}\quad(a)$$

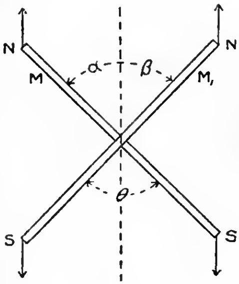


Fig. 124.

I. The simplest case is when $\theta=90°$. Then we have

$$\alpha+\beta=90°$$

$$\text{or}\ \sin\alpha=\cos\beta$$

$$\therefore\quad \frac{M}{M_1}=\frac{\sin\beta}{\cos\beta}=\tan\beta$$

from which both angles are determined.

II. When θ is not 90°, equation (a) indicates a graphic solution. Draw two lines (Fig 125), enclosing the angle θ, and mark off along them, to any convenient scale, lengths OA and OB proportional to M and M_1 respectively. (Notice that OA and OB do not represent the *lengths* of the magnets, in fact OB may refer to the *shorter* magnet, although OB>OA.) Complete the parallelogram and draw the diagonal OR, then AOR $=\alpha$ and BOR $=\beta$.

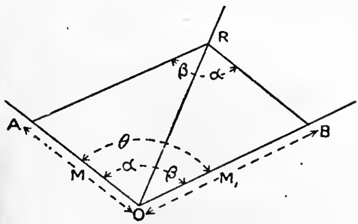


Fig. 125.

From the figure

$$\frac{AO}{AR}=\frac{\sin ARO}{\sin AOR}=\frac{\sin\beta}{\sin\alpha}$$

$$\text{but}\quad \frac{AO}{AR}=\frac{M}{M_1}$$

$$\therefore\quad \frac{M}{M_1}=\frac{\sin\beta}{\sin\alpha}$$

and OR represents the moment of the single resultant magnet equivalent to M and M_1

Astatic Needle.—The above reasoning shows that the magnetic moment of a combination of magnets is the vector sum[1] of the moments of the component magnets. Hence, if in Fig. 124 the two magnets are of equal moment and their axes are oppositely directed and *parallel* to each other, the resultant magnetic moment will be zero. As a whole, the system will remain in any position, and will be quite free from any directive action due to the earth's field (or any other uniform field).

Such an arrangement, which may be realised in various ways, is said to be *astatic*. Fig. 126 shows a well-known form of the type once largely used in the construction of the astatic galvanometer (see p. 291). It consists of two magnets fixed one above the other to a vertical spindle, which is suspended by a silk fibre or mounted on bearings. It will be understood that the magnets may be any distance apart, provided that they are rigidly connected together.

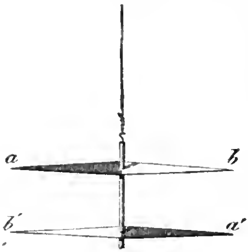


FIG. 126.

It is almost impossible to make a perfectly astatic pair. This will be understood from the following considerations:—

It is very difficult (1) to magnetise two needles to exactly the same strength; (2) to fix them parallel; (3) to fix them so that their axes lie in the same vertical plane, *i.e.* so as not to cross one another.

A. If the needles are of unequal length, they will, owing to the action of the earth's magnetism, tend to move into the magnetic meridian.

B. If the magnets are of equal size and strength, but their axes are not quite in the same vertical plane, they will set themselves at right angles to the magnetic meridian, *i.e.* east and west. The reason of this will be easily understood, for a glance at Fig. 125 will show that two magnets of nearly equal moment, having their axes oppositely directed, and not quite parallel to each other, will be equivalent to a single magnet of very small moment inclined practically at right angles to the combination.

2. The centres of two short magnets AB, CD, are at a distance r apart. AB lies along the line joining their centres, and CD at

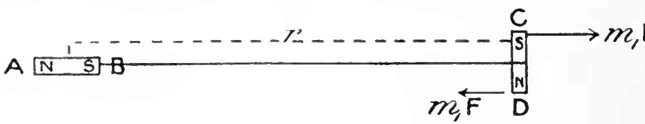


FIG. 127.

right angles to it. Show that the couple due to AB tending to make CD twist round is $\frac{2MM_1}{r^3}$ where M and M_1 are the moments of the magnets. (B. of E, 1898.)

[1] The "vector sum" is the resultant obtained by the method generally known in another connection as the "parallelogram of forces," and is applied in the previous example.

Let the moment of AB be M; the moment of CD, M_1, and its pole strength m_1. As the magnets are short compared with r, we may regard CD as being in a uniform field due to AB and acting along the axis, and if F is the strength of this field, the force in dynes on each pole is m_1F and the moment of the couple tending to rotate CD is

$$m_1F \times CD = M_1F$$

$$\text{Also } F = \frac{2M}{r^3} \quad \text{(see p. 140)}$$

$$\therefore \text{ Moment of couple} = \frac{2MM_1}{r^3}$$

(The student can show, in the same way, that the couple due to CD tending to rotate AB is $\frac{MM_1}{r^3}$).

3. A bar magnet is suspended horizontally by a wire. When the top of the wire is twisted through 120°, the magnet is deflected through 30°. How much must the top be twisted in order to deflect the magnet through 90°?

When the magnet is deflected through some angle θ by twisting the wire, it is under the influence of two opposing couples—(1) a deflecting couple due to the torsion, whose moment is within certain limits proportional to the angle of twist, and (2) a restoring couple due to the earth's magnetic field, whose moment is MH sin θ. When in equilibrium, the moments of these couples must be equal.

$$\text{Angle of twist} \propto MH \sin\theta$$

$$\therefore\ 120^\circ - 30^\circ \propto MH \sin 30^\circ \qquad \text{i.}$$

Let t° = angle of twist when the deflection is 90°

$$\text{then } t^\circ \propto MH \sin 90^\circ \qquad \text{ii.}$$

$$\therefore\ \frac{90^\circ}{t^\circ} = \frac{\sin 30^\circ}{\sin 90^\circ}$$

$$\therefore\ t^\circ = 180^\circ$$

But this is the actual twist on the wire, when the deflection is 90°, therefore the top of the wire must be turned through $180^\circ + 90^\circ = 270^\circ$.

The above argument indicates a method of comparing the moments of two magnets, which is of great theoretical importance. Its principle will be sufficiently shown by the following example:—

4. Two magnets, A and B, are in turn suspended horizontally by a vertical wire so as to hang in the magnetic meridian. To deflect the magnet A through 45°, the upper end of the wire has to be turned once round. To deflect B through the same angle, it has to be turned

round one and a half times. Compare the moments of the two magnets.

We have, generally,

$$\text{Angle of twist} \propto MH \sin\theta$$

$$\therefore\ 360^\circ - 45^\circ \propto M_1 H \sin 45^\circ$$

$$\text{and } 540^\circ - 45^\circ \propto M_2 H \sin 45^\circ$$

$$\text{Whence } \frac{M_1}{M_2} = \frac{315}{495} = \frac{7}{11}$$

5. Find approximately the force of attraction or repulsion between two short bar magnets of moments 10 and 20 C.G.S. units respectively, with their centres at a distance of 20 centimetres apart and their axes pointing in the same direction along the same line.

(B. of E., 1909.)

Let us first consider the general case shown in Fig. 128.

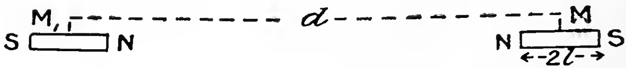


FIG. 128.

The magnet M (of pole strength m and length $2l$) is placed in a field due to the magnet M_1, then

(a) the near pole of M is in a field whose strength is $\frac{2M_1}{(d-l)^3}$, and the repulsive force upon it is $\frac{2M_1}{(d-l)^3} \times m$ dynes

(b) the further pole is a field of strength $\frac{2M_1}{(d+l)^3}$ and the attractive force is $\frac{2M_1}{(d+l)^3} \times m$ dynes.

$$\therefore \text{Nett repulsive force} = \frac{2M_1 m}{(d-l)^3} - \frac{2M_1 m}{(d+l)^3}$$

$$= 2M_1 m \left\{ \frac{1}{(d-l)^3} - \frac{1}{(d+l)^3} \right\}$$

$$= 2M_1 m \left\{ \frac{(d+l)^3 - (d-l)^3}{(d^2-l^2)^3} \right\}$$

$$= 2M_1 m \cdot \frac{6d^2 l + 2l^3}{(d^2-l^2)^3}$$

but, as the magnets are *short*, we may neglect l^2 and higher powers, so that the expression becomes

$$\text{Repulsion} = \frac{2M_1 m \times 6d^2 l}{d^6} = \frac{6MM_1}{d^4}$$

(If the student considers the case when like poles are not opposed to each other, he will find that the force is one of attraction and of the same value.) Now, substituting the numbers given in the question,

$$\text{Repulsion} = \frac{6 \times 10 \times 20}{20^4}$$

$$= \cdot 0075 \text{ dyne.}$$

6. A compass-needle is placed on a table, and a bar magnet is laid on the floor below it, the centre of the bar magnet being directly underneath the centre of the needle. When the N pole of the bar magnet is northward, the compass-needle, after being disturbed, makes 100 oscillations in 16 minutes. When the N pole is southwards, the needle makes 100 oscillations in 12 minutes. When the bar magnet is removed, the needle oscillates under the earth's influence alone; how long will it take to make 100 oscillations?

The compass-needle is vibrating in a field due partly to the bar magnet and partly to the earth, both acting in the same direction. It is implied that the earth's field is the greater and is weakened, but not reversed, by that due to the magnet when acting in opposition.

Let E = strength of field due to earth,
,, F = ,, ,, ,, ,, ,, magnet,
,, t_1 = time in minutes of making 100 vibrations when both fields coincide in direction,
,, t_2 = time in minutes of making 100 vibrations when the two fields are in opposition,
,, t = time in minutes of making 100 vibrations under earth's influence alone.

$$\text{Then } E + F \propto \frac{1}{t_1^2} \propto \frac{1}{(12)^2} \qquad \text{i.}$$

$$E - F \propto \frac{1}{t_2^2} \propto \frac{1}{(16)^2} \qquad \text{ii.}$$

$$E \propto \frac{1}{t^2} \qquad \text{iii.}$$

From i. and ii. $$\frac{E+F}{E-F} = \frac{256}{144}$$

$$\therefore \quad \frac{E}{F} = \frac{25}{7} \qquad \text{iv.}$$

From i. and iii. $$\frac{E}{E+F} = \frac{144}{t^2}$$

$$\therefore \quad \frac{E}{F} = \frac{144}{t^2 - 144} \qquad \text{v.}$$

Equating iv. and v. $\frac{144}{t^2-144}=\frac{25}{7}$

From which $t^2=184{\cdot}3$

$\therefore\ t=13{\cdot}56$ minutes.

7. Prove that the work done in twisting a magnet of moment M through 90° from the meridian in a field of strength H is given by the product MH. (B. of E., 1904.)

When the magnet is deflected through angle θ (Fig. 129), the pole N moves through an arc AN, which is equivalent to the distance AB measured along the direction of the earth's field. Hence, work is done in moving the pole against the magnetic force, and as work = force × distance, we have,

Work done on N pole $= m\text{H} \times \text{AB}$.

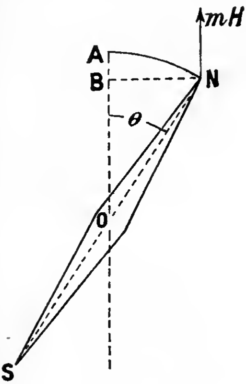


Fig. 129.

Now, an exactly equal amount of work is done on the other pole, and

$$\begin{aligned}\therefore\ \text{Total work} &= 2m\text{H}\times\text{AB}\\ &= 2m\text{H}(\text{AO}-\text{BO})\\ &= 2m\text{H}\left(\frac{l}{2}-\frac{l}{2}\cos\theta\right)\\ &= ml\text{H}(1-\cos\theta)\\ &= \text{MH}(1-\cos\theta)\end{aligned}$$

So that when $\theta=90°$, work done is MH dynes.

Intensity of Magnetisation.—The intensity of magnetisation of a magnet is the magnetic moment per unit of volume. For instance, in the case of a uniform bar of pole strength m, area of cross section A, and length l, we have

$$\text{Intensity}=\frac{\text{M}}{\text{V}}=\frac{ml}{\text{A}\times l}=\frac{m}{\text{A}}$$

or, the intensity would be the number of unit poles per square centimetre of end surface, if the magnetisation were strictly confined to the ends.

This method of representing the strength of a magnet is occasionally convenient in theoretical discussions, and is employed later in Chapter XXV., although at present it is merely necessary to indicate its meaning.

Magnetic Potential.—The idea of potential in connection with an electrostatic field has been discussed in Chapter IV. Somewhat similar ideas can be applied to the phenomena of magnetism, and the definition already given will still hold good, provided that we write "magnetic pole" in place of "electric charge." It, therefore, follows that the magnetic P.D. between any two points is measured by the work done (in ergs) when a unit pole is carried from one point to the other against a magnetic force; also that when we speak of the "magnetic potential *at a point*," we mean the P.D. between that point and a point at infinite distance, as measured by carrying a unit pole from infinity to the point in question against the force.

On the other hand, there is no "flow" of magnetism analogous to the "flow" of current, produced (say) by joining points at different magnetic potentials, and, in consequence, the idea of potential is not so generally useful in magnetism as it is in electrostatics.

Potential at any Point near a Magnet Pole.—The argument given on p. 36 will apply to this case, provided that we write $\frac{m}{\mu d^2}$ for the "magnetic force" at distance d from an isolated magnetic pole (see p. 136). Hence, we find that the potential at distance d from a single pole of strength m is $\frac{m}{\mu d}$, and we may agree to regard it as positive, when the pole is a N pole, and negative when a S pole.

Potential at any Point near a Bar Magnet.—In this case we have simply to superpose the effects of the two poles, considered as acting independently.

Let NS (Fig. 130) be the magnet, of pole strength m; and let P be the point in question. Then if O be the middle point of the magnet, the position of P is defined in terms of length OP and the angle, θ, which OP makes with the axis of the magnet.

$$\text{Potential at P due to N pole} = +\frac{m}{\mu.\text{NP}}$$

$$\text{Potential at P due to S pole} = -\frac{m}{\mu.\text{SP}}$$

Now, potential is not a vector quantity, and hence we have

$$\text{Actual potential at P} = \frac{m}{\mu.\text{NP}} - \frac{m}{\mu.\text{SP}}$$

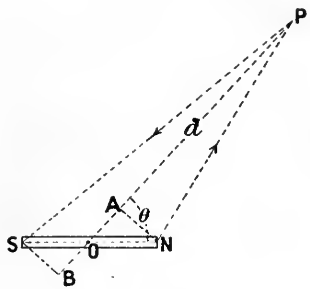


FIG. 130.

Case of a Short Magnet.—When OP is great compared with the length of the magnet, this result can readily be simplified. Draw NA and SB (fig. 130) perpendicular to PO (or its direction produced). Then, as the angles NPO and SPO are very small, we have approximately NP = OP − OA, and SP = OP + OB, where OA = OB.

$$\therefore \quad \text{Potential at P} = \frac{m}{\mu}\left\{\frac{1}{\text{OP}-\text{OA}} - \frac{1}{\text{OP}+\text{OB}}\right\} = \frac{m}{\mu} \cdot \frac{2\text{OA}}{\text{OP}^2-\text{OA}^2}.$$

Now OA is very small compared with OP, and, therefore, the denominator will not differ appreciably from OP^2.

$$\text{Again, in the triangle OAN we have } \frac{\text{OA}}{\text{ON}} = \cos\theta$$

$$\therefore \quad \text{the expression becomes } \frac{m}{\mu} \cdot \frac{2\text{ON}\cos\theta}{\text{OP}^2}$$

But 2ON = length of magnet

$$\therefore \quad \text{Potential at P} = \frac{\text{M}\cos\theta}{\mu d^2}, \text{ where } d = \text{OP}.$$

Exercise VIII

1. Two long magnets are placed vertically with their north poles (A and B) on the same level as the north pole (C) of a compass-needle, one being magnetic east and the other magnetic west of C. If the compass-needle is not deflected when the distance AC is twice BC, and if all the magnets are so long that the effects of the south poles may be neglected, show what are the relative pole strengths of A and B.

2. A straight piece of watch-spring, 6 inches long, is magnetised and laid on a flat cork floating on water. The spring is now bent until its ends are two inches from each other, and they are fixed at that distance by a piece of thread; the spring is then replaced upon the cork. Compare the couples with which the spring tends to make the cork take a definite direction in each case.

3. A uniformly magnetised bar of brittle steel is broken into two pieces, one twice as long as the other, and the pieces are fastened together at right angles to each other. How would the combination thus formed set itself, under the action of the earth's magnetic force, if made to float on water?

4. A magnetic needle is suspended horizontally in the magnetic meridian. It is then drawn out of the meridian (α) through 30°, and afterwards (β) through 45°. Compare the couples which act upon the needle to bring it again into the meridian.

5. As in question 3, if $\alpha=30°$ and $\beta=60°$. Compare the couples.

6. If $\alpha=30°$, $\beta=90°$. Compare the couples.

7. A magnet pole of strength 6 is placed in a magnetic field of strength ·42: what will be the force acting on the pole?

8. A magnet pole of strength 7 experiences a force of 2·9 dynes. What is the horizontal component of the magnetic field?

9. A very long vertical magnet of pole strength 150 is placed at a perpendicular distance of 10 centimetres from the centre of a horizontal magnetic needle of length 5 centimetres and pole strength 30. Find the moment of the couple acting upon the needle.

10. A very long vertical magnet is placed at a perpendicular distance of 12 centimetres from the centre of a horizontal magnetic needle of length 10 centimetres and strength 13. The moment of the couple acting upon the needle is 60. Find the strength of the pole of the long magnet.

11. Two bar magnets, the moments of which are in the ratio of 8 to 27, are placed with their centres 3 feet apart, their magnetic axes being in the same straight line, which is perpendicular to the magnetic meridian. If their north poles are turned towards each other, find the position which a small compass-needle must occupy on the line joining the magnets in order that it may point in the same direction as if the magnets were not there.

12. A bar magnet suspended by a fine wire points north and south (magnetic) when the wire is not twisted. When the upper end of the wire is turned through 100° the magnet is deflected 30° from the magnetic meridian. Show how much the upper end of the wire must be turned to deflect the magnet 90° from the meridian.

13. The lower end of a fine wire, which hangs vertically, is fastened to the middle of a straight steel magnet, so that the magnet is suspended horizontally by the wire. When the wire is without twist, the magnet comes to rest in the magnetic meridian, but when the upper end of the wire is turned once round, the magnet is deflected from the meridian through 30°; how much must the top of the wire be turned to make the magnet set at right angles to the meridian?

14. Two magnets, A and B, are in turn suspended horizontally by a vertical wire so as to hang in the magnetic meridian. To deflect the magnet A through 45° the upper end of the wire has to be turned once round. To deflect B through

the same angle it has to be turned round one and a half times. Compare the moments of the two magnets.

15. Two straight pieces, one 3 inches and the other 5 inches long, are cut from the same narrow strip of steel. After being equally magnetised they are hung horizontally one at a time by the same fine glass thread so as to rest in the magnetic meridian when the glass thread is not twisted. On turning the upper end of the thread half round (through 180°) the shorter magnet is deflected 10° from the meridian. Show how much the upper end of the thread must be turned to deflect the larger magnet 10°.

16. A long bar magnet lies in the magnetic meridian, with its N pole towards the south. A horizontally suspended compass-needle is placed in the line obtained by producing the axis of the magnet. What effect will the sliding of the magnet towards the needle have on the time of vibration?

17. A glass tube containing four similar pieces of hard steel, which just fill it when placed end to end, is suspended so that it can oscillate about its central point in a horizontal plane. What will be the nature of the difference (if any) in the times of oscillation when (1) the two outer pieces only, (2) the two inner pieces only, are magnetised, unlike poles being in both cases nearest together? Neglect the effects of induction, and give reasons for your answer.

18. A magnetic needle, balanced horizontally at its centre upon a fine pivot, makes 11 vibrations in 2 mins. 1 sec. at a place A, and 12 vibrations in 2 mins. at a place B. Compare the strength of the earth's horizontal force at the two places, explaining clearly how you arrive at your result.

19. A magnetic needle was suspended in a paper stirrup by means of a fibre of unspun silk, and made 12 oscillations in 2 mins. It was removed, and remagnetised. When suspended as before, and moved from its position of rest, it made 45 oscillations in 3 mins. Compare its magnetic moments in the two cases.

20. A bar magnet, which can move only in a horizontal plane, is caused to vibrate at three different stations, A, B, and C. At A it makes 20 vibrations in 1 min. 30 secs.; at B, 25 vibrations in 1 min. 40 secs.; at C, 20 vibrations in 2 mins. Find three numbers proportional to the forces which act upon the magnet at the three places.

21. A small magnetic needle. suspended horizontally by a fibre of raw silk, makes 10 oscillations in 1 min. when under the influence of the earth's action. When the S pole of a long magnet A is placed 3 inches from the N pole of the needle, it makes 32 oscillations in a minute. Afterwards the S pole of another magnet B is similarly placed, and then the needle makes 25 oscillations in a minute. Compare the pole strengths of A and B.

22. A small magnetic needle, suspended horizontally by a fibre of unspun silk, makes 97 oscillations in 8 mins. 5 secs. under the earth's influence. When the S pole of a long magnet A is placed a few inches from the N pole of the needle, it makes 160 oscillations in 5 mins. 20 secs.; when, however, the S pole of a magnet B is similarly placed, it makes 170 oscillations in 7 mins. 5 secs. Compare the pole strengths of A and B.

23. A magnetic needle made 50 oscillations in 2 mins. 5 secs. under the earth's influence. When the S pole of a long bar magnet was brought 4 centimetres from the N pole of the needle, 120 oscillations were made in 3 mins. 20 secs. When the distance between the poles of the needle and magnet was 12 centimetres, it made 65 oscillations in 2 mins. 10 secs. Compare the field due to the bar magnet in the two positions.

24. A small magnetic needle, suspended horizontally by a silk fibre, makes 100 vibrations in 5 mins. 36 secs. under the influence of the earth's magnetism only, and 100 vibrations in 4 mins. 54 secs. when a horizontal bar magnet is placed with its centre vertically below the needle, and with its axis in the magnetic meridian. Compare the field due to the bar magnet with that due to the earth.

25. A uniformly magnetised steel wire, 6 inches long, is laid upon a table; a very short bit of soft iron wire is supported, so as to be free to turn about its centre, at a distance of 6 inches from one end and 3 inches from the other end of the magnetised wire. Show how to draw a figure which would give the direction taken up by the bit of soft iron. [The effect of the earth's magnetism is to be neglected.]

26. A bar magnet is suspended horizontally in the magnetic meridian by a wire without torsion. To deflect the bar 10° from the meridian the top of the wire has to be turned through 180°. The bar is removed, remagnetised, and restored, and the top of the wire has now to be turned through 250° to deflect the bar as much as before. Compare the magnetic moments of the bar before and after remagnetisation. (B. of E., 1894.)

27. A short bar magnet is placed on a table with its axis perpendicular to the magnetic meridian, and passing through the centre of a compass-needle. In London, the compass-needle is deflected through a certain angle when the centre of the magnet is 25 inches from the centre of the needle. If the experiment be repeated in Bombay, the magnet must be moved 5 inches nearer to the needle to produce the same deflection. Use these data to compare the horizontal forces in London and Bombay. (B. of E., 1895.)

28. Prove that the magnetic force exerted by a short magnet at a point A on the line passing through its centre and perpendicular to its axis, is the same as the force exerted at a point on the axis, the distance of which from the centre of the magnet is $\sqrt[3]{2}$ times the distance of A from the centre. (B. of E., 1896.)

29. A magnet placed due east (magnetic) of a compass-needle deflects the needle through 60° from the meridian. If at another station where the horizontal force of the earth's magnetism is three times as great as at the first, the same magnet be similarly placed with respect to the compass-needle, what will be the deflection of the latter? (B. of E., 1897.)

30. Two short bar magnets, the moments of which are 108 and 192 respectively, are placed along two lines drawn on the table at right angles to each other. Find the intensity of the magnetic field due to the two magnets at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 centimetres from this point. (B. of E., 1900.)

31. A short bar magnet is placed, at Gibraltar, perpendicular to the magnetic meridian, and "end-on" towards a compass-needle from which it is distant 100 centimetres. When the experiment is repeated at Portsmouth, the magnet has to be placed at a distance of 110 centimetres from the compass to produce the same deflection of the needle. Compare the horizontal forces of the earth's magnetism at Gibraltar and Portsmouth. (B. of E., 1900.)

32. A uniformly magnetised bar magnet 10 centimetres long, having a moment of 200 C.G.S. units, is placed in a horizontal position with its axis in the magnetic meridian. A small compass-needle placed at a distance of 10 centimetres east of the centre of the bar is observed to be in neutral equilibrium. Find the horizontal intensity of the earth's field. (B. of E., 1901.)

33. A small compass-needle makes 10 oscillations per minute under the influence of the earth's magnetism. When an iron rod 80 centimetres long is placed vertically with its lower end on the same level with and 60 centimetres from the needle, and due (magnetic) south of it, the number of oscillations of the needle is 12 per minute. Calculate the strength of the pole of the iron rod, (1) neglecting, (2) taking account of, the influence of the upper end. (B. of E., 1901.)

34. A magnet 10 centimetres long is placed in the magnetic meridian, the "north" end of the magnet being to the south. The force due to this magnet just counterbalances the earth's horizontal force (0·18 C.G.S. units) at a place 35 centimetres from the centre of the magnet (along its axis produced). Find the strength of each pole of the magnet. (B. of E., 1905.)

35. Explain how to determine the intensity of a magnetic field by observing the time of oscillation of a needle. A magnetic needle under the influence of the earth alone makes 10 oscillations per minute. When placed at a point A in the magnetic field it makes 25 oscillations per minute. Find the intensity of the field at A in terms of the earth's field. (B. of E., 1905.)

36. Two bar magnets, respectively 10 and 12 centimetres long, are placed at right angles to the magnetic meridian; the first 10 centimetres north and the other 12 centimetres south of a small compass-needle. Compare the moments of the magnets if the needle is not deflected. (B. of E., 1908.)

37. What is meant by a neutral point in a magnetic field? There is found to be a neutral point on the prolongation of the axis of a bar magnet at a distance of 10 centimetres from the nearest pole. If the length of the bar be 10 centimetres, and $H = 0{\cdot}18$ C.G.S. units, find the pole strength of the magnet. (Camb. Local, Senior, 1902.)

38. Two magnets have the same pole strength, but one is twice as long as the other. The shorter magnet is placed in the "end-on" position with its centre at a distance of 20 centimetres from the axis of suspension of a magnetometer needle. Where may the other magnet be placed in order that there may be no deflection of the needle? (Camb. Local, Senior, 1908.)

39. A rectangular bar magnet, 10 centimetres long, is magnetised until the strength of its poles is 150. Find the intensity of magnetisation if it has a sectional area of 1 square centimetre.

40. Two short magnets, with their axes horizontal and perpendicular to the magnetic meridian, are placed with their centres 30 centimetres east and 40 centimetres west respectively of a compass-needle. Compare the moments of the magnets if the needle remains undeflected, and show how to derive the formula employed in the calculation. (B. of E., 1911.)

CHAPTER XIII

TERRESTRIAL MAGNETISM

Exp. 123. Place a glass beaker (to exclude draughts) over a pivoted compass-needle two or three inches long. Hold a soft iron bar two or three feet long (*e.g.* a poker), so that it points roughly east and west, and tap it with a piece of wood. Then test it for polarity. It may prove to be very weakly magnetised, or not sensibly magnetised at all. Hold it in the plane of the meridian, sloping down at an angle of 60° or 70°, and having its lower end at the same level as the N pole of the compass-needle and at the *side* of it. Tap the bar with the wood, and notice that the needle is suddenly deflected, the N pole being repelled, which shows that the lower end of the bar has become a N pole. Carefully invert the bar, and show that the other end is a S pole. Now tap it again, and observe that the attraction suddenly becomes a repulsion, showing that the magnetism of the bar has been reversed.

Repeat the experiment with the bar (1) vertical, (2) horizontal and in the meridian. Notice that the general effect is the same in each case, although it is perceptibly weaker in the second position. Repeat with the bar horizontal and at right angles to the meridian, and notice that no effect is obtained.

If a reflecting magnetometer be used, larger deflections will be obtained, and it will then be possible to show that the greatest effect is produced when the bar is inclined at an angle of about 70° to the horizontal. Hence, the earth's lines of force slope downwards in this part of the world, and *the angle they make with a horizontal line in the magnetic meridian is known as the angle of inclination or dip.*

It follows that a freely suspended magnet tends to place itself in this position, so that a compass-needle pivoted at its centre of gravity must be slightly weighted to keep it truly horizontal (or its point of suspension must be slightly removed from its centre of gravity towards the N pole).

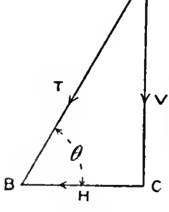


FIG. 131.

Let the angle ABC (Fig. 131) be equal to the angle of dip. Let AB represent the total strength of the earth's field, (T), drawn to any convenient scale, then if AC is the perpendicular on BC, AC represents the vertical component of the earth's field (V), and BC the horizontal component (H), on the same scale.

Remembering that the strength of the field is measured by the number of lines of force per square centimetre, the meaning of these terms may be illustrated as follows: Suppose that a piece of card, containing a hole 1 square centimetre in area, is held at right angles to the actual direction of the earth's field, *i.e.* at right angles to AB. Then the number of lines passing through the aperture is the value of T. (The earth's field is so weak that this number is less than unity, see p. 133.) If the card be held vertically and at right angles to the meridian, *i.e.* at right angles to BC, the lines will slope through the aperture, and the smaller number passing through will be the value of H. Similarly, if the card be laid horizontally on the table, the number passing through will be the value of V.

From what has been already said in several places, it follows that the same numbers also represent the respective forces in dynes acting on a unit pole, and this is, in fact, the usual meaning to be attached to these terms in dealing with the earth's field.

It follows at once from Fig. 131, that if any two of the four quantities, T, V, H, and θ, are known, the other two may be calculated, for we have

$$T^2 = V^2 + H^2$$

$$H = T \cos \theta$$

$$\frac{V}{H} = \tan \theta, \text{ \&c.}$$

In practice, it is usually convenient to measure H and θ. Now, H is the effective component acting on a magnet free to move only in a horizontal plane, and has, therefore, entered into various formulæ in the preceding chapters.

It will be seen from the figure that as the angle of dip increases, the horizontal component decreases. For this reason, the movements of a compass-needle become sluggish near the magnetic poles, and its readings are more affected by friction, for the moment of the restoring couple, MH sin θ, is small. On the other hand, in the regions near the magnetic equator, it is the vertical component which becomes smaller, and in certain places vanishes.

Declination.—A compass-needle does not usually point to the true north and south. *The angle contained between the magnetic and the geographical meridians at any given place is called the* **angle of declination** *at that place.* In this country it is at present about 16° west of true north. It follows that the magnetic poles of the earth do not coincide with the geographical poles. In 1831, Sir James Ross found that the magnetic north pole was situated in Boothia Felix, 96° 46′ W. longitude, and 70° 5′ N. latitude. The south magnetic pole was reached in the year 1909. It is situated at 154° E. longitude and 72° 25′ S. latitude.

Roughly speaking, we may represent the magnetic state of the earth by supposing that a comparatively short bar magnet is placed at its centre, having its axis slightly inclined to the axis of rotation (see Fig. 79). Of course, there is no suggestion that such is the case: the fact that iron becomes non-magnetic at a red heat, taken in conjunction with the high temperature of the interior of the earth, suggests that we must look in the surface layers to a moderate depth for the seat of terrestrial magnetism, and the great local irregularity in distribution adds support to this view.

Magnetic Elements.—The magnetic state at any given place is completely determined if we know the values of

(1) **The declination**
(2) **The inclination** } at that place.
(3) **The total intensity (T)**

These are known as the *terrestrial magnetic elements* of the place.

As regards the third one, it is usual to measure H, and deduce T from H and the angle of dip.

To Find the Direction of the Magnetic Meridian.—The actual direction of the magnetic meridian at a given place (*e.g.* inside a laboratory) may be roughly determined by means of a freely

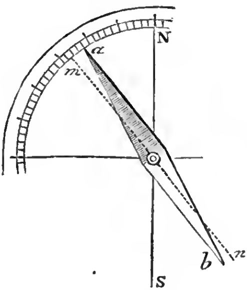


FIG. 132.

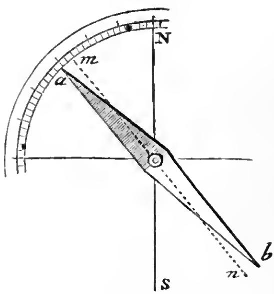


FIG. 133.

suspended or pivoted magnet shaded from draughts. It is, however, important to remember that the axis of magnetism does not of necessity exactly coincide with the axis of length, and it is therefore usual to make a second determination with the magnet turned over, *i.e.* the upper side is now downwards. The two directions will not, as a rule, coincide with the true magnetic meridian, *e.g.* if, in Figs. 132 and 133, *ab* is the axis of figure, and *mn* the magnetic axis, the first position (Fig. 132) gives the reading N*a* too small, the true declination being N*m*. After reversing the needle (Fig. 133), the reading N*a* is too great, the true declination being N*m*. The true declination is, therefore, the mean of the two readings.

These diagrams are inserted here in order to show clearly the nature of the error, but it must be remembered that in such an experiment we usually wish to mark out the direction of the meridian in space, *e.g.* across a table, or along the floor of the laboratory, and for this purpose readings on a scale of degrees are useless. For a rough determination, four knitting-needles may be held vertically—two at each end of the magnetic needle, and as far apart as convenient—and their positions adjusted until they are exactly in a line with the needle. This direction is then marked on the table, the needle inverted and the process repeated. Finally, the angle between the two directions thus obtained is bisected by a third line, which is the meridian required.

Measurement of Declination.—To measure the declination, it is evidently necessary to determine the directions of both magnetic and geographical meridians, and for the latter purpose an astronomical observation of some kind is required.

In the Kew pattern declination instrument, shown in Fig. 134 (taken from Carey Foster and Porter's *Electricity and Magnetism*), the magnet is in the form of a hollow steel tube, suspended inside a glass case by means of a silk fibre attached to S. This magnet can be viewed end-on by means of the telescope T. In the end of the tubular magnet farthest away from the telescope, a small glass-scale is fixed, and at the other end a convex lens, having a focal length equal to the length of the tube. This enables the scale to be seen distinctly through the telescope. The stand carrying the telescope and case is turned round until the middle division of the scale is seen on the cross wires inside the telescope, and the direction of the latter is noted on the graduated circle C. Then the tubular magnet is turned over and the observations repeated, the mean of the two readings giving the true direction of the magnetic meridian. It will be seen that the instrument merely enables the observations, previously mentioned, to be made with much greater accuracy.

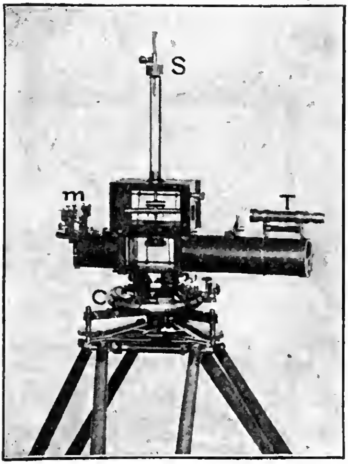


FIG. 134.

For the determination of the direction of the geographical meridian, the adjustable mirror *m* is provided, by means of which it is possible to view any convenient star (or the sun) through the telescope, the telescope and star (or sun) being then in the same vertical plane. The exact time at which the observation is made is noted. The position of the star or of the sun at that instant with reference to the geographical meridian is then found by referring to the *Nautical Almanac*, and thence the actual reading of the geographical meridian on the instrument becomes known.

Inclination or Dip.—*The angle which a line drawn in the actual direction of the earth's field makes with the horizontal line, is called the angle of inclination or dip.*

Exp. 124. Fasten a fibre of untwisted silk to the middle of a steel knitting-needle. Cause the needle to hang horizontally, filing one end if necessary. Magnetise it by rubbing it with a magnet, and observe that it sets itself in the magnetic meridian with its N pole dipping downwards.

Evidently, the needle takes up this position owing to its tendency to point along the actual direction of the earth's field, but in order that its direction may accurately coincide with that of the field, it is necessary to ensure that no other couple is acting upon it.

Instruments for measuring dip are known as dip circles.

Dip Circle.—One form of the instrument for ascertaining the inclination is shown in Fig. 135, which is, however, not sufficiently delicate for very accurate measurements. It consists of—

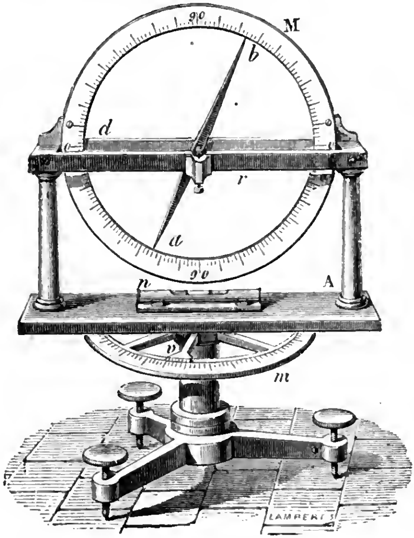


FIG. 135.

(*a*) A graduated horizontal brass circle *m*, supported on three legs, which are provided with levelling screws;

(*b*) above this is a plate A, moving about a vertical axis, which supports

(*c*) a vertical graduated circle M, upon which the dip is measured. This moves with the plate A, and is therefore capable of horizontal rotation.

(*d*) A magnetic needle *ab*, supported on a horizontal axis (*which passes through its centre of gravity*) at the centre of the graduated circle M, so that it is capable of moving in a vertical plane.

(*e*) A spirit-level *n*, fixed to the plate A.

Method of Determining the Dip.—(1) Level the instrument. This is done by turning the screws until the air-bubble lies in the middle of the spirit-level.

(2) Turn the plate A on the circle *m* until the needle is vertical. The plane of the needle is now at right angles to the magnetic meridian, because the horizontal component now acts at right angles to the plane in which the needle moves, so that its only effect is to increase the pressure on one support of the needle, and to diminish it on the other. The vertical component, therefore, acts alone, and causes the needle to rest vertically.

(3) Turn A through 90° on the circle *m*. This, of course, brings the needle into the meridian.

Too much reliance must not, however, be placed on this adjustment, as the determination of the true vertical position requires correction for various errors, *e.g.* 1, 2, and 3 below.

(4) (*a*) Read the angles between both poles and the horizontal line passing through its point of suspension.

(*b*) Turn the needle so that the faces are reversed, and again read the two angles.

(*c*) Turn the plate A through 180°, so that the readings are now on the other side of the scale. Take the two readings.

(*d*) Turn the needle so that the faces are reversed, and again take two readings, making eight readings in all.

(*e*) Remagnetise the needle so that its polarity is reversed, and repeat operations (*a*) (*b*) (*c*) (*d*), thus obtaining eight more readings.

The true dip is found by taking the mean of the sixteen readings.

Sources of Error.—The reason of taking the eight readings given in (*a*) (*b*) (*c*) (*d*) is due to the fact that there are several sources of error to which the instrument is liable.

(1) *The error of centering*, which is due to the axis, about which the needle moves, not passing through the centre of the vertical circle. This error is easily obviated by reading the angle at both ends of the needle. The reason of this will be easily understood by referring to Fig. 136, in which a simple example is shown.

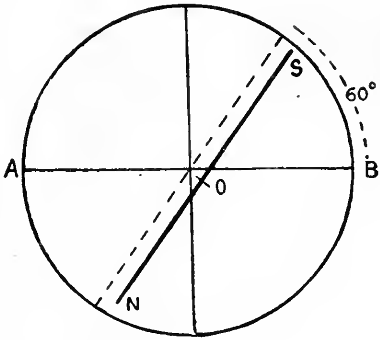


FIG 136.

Suppose that the true dip is 60°, as shown by the dotted line. If, however, the axis pass through the circle at O, then the needle will rest in the position NS. If the angle BOS be 59°, then the angle AON will be 61°, the mean of which is 60°.

(2) The axis of figure may not coincide with the magnetic axis, *i.e.* the geometrical axis may not contain the poles. This error is corrected by reversing the needle so that the face which lies towards the observer is turned towards the instrument. This will be understood from Figs. 132 and 133, and the example thereon.

(3) The divided circle M may not be accurately set, *i.e.* the line joining the 90° marks may not be truly vertical. The error thereby caused is eliminated by turning the instrument through 180°, and thus obtaining readings on the opposite sides of the scale.

(4) The axis, about which the needle moves, may not pass through its centre of gravity. This causes the angle *da* on the graduated scale (Fig. 135) to be either too large or too small; if the centre of

gravity is below the axis of suspension, the force of gravity acting on the needle will make the angle too large, as in Fig. 137; if it be *above* the axis of suspension the angle will be too small, as shown in Fig. 138.

To correct this defect the polarity must be reversed by remagnetisation, each end of the needle receiving opposite magnetism to that which it had at first. The dip is again observed, and the mean of the readings taken.

(5) Friction between the supports and the axle of the needle must

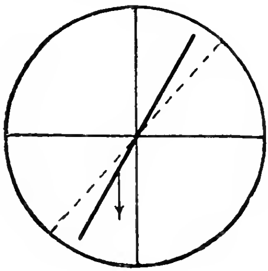


FIG. 137

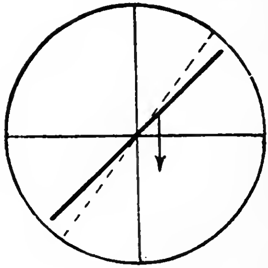


FIG. 138.

be diminished as much as possible. Agate knife-edges are generally employed for bearings.

It will be noticed that everything depends on the exact adjustment of the needle in the magnetic meridian. If this condition is not satisfied, an error will be introduced which cannot be eliminated. For this reason, it is sometimes preferable to avoid the difficulty by determining the apparent dip in two positions lying on each side of the meridian and separated by 90°. It is then only necessary to turn the instrument through exactly a right angle, and this can be done with great accuracy. But it involves taking two complete sets of readings.

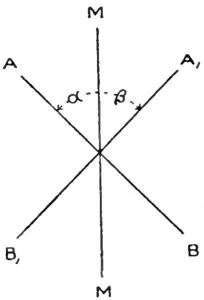


FIG. 139.

In Fig. 139 let MM be the direction of the meridian, and let θ_1, θ_2 be the measured values of dip when the needle moves in the planes AB and A_1B_1 respectively, making angles α and β with the meridian; where $\alpha+\beta=90°$. Let θ be the true angle of dip.

The couple due to the vertical component V is not affected by the displacement, but that due to the horizontal component is now partly acting as a twist on the bearings, and its effective portion resolved along the line AB is H cos α.

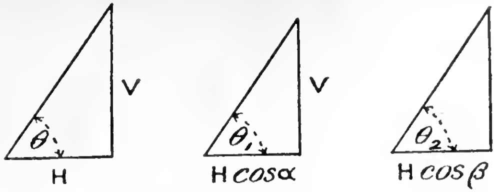


FIG. 140.

Hence, we obtain the three triangles shown in Fig. 140, from which we have—

$$\frac{V}{H} = \tan \theta \qquad (1)$$

$$\frac{V}{H \cos \alpha} = \tan \theta_1 \qquad (2)$$

$$\frac{V}{H \cos \beta} = \tan \theta_2 \qquad (3)$$

From (1) and (2) $\cos \alpha = \dfrac{\tan \theta}{\tan \theta_1}$

From (1) and (3) $\cos \beta = \dfrac{\tan \theta}{\tan \theta_2} = \sin \alpha$

$$\therefore \quad \cos^2 \alpha = \left(\frac{\tan \theta}{\tan \theta_1}\right)^2$$

and $\sin^2 \alpha = \left(\dfrac{\tan \theta}{\tan \theta_2}\right)^2$

but $\sin^2 \alpha + \cos^2 \alpha = 1$

$$\therefore \quad 1 = \left(\frac{\tan \theta}{\tan \theta_1}\right)^2 + \left(\frac{\tan \theta}{\tan \theta_2}\right)^2$$

or $\dfrac{1}{\tan^2\theta} = \dfrac{1}{\tan^2\theta_1} + \dfrac{1}{\tan^2\theta_2}$

i.e. $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

Construction of a Simple Dip Circle (Fig. 143).—A dip circle may be constructed on the following plan, and although the results obtained with it are merely approximate, the experiments will give us a practical insight into the methods usually adopted in accurately determining the dip at a place.

(1) Cut two circular discs of cardboard, 10 inches in diameter. On *each* draw two diameters, AB, CD (Fig. 141), at right angles to each other. With

the centre O, describe two arcs, EF, GH; the outer one being 1 inch, and the inner one $1\frac{3}{4}$ inches from the circumference.

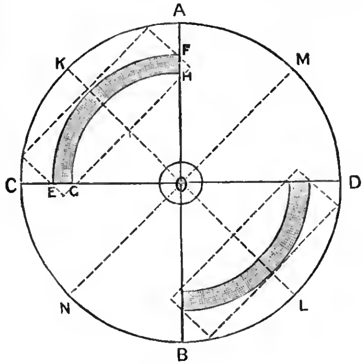


FIG. 141.

Describe similar arcs in the vertically opposite quadrant; and then with a very sharp knife cut out the portions shaded in the figure. Draw two diameters, MN, KL, to bisect the angles AOD, AOC. Now graduate the circumference AC, BD of the two quadrants as accurately as possible into 1-degree spaces, marking the point 90° where the dotted line KL meets the circumference. The points A, C are thus marked 45°, and M, N, 0° (see also Fig. 143).[1]

(2) Cut four pieces of looking-glass (6 inches long and 2 inches broad), and glue them to the back of the cardboard, in the position shown by the dotted parallelograms. By this means the shaded space EGHF and the corresponding one in the vertically opposite quadrant are completely covered by them; the mirror side, of course, is towards the graduated faces. Now glue the two discs, back to back, so that the diameters AB, CD on one face are similarly situated to those on the other.

(3) Bore a hole through two thin small corks, and insert in each hole a small piece of narrow glass tubing, having previously heated each end to remove the rough edges. Glue the corks to the cardboard circles so that the glass tubes are exactly at their centres.

(4) Cut a board (12 inches long, 6 inches wide, and $\frac{3}{4}$ inch thick) to form the base of the instrument. Also cut two strips of wood (square in section) 6 inches long and $\frac{3}{4}$ inch wide. Make two square holes in the base—the outer edges of which are $\frac{1}{2}$ inch from each end of the board—to admit one end of the strips. Having cut a groove down one side of each strip ($1\frac{1}{2}$ inches long and about $\frac{1}{4}$ inch deep) to admit and support the cardboard disc as shown in Fig. 143, glue them in their places.

(5) Now fix the cardboard on the pillars so that the zero points are in a horizontal line.

(6) Take two pieces of glass rod, soften one end of each piece in a blow-pipe flame, and while hot make an indentation as shown in Fig 142. Fix these in the base, one on each side of the cardboard, so that if one end of a sewing-needle be placed in the glass tubing at the centre and the other rests in the indentation, the needle may be exactly horizontal and at right angles to the cardboard circles.

FIG. 142.

(7) Cut a piece of clock-spring to form a lozenge-shaped needle, $9\frac{1}{2}$ inches long. Drill a hole as nearly as possible at its centre of gravity. Using a small piece of solder, fasten a fine sewing-needle through the hole, one half projecting on each side—this forms an axis at right angles to the strip. It will probably be found necessary to file one end of the lozenge-shaped needle, until, *before being magnetised*, it remains at rest on its axis *in any position whatever*. When this is the case, its centre of gravity coincides with its axis of rotation.

[1] Cardboard scales can be bought at the cost of a few pence.

(8) Bore a hole in the wooden base in order to insert the axis (the sewing-needle), while the strip of steel is being magnetised.

(9) Magnetise the lozenge-shaped needle by the method of separate touch.

Exp. 125, *to show the method of obtaining the dip with this instrument.* Place the instrument in the magnetic meridian, and repeat the sixteen observa-

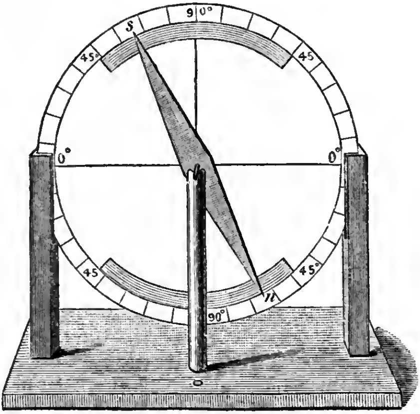


FIG. 143.

tions given on p. 179. The following table, which represents the results of a particular experiment, is given as a guide. A is one face of the needle, B the other.

Position of Magnetic Needle.	Position of Face of Instrument.	Deflections, Upper, Lower.	Mean.	True Dip.
A	East	66·5°, 67·5°	67°	
B	,,	68°, 69°	68·5°	
A	West	66°, 67°	66·5°	
B	,,	67·5°, 68·5°	68°	
After remagnetisation				67° nearly.
A	East	67°, 68°	67·5°	
B	,,	66·5°, 66°	66·25°	
A	West	67·5°, 68°	67·75°	
B	,,	66°, 65·5°	65·75°	

Exp. 126, *to find the angle of dip by means of a reflecting magnetometer.* After adjusting the scale so that the spot of light is at zero, place a rod of soft iron, about 3 feet long and ¼ inch in diameter, horizontally, as shown in Fig. 144, so that it lies in the magnetic meridian with one end immediately behind the magnetometer mirror—the best distance from the magnetometer will be found by trial. Notice that there is a deflection, but before taking a reading, tap the rod several times with a piece of wood. This will help the rod to

acquire the full amount of magnetisation possible under the influence of the horizontal component of the earth's field. Read the deflection, say d divisions, and notice that its direction indicates that A is a N pole. Raise the other end, B, until the rod is exactly vertical, as determined by a simple plumb-line. Do this without altering the position of A, and, hence, to obtain good results the end A should be held by a pin or by some simple device which keeps it in position when the end B is raised. The rod is now under the influence of the vertical component of the earth's field. Tap it again with the wood, and read the deflection, say d_1 division, which will be greater than before. Repeat the operations a number of times until fairly consistent readings are obtained, and take the average of the best results.

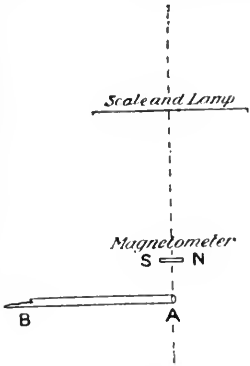


FIG. 144.

Then we have $V \propto d_1$

$H \propto d$

and $$\frac{V}{H} = \tan \text{dip} = \frac{d_1}{d}$$

To Determine the Total Intensity.—As already stated, to determine the third magnetic element, T, it is usual in magnetic surveys to measure H, and then use the relation $H = T \cos \theta$, where θ is the angle of dip. The measurement is carried out by means of a portable instrument, which enables the ratio $\frac{M}{H}$ to be obtained by a slightly modified form of the method explained on pp. 159–161. The time of vibration of the deflecting magnet is also taken in order to determine the value of the product MH.

Magnetic Variations—(*a*) **Variations in Declination.**—The declination varies at different places on the earth's surface. At present it is west in Europe, but east in many places in Asia and in North and South America. At some places there is no declination, *i.e.* the geographical and magnetic meridians coincide.

The declination is, moreover, never constant even at the same place, in fact, the needle of an exceedingly delicate instrument is never at rest.

Such variations are of two kinds:—

(1) Regular, including secular, annual, and diurnal;
(2) Irregular or accidental.

Secular Variation.—There is a gradual change in the direction of the compass-needle at any particular place; the needle at one time pointing to the west, and at another time to the east, of the true north.

The table at top of p. 185 shows the secular variation at Greenwich.

From it we see that before the year 1657 the declination was east; in that year the geographical and magnetic meridians coincided; afterwards the declination became west; and that the greatest westerly declination was reached in 1816. At the present time the declination is slowly decreasing, but only at the rate of about 7′ per year.

Year.	Declination.	Year.	Declination.
1580	11° 17′ E.	1890	17° 29′ W.
1634	4° 0′ E.	1893	17° 11′ W.
1657	0° 0′	1895	16° 57′ W.
1705	9° 0′ W.	1898	16° 39′ W.
1760	19° 30′ W.	1899	16° 34′ W.
1816	24° 30′ W.	1902	16° 22′ W.
1868	20° 33′ W.	1906	16° 3′ W.
1882	18° 22′ W.	1909	15° 47′ W.
1889	17° 35′ W.		

Annual Variation.—The needle is also subject to small annual variations. In London, this variation is greatest at the vernal equinox, and smallest at the summer solstice, after which it gradually increases during the next nine months.

Diurnal Variation.—With very sensitive instruments the needle is observed to have a daily motion. In England the N pole of the needle moves westward every day from 7 A.M. to about 1 P.M. It then begins to move eastward, and continues to do so until about 10 P.M. It approximately retains this position until sunrise.

Irregular Variations.—It sometimes happens that the needle is suddenly disturbed. These irregular and accidental disturbances are known as **magnetic storms**, and frequently accompany such natural phenomena as volcanic eruptions, earthquakes, and the aurora borealis.

Isogonic and Agonic Lines.—Charts have been prepared on which places having equal declination are joined by a line. Such lines are called *lines of equal declination*, or *isogonic lines*. Similarly, the line joining places where there is no declination is called the *agonic line* (see Map, p. 186).

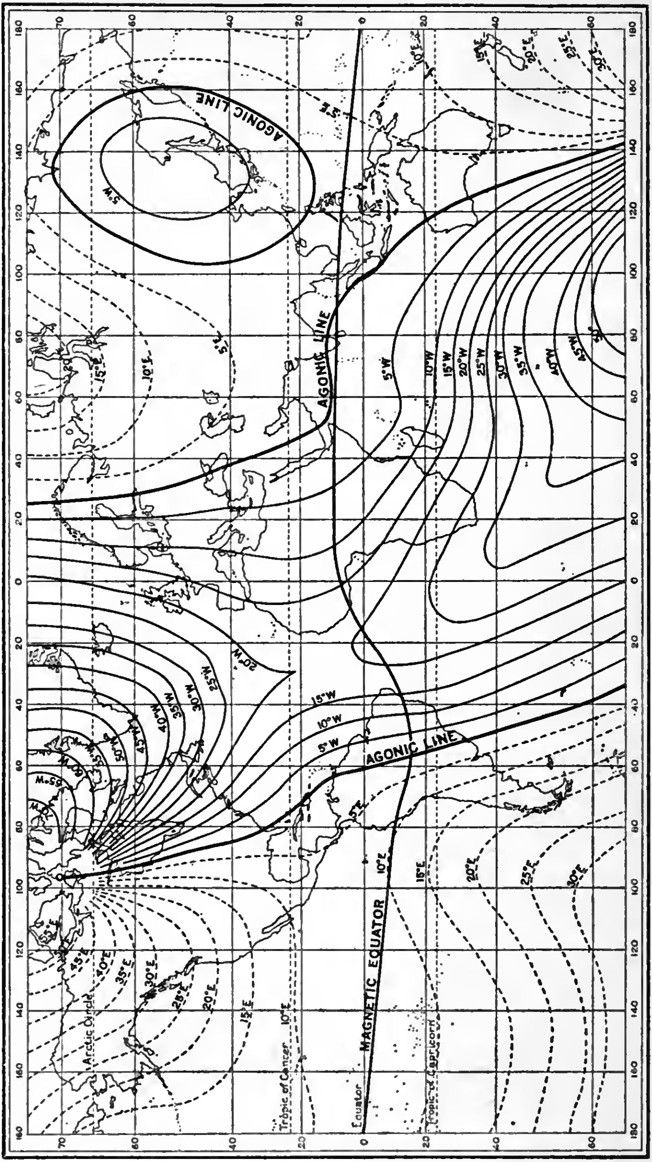
(*b*) **Variations in Dip.**—The value of the dip, like that of declination, varies in different places on the earth's surface. At the magnetic poles the needle is vertical, *i.e.* the dip is 90°. In London (lat. 51° N.) in 1912, the dip was 67° nearly. At the magnetic equator there is no dip. In the southern hemisphere the inclination of the needle is similar to that in the northern hemisphere, but the S pole is downwards.

Dip is also subject to **secular** change, *i.e.* its value alters during a very long period of time.

The following table shows its alterations near London :—

Year.	Inclination.	Year.	Inclination.
1576	71° 50′	1890	67° 23′ 0″
1676	73° 30′	1895	67° 15′ 0″
1723	74° 42′	1898	67° 12′ 0″
1800	70° 35′	1899	67° 10′ 0″
1828	69° 47′	1903	67° 0′ 51″
1854	68° 31′	1906	66° 55′ 0″
1874	67° 43′	1910	66° 52′ 36″

LINES OF EQUAL DECLINATION, 1907.



GEORGE PHILIP & SON L^TD

Longmans, Green & Co., London, New York, Bombay & Calcutta.

Reproduced by permission from the Admiralty Chart.

Isoclinic and Aclinic Lines.—The imaginary lines which join places at which the dip is of the same value, are called *isoclinic lines.* The line which joins places at which there is no dip, is called the *aclinic line,* or *magnetic equator* (see Map, p. 188).

Magnetic Elements at Various Places.—The following table, selected from the Report for the year 1909 of the Observatory Department of the National Physical Laboratory, shows the mean values of the magnetic elements at various places for the year specified :—

Place.	Year.	Declination.	Inclination.	Horizontal Force in C.G.S. Units.	Vertical Force in C.G.S. Units.
Greenwich . .	1908	15 53·5 W.	66 56·3 N.	·18528	·43518
Kew . . .	1909	16 10·8 W.	66 59·7 N.	·18506	·43588
Hamburg . .	1903	11 10·2 W.	67 23·5 N.	·18126	·43527
Valencia (Ireland).	1909	20 50·3 W.	68 15·1 N.	·17877	·44812
Uccle (Brussels) .	1908	13 36·7 W.	66 1·6 N.	·19061	·42867
Munich . .	1906	9 59·5 W.	63 10·0 N.	·20657	·40835
Baldwin (Kansas) .	1906	8 30·1 E.	68 45·1 N.	·21807	·56081
Athens. . .	1908	4 52·9 W.	52 11·7 N.	·26197	·33613
Hong Kong . .	1908	0 3·9 E.	31 2·5 N.	·37047	·22292
Alibag (Bombay) .	1908	1 2·2 E.	23 21·8 N.	·36857	·15922
Melbourne . .	1901	8 26·7 E.	67 25·0 S.	·23305	·56024
Manila. . .	1904	0 51·4 E.	16 0·2 N.	·38215	·10960
Mauritius . .	1908	9 14·3 W.	53 44·9 S.	·23415	·31932
Christchurch (N.Z.)	1903	16 18·4 E.	67 42·3 S.	·22657	·55259

The lines connecting places where the force is of the same value are called *isodynamic lines,* (See Map, p. 189.)

Recording Instruments.—In magnetic observatories, instruments are used which give a continuous photographic record of changes in H, V, and declination, from which the changes in dip and total intensity may be deduced.

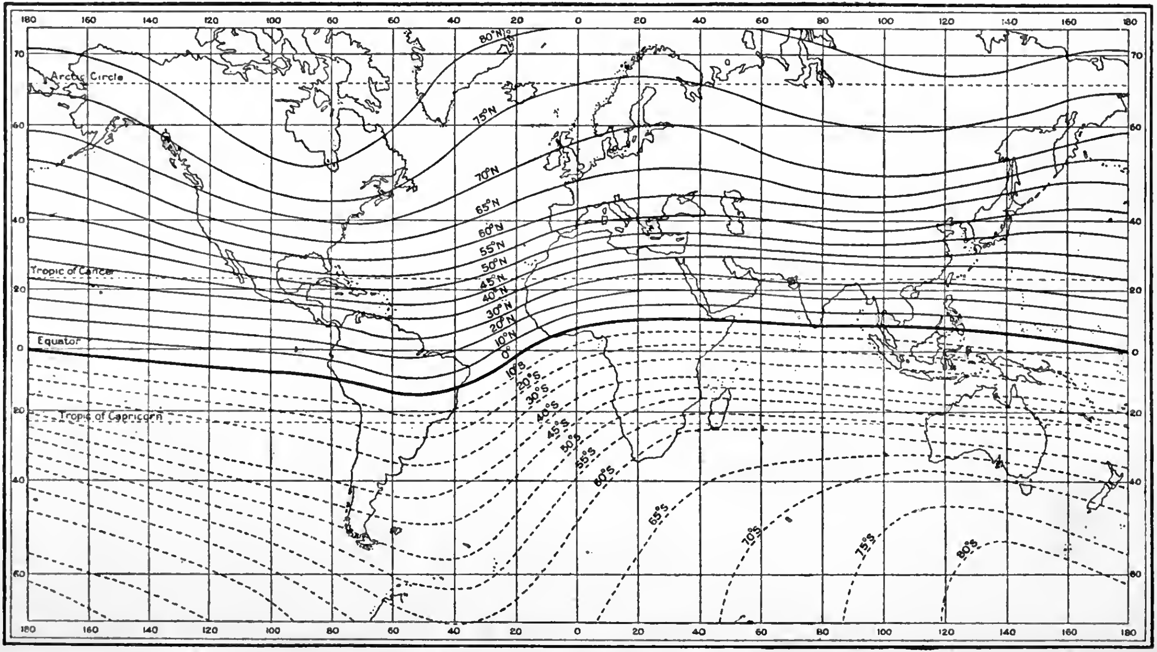
The declination instrument is a delicately suspended horizontal bar magnet (usually made of several small magnets fixed together, see p. 119), to which a mirror is attached, and a spot of light is reflected from this mirror upon a slowly moving strip of photographic paper.

The instrument for measuring the horizontal force consists of a similar horizontal bar magnet having a bifilar suspension attached to a torsion head. This head is rotated until the magnet is at right angles to the meridian, in which position the restoring couple MH due to the earth's field is balanced by the couple due to the twist on the suspension. The latter may be regarded as constant, and hence any change in H will cause a small displacement of the bar. Such displacements are recorded as mentioned above.

It is not easy to convert a dip needle into an automatic recording instrument, and hence the vertical force is measured instead. In this instrument, the bar magnet is supported horizontally on knife-edges—not unlike the arms of a balance. Evidently it tends to dip, and this tendency is balanced by placing weights in a scale-pan. Any alteration in V will upset the equilibrium, and will cause a slight displacement of the magnet in a vertical plane.

In each instrument, a simple device is used in order to obtain a constant zero line from which the displacements can be measured. The mirror is fixed below the moving part, and is semicircular in shape. A similar semicircular mirror

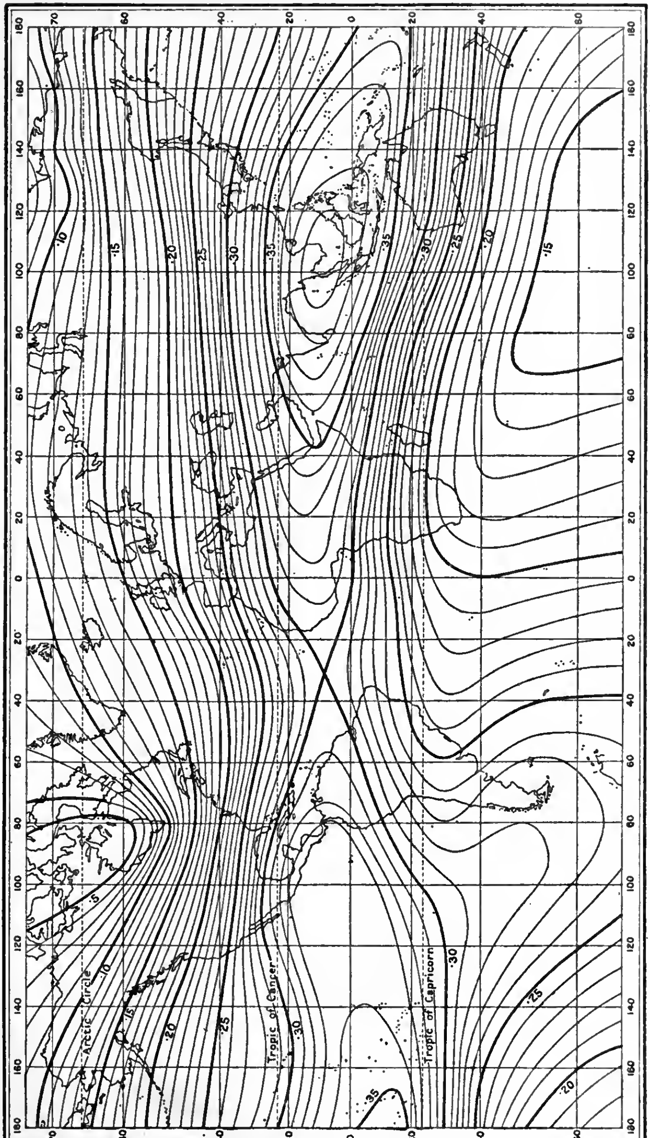
Lines of Equal Magnetic Dip, 1907.



Longmans. Green & Co. London. New York. Bombay & Calcutta.

Reproduced by permission from the Admiralty Chart.

MEAN LINES OF EQUAL HORIZONTAL FORCE, 1907.



Longmans, Green & Co., London, New York, Bombay & Calcutta.

Reproduced by permission from the Admiralty Chart.

(the other half of an ordinary circular mirror) is rigidly fixed exactly below it. Light from a suitable source falls upon both halves, each of which forms an independent spot of light upon the sensitive paper. When there is no disturbance, the trace will be two straight lines, but generally one will be a wavy line, and the amplitude of its excursion is determined by reference to the straight comparison line.

The Mariner's Compass.—Terrestrial magnetism has a most important influence on navigation. As a magnetic needle always points to the magnetic north and south poles, and as declination charts have been prepared, the mariner is enabled to guide his vessel from port to port by its indications.

A simple form of compass for this purpose consists of (*a*) a flat circular card, on the *under* surface of which one or more light magnetic needles are fastened, which are capable of moving horizontally on a pivot of steel or iridium, fitting into an agate cap.

The card is divided into thirty-two divisions, which are called *rhumbs* or *points of the compass.* These divisions are obtained as follows:—

The circle (Fig. 145) is divided (1) into four quadrants by means of two diameters at right angles. The extremities of these diameters are marked N, S, E, and W (north, south, east, and west).

(2) The four right angles thus formed are bisected by lines, the extremities of which form the NE, NW, SE, and SW points. They are named by placing together the two letters at the extremities of the bisected quadrant; *e.g.* NE is the point midway between N and E.

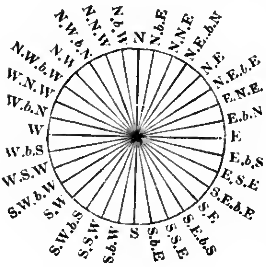


FIG. 145.

(3) These eight angles are bisected; the extremities of the lines are named by placing together the letters at the ends of the arms of the bisected angles, remembering, however, to put the name of the cardinal point *first*, and then the name of the other point: *e.g.* the point midway between the N and NE points is marked NNE (north-north-east); that between S and SW is named SSW, and so on.

(4) The sixteen angles are again bisected; any one of the points thus formed is named by placing (*a*) the name of *one* of the nearest points to it (precedence being given to the point *first obtained* in the above process—*i.e.* N takes precedence of NNE; NE takes precedence of NNE or ENE); and (*b*) the name of the other nearest cardinal point, the two names being separated by the letter *b* (by); *e.g.*

The point	between	N	and	NNE	is marked	N *b* E.
,,	,,	NE	,,	ENE	,,	NE *b* E.
,,	,,	E	,,	ESE	,,	E *b* S.
,,	,,	SW	,,	WSW	,,	SW *b* W.
,,	,,	NW	,,	WNW	,,	NW *b* W.

(*b*) The needle and card are enclosed in a copper bowl provided with a strong glass cover.

(*c*) The bowl is supported on gimbals, *i.e.* two concentric brass rings, one of which, fastened to the bowl, moves about two pivots. These two pivots are fastened into the outer ring, which rests, by means of two similar pivots placed at right angles to the former, on a support.

By means of the gimbals, the bowl always remains horizontal, but in order to obtain steadiness and freedom from the oscillations set up by the motion of the ship, the time of vibration of the magnet system must be great compared with the average period of the ship's motion. As $t=2\pi\sqrt{\frac{I}{MH}}$, and as H is constant, we see that t may be made large either by increasing I or by diminishing M. The simplest way

FIG. 146.

of increasing I is to increase the mass of the system, and in the older forms of compass, this plan was usually adopted—a comparatively heavy magnet (or system of magnets) being employed. On the other hand, it is important that the friction on the point should be small, a condition which is satisfied when the moving system is as light as possible.

Lord Kelvin showed how to reconcile these apparently conflicting conditions. His pattern of compass-needle is shown in Fig. 146. It consists of an aluminium ring to which the central jewelled bearing for the pivot is attached by silk threads. Eight short pieces of steel, like pieces of knitting-needles, are fastened to two silk threads, which are attached to the radiating silk threads. The whole arrangement for a 10-inch circle weighs less than half an ounce. The magnetic moment is small, and so also is the moment of inertia, but as the construction places the greater part of the mass in the rim, where it is most effective in producing moment of inertia, I is large relatively with respect to M, which is exactly the condition required to make t great.

Effect of Iron Ships on the Compass.—The magnetism of iron ships may be roughly ascribed to two general causes: (1) The ship acts like a weak *permanent* magnet, due to the magnetism acquired during construction, its direction and strength depending on the ship's position during that time. This does not alter in direction with the movements of the vessel. (2) The ship may be regarded as a mass of iron acted upon inductively by the earth's field. The direction of this induced magnetism will change with each movement of the ship, and will also depend on its position on the earth's surface.

These errors are detected and compensated by the operation of "swinging" the ship, *i.e.* the vessel is turned completely round so that its bow is directed in succession to every point of the compass, and in each position the compass reading is compared with the known direction of the meridian.

It will be seen that the effect of the permanent magnetism will resemble that due to rotating a bar magnet under a compass-needle. There will be no deviation when the bar magnet points N and S, and the greatest deviation when it points E and W. Hence, the error will reach a maximum twice in each revolution (*i.e.* once in each semicircle), and such deviation is, therefore, known as the **semicircular deviation.** As the permanent magnetism is not usually lengthways, it is conveniently regarded as consisting of three rectangular components—one lengthways, one sideways, and one vertical (the latter only affecting the compass-needle when the vessel rolls)—and compensation is obtained by placing three sets of permanent magnets inside the binnacle below the compass, pointing forwards, sideways, and vertically respectively, each set being separately adjusted by trial to neutralise the deviation due to the corresponding component.

Again, the *induced* magnetism will, as the ship swings round, produce an effect similar to that obtained by rotating a bar of soft iron beneath a compass-needle. There will be no deviation when the bar points N and S, and, also, none when it points E and W, because in the latter position it is at right angles to the earth's field, and is, therefore, not magnetised. But in the four intermediate positions there will be deviations, reaching a maximum when the vessel points NE, SW, NW, and SE, and hence this is known as **quadrantal deviation.**

A large mass of soft iron symmetrically placed with respect to the needle would not affect it in any way; for instance, if two soft iron bars, fixed together at right angles, were rotated beneath it, one bar would tend to neutralise the deviation produced by the other. The quadrantal error is, therefore, eliminated by fixing spheres of soft iron on each side and very close to the compass, their exact distance being adjusted by trial. In this way, the effect of the lengthways distribution of iron in the vessel is compensated. Lord Kelvin first made this method of correction practicable by introducing his small magnetic needles, for the older type would have required inconveniently large spheres of iron.

Thus far, we have merely considered the induced magnetism due to the horizontal component, but the vertical component produces its effect, the result being another semicircular error quite distinct in nature from the one already mentioned, as well as a disturbance when the vessel rolls. Without going into details, it will be sufficient to say that the former effect is neutralised by placing a vertical soft iron bar in some definite position—usually in front of the binnacle—and the second, partly in this way and partly by the influence of the permanent magnets used for the other corrections.

It is found that a large amount of the magnetism, induced in the vessel during its construction, is lost during a long voyage, due, no doubt, to the buffeting of the ship by the waves. In course of time, however, the conditions of the vessel with respect to its magnetism become constant, so that it needs no further correction. The magnetism, which is then retained in the ship, is called *permanent* magnetism; that which it loses, is called *sub-permanent.*

The Anschutz Gyro-Compass.—It is worth mentioning that a novel form of mariner's compass, which does not depend upon magnetism, has recently been introduced. It is a most interesting and important application of the principle of the gyroscope.

A heavy wheel, driven at very high speed (20,000 revolutions per minute), by an electric motor, is mounted in the binnacle, and under the influence of its own rotation and that of the earth, tends to set its axis of rotation parallel to the earth's axis.

Such compasses are now being introduced into the British and other navies. They are unaffected by the presence of iron, and possess the great advantage of indicating the true or geographical meridian. An obvious disadvantage is the dependence on rotation, for if the motor fails the compass becomes useless.

Exercise IX

1. How may the magnetic meridian be determined by means of a dip circle?

2. Explain why, in determining the magnetic dip at any place, it is necessary to reverse the magnetism of the needle so as to make each end of it dip in turn.

3. The N poles of two equal and equally magnetised magnets are attached to the ends of a light bar of wood, so that the magnets are parallel to each other and at right angles to the bar, and the S poles are on opposite sides of it. If the whole be suspended by a thread, so that the bar and the magnets lie in a horizontal plane, what position will the bar take up with respect to the magnetic meridian? Give reasons for your answer.

4. An iron ball is held due north of a compass-needle. Describe the motion of the needle as the ball is carried round it in a circle in the directions north, east, south, west, north.

5. What effect (if any) is produced (1) on the weight, (2) on the position of the centre of gravity, of a piece of steel by magnetising it? Give reasons for your answer.

6. A small magnet hanging by a silk fibre makes ten oscillations in a minute when acted on by the earth's magnetic force alone. When equal masses of iron are placed at equal distances from it, one to the north and the other to the south, the magnet makes more than ten oscillations in a minute; but when the same pieces of soft iron are placed at equal distances east and west of the magnet, it makes less than ten oscillations in a minute. Why is this?

7. Describe and explain some method by which the intensity of the horizontal component of the earth's magnetism at two different places can be compared.

8. A straight bit of straw is hung vertically by a fine silk fibre, and two magnetised sewing-needles are thrust through it horizontally. Show what are the essential conditions in order (*a*) that the needles may point in the same direction as each would alone; (*b*) that they may point equally well in any direction.

9. Two soft iron rods are placed vertically, one east and the other west of the centre of a compass-needle; the lower end of the rod on the east and the upper end of the rod on the west being level with the compass. Describe and explain the effect on the compass.

10. A bar of very soft iron is set vertically. How will its upper and lower ends respectively affect a compass-needle? Would the result be the same at all parts of the world as it is in this country? If not, state generally how it would differ at different places.

11. Find the total magnetic force of the earth at a place where the horizontal component is ·18 dyne and the dip 45°.

12. Describe, by the help of diagrams, the general character of the distribution of the lines of magnetic force over the earth's surface. (B. of E., 1911.)

13. Supposing an iron ship to behave like a permanent magnet, with a north pole at the bow and a south pole at the stern, explain how a compass on board would behave as the ship swings through 360°. Would the effect be the same in England and at the equator? (B. of E., 1899.)

14. What is meant by (1) declination, (2) agonic lines, (3) magnetic equator? Draw a map showing the general position of the agonic lines, and indicate the direction of the declination in the regions into which these lines divide the earth's surface. (B. of E., 1903.)

15. What is an *isogonal line?* Describe the general form of the isogonals over the surface of the earth. How are the observations made which are used in determining the isogonals? (B. of E., 1906.)

16. A thin uniform magnet, 1 metre long, is suspended from the north end so that it can turn freely about a horizontal axis which lies magnetic east and west. The magnet is found to be deflected from the perpendicular through an angle θ ($\sin\theta = \cdot 1$; $\cos\theta = \cdot 995$). If the weight of the magnet is 10 grams, the horizontal component of the earth's field is ·2 C.G.S. units, and the vertical component ·4 C.G.S., find the moment of the magnet. (B. of E., 1906.)

VOLTAIC ELECTRICITY

CHAPTER XIV

VOLTAIC CELLS

If by some means the ends of a wire are kept at different potentials, we produce what is known as an *electric current* through the wire. Such a difference of potential is often produced by chemical action; but before discussing any method by which it can be done, it will be advantageous to perform a few preliminary experiments.

Exp. 127. (*a*) Pour a little *strong* sulphuric acid into a dry test-tube, and drop in a small piece of zinc. Notice that the zinc is unattacked—although there may be a few small bubbles of gas evolved at first.

(*b*) Repeat the experiment, using *dilute* acid, and notice the rapid evolution of gas. If the gas be collected and examined, it will be found to be hydrogen.

(*c*) After the action has continued for a short time, pour a little of the liquid into a porcelain evaporating dish and then evaporate it to dryness over a Bunsen's or a spirit-lamp flame. A white solid—sulphate of zinc—remains, which has been formed by the zinc dissolving in the acid.

If the weight of hydrogen be computed, and the loss of weight of the zinc be determined, the two weights will be constant in proportion, viz. one part of hydrogen will be evolved, when 31·5 parts of zinc are dissolved. These numbers are to one another as the chemical equivalents of the elements (p. 326).

This is an example of chemical action, and is represented by the equation—

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

Exp. 128. (*a*) Immerse a piece of *pure* zinc in dilute sulphuric acid. Observe that it is unattacked. It must, however, be mentioned that pure zinc is very difficult to obtain, so that some action will probably occur, although it will be less than when commercial zinc is used.

(*b*) Touch the zinc with a copper wire, under the surface of the liquid. Notice that a greatly increased evolution of gas takes place, and also that the bubbles emanate from the immersed portion of the copper wire.

(*c*) Repeat the experiment with commercial zinc, using very weak acid so that the action is not great at first. Notice that, when contact is made with the copper wire, the rate of evolution of the gas is increased, and that it is given off at *both* the copper and the zinc.

The Simple Cell.—**Exp. 129.** (*a*) Immerse plates of zinc and copper, side by side, but *without contact*, in dilute sulphuric acid. Notice that evolution of gas takes place from the zinc at once; the copper plate being so far inert.

(*b*) Let the plates touch beneath the surface. At once there is an increased evolution of gas, which now comes off from *both* plates.

(*c*) Instead of making contact in the liquid, attach a wire to each plate (Fig. 147), and notice that exactly a similar effect is produced when the two wires are brought into contact.

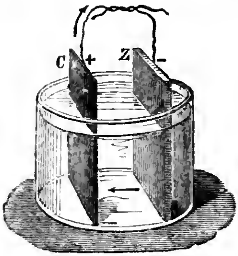


FIG. 147.

(*d*) Put the ends of the two wires on the tongue, but without letting them touch. A peculiar taste is perceived.

(*e*) Hold one wire immediately above a compass-needle, in the direction of its length, and then touch the end of that wire with the other wire. Notice that the needle is deflected.

Experiment 129, (*d*) and (*e*), shows that the connecting wire possesses new and peculiar properties.

This arrangement constitutes what is known as a simple voltaic cell, and the wire is said to carry an electric current.

From a practical point of view, the cell has one obvious defect—the zinc dissolves rapidly and gives off gas during the whole time it is in the liquid, whether a current is being taken out of the cell or not. We see that this loss might be stopped by removing the zinc when not in use, or more conveniently by using a plate of *pure* zinc, which would only be attacked when the wires are joined, *i.e.* when the *circuit* is closed. The use of pure zinc is out of the question, but there is a simple method—termed amalgamation—by which ordinary zinc may be made to act like pure zinc.

Exp. 130. Amalgamate the strip of zinc used in the last experiment. This consists of two operations, of which, with this particular strip, the first has been performed: (1) Cleaning the zinc by dipping it in dilute sulphuric acid, and (2) coating its surface with mercury, which is easily done by pouring a little mercury on the zinc, and immediately spreading it over the surface with a rag or brush. The mercury unites with the zinc, forming a silvery-white amalgam of mercury and zinc. The unused mercury ought to be collected and put in a bottle for future use.

Put the strip back in its place, and notice that no gas is evolved at *either* plate until the plates (*a*) touch in the liquid, or (*b*) are connected by a wire outside the liquid. Then, gas is evolved in quantity, but it comes *entirely from the copper plate*. In fact, the experiment strongly suggests that the copper plate is being attacked and that the zinc is unattacked. If, however, the plates be weighed before and after the experiment, it will be found that it is the zinc which has lost weight and not the copper.

This evolution of gas at the inert plate is characteristic of voltaic action, and when it occurs it is certain that a current is flowing in some circuit.[1]

[1] On this point beginners are apt to get confused. They have learnt in chemistry to associate the evolution of gas with chemical action, and thus regard the hydrogen as *really* being evolved at the zinc plate, but in some mysterious way coming *apparently* from the copper, although such an idea contradicts the evidence of their own eyes. As a matter of fact, the hydrogen evolved during any experiment of moderate length has never been near the zinc, and later (p. 204) it will be seen that the evolution of gas at the zinc surface is not real, but only apparent.

Now, if an electric current flows through the wire connecting the plates, a P.D. must have existed between them *before* they were connected; and, as already stated, this is only another way of saying that they are connected by *lines of electric force*, which again implies that one plate is charged positively and the other negatively. If the current persists indefinitely *after* connection, it can only be due to the fact that these charges are replaced as fast as they are removed by conduction in the wire. It should also be noticed that a flow of current implies a *motion* of lines of electric force in the dielectric outside the conductor.

If either plate is connected to the cap of an electroscope, not the slightest effect will be observed, because the instrument is not sufficiently sensitive to respond to such feeble charges. If, however, a quadrant electrometer is available, a large deflection will be obtained on connecting up the cell to its terminals, and by observing the direction of the deflection it can be shown that the copper plate is charged positively, and the zinc plate, negatively.

A similar result can be obtained in a simpler way by using a condensing electroscope (p. 57), but, with an instrument of ordinary sensitiveness, there may be a difficulty in obtaining a sufficiently marked effect if only one cell is used. We must, therefore, anticipate matters a little, and use about half a dozen Leclanché or dry cells (p. 208) connected up in series (p. 216).

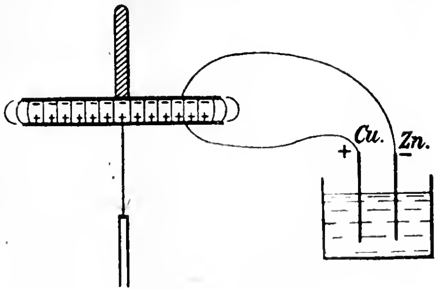


Fig. 148.

Exp. 131. (*a*) Bring the free ends of the wires in contact with the cap and the disc, as shown in Fig. 148, and notice that there is no deflection, either at the moment of contact or when the wires are removed. (In the figure only one cell is drawn for convenience, and the cap and disc are shown slightly separated in order to mark the charges. Really they are in contact except for the layer of varnish.) Lift the upper disc, and observe that the leaves diverge. Show, in the usual way, that the charge is positive.

(*b*) Repeat the experiment with the wires reversed, *i.e.* the one from the zinc touching the electroscope. Show that the charge in the leaves is negative.

We know from previous experiments that the presence of the oppositely charged disc greatly increases the capacity of the cap of the electroscope, and hence a much greater charge flows into it than would otherwise be the case. On removing the disc, the capacity diminishes, and hence, the quantity remaining the same, the potential rises sufficiently to cause a divergence of the leaves.

Properties of a Conductor carrying a Current.—The new properties acquired by a conductor, whilst carrying an electric current, may be summarised as follows:—

(1) A magnetic field is produced in the space around the conductor.

(2) Heat is produced in it during the whole of the time the current is flowing.

(3) Chemical action takes place in certain conducting liquids when the current is passed through them.

On the first property mentioned above depends the action of various forms of **galvanometer**, which may be regarded as an instrument primarily intended to detect the existence of an electric current. Detailed information upon this subject will be given later; at present it is sufficient to assume that such an instrument is available, its method of action being immaterial. A very simple form may be made by the student as follows:—

Simple Galvanometer.—(*a*) Make a wooden framework about 6 inches long, $1\frac{1}{2}$ inch wide, and $1\frac{1}{2}$ inch deep. As it is preferable not to have a top to the framework, support the wooden sides by rectangular wooden blocks, as shown in Fig. 149. Cut a groove about an inch wide underneath the bottom.

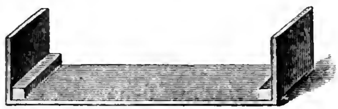


FIG. 149.

(*b*) Wind silk-covered copper wire ten or twelve times round the frame, so that it lies evenly in the groove.

(*c*) Fasten the frame to a wooden base, A (Fig. 150), having first cut a groove in it similar to the one in the bottom of the frame.

(*d*) Attach the ends of the wires to two binding-screws, B, C, fixed to the base.

(*e*) Having graduated a circular piece of cardboard, D, in degrees, glue it to the bottom of the frame, taking care that the zero of the scale is under the middle wire.

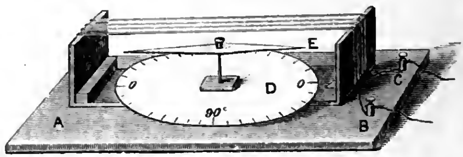


FIG. 150.

(*f*) Fix a sewing-needle vertically in a small cork so that the point projects about a quarter of an inch, and then glue the cork so that the needle forms a pivot at the centre of the card.

(*g*) Place a magnetic needle, E (two inches or so long), on the pivot.

Exp. 132. Connect the wires from a cell to the terminals of the simple galvanometer, and observe that, when the current passes above the needle from north to south, the N pole of the needle is deflected towards the east, and when it passes from south to north, the N pole is deflected towards the west.

Thus, it is possible to define current direction, although previous work has shown that there is really—so far as our knowledge goes at present—a flow of a positive charge one way and of an equal amount of negative the other way. It is simply for convenience that we agree to make the term "direction of current" mean *the direction in which the positive charge flows.* Outside the cell, it is from copper to zinc.

Electromotive Series.—It must not be supposed that the production of a P.D. is peculiar to zinc and copper immersed in dilute sulphuric acid, or that it is exceptionally strong with these materials. Practically, any two metals will give the effect, but it is greater in proportion to the difference in the action of the acid upon them. Further, liquids of various kinds may be used, *e.g.* other acids, a solution of common salt or of other salts, the only change being in the *magnitude* of the P.D. set up. The maximum value of the P.D. produced (which is the value when the plates are on "open circuit," *i.e.* when they are not connected by a wire) is called the **electromotive force** of the cell. Experiment shows that its value is independent of the shape or size of the cell and is constant for the same materials, *i.e.* it depends only upon the nature of the chemical action involved (see p. 311).

Exp. 133. Attach the wires from a copper-and-zinc cell to the binding-screws of the simple galvanometer, and note the direction of deflection.

Exp. 134. Replace the copper by plates of platinum, iron, silver (half a crown), and carbon respectively. Having attached the wires to the binding-screws, observe that, in each case, the deflection of the needle is in the same direction as when copper was used. We, therefore, conclude that the current passes through the wire from each of these substances to the zinc.

Exp. 135. Repeat the experiment with iron and carbon plates. Notice that the deflection shows that the current flows through the wire from the carbon to the iron.

From experiments similar to these, the following substances, when partially immersed in dilute sulphuric acid, have been arranged, so that, any two being used, the current flows through the connecting wire from the latter to the former :—

1. Zinc
2. Cadmium
3. Tin
4. Lead
5. Iron
6. Nickel
7. Bismuth
8. Antimony
9. Copper
10. Silver
11. Gold
12. Platinum
13. Carbon

The position of any substance on the list varies considerably with (*a*) its condition ; (*b*) the strength of the liquid ; and (*c*) the nature of the liquid. The greater the distance on the list between any two bodies, the greater the difference of potential.

The substance from which the current flows through the wire is called *electro-negative* to the other, which is *electro-positive.*

The terms electro-positive and electro-negative *substances* must not be confounded with positive and negative *poles*. The following consideration will make this clear: The starting-place of the current is the portion of the electro-positive plate (*e.g.* the zinc) immersed in the liquid. This portion is therefore positive, while, as we have proved in Experiment 131, the external portion is negative. Now, the parts *outside* the liquid are the *poles* of the cell; thus we learn that the *negative pole* is the external portion of the *positive plate;* and *vice versâ.*

A current may, however, be produced by one metal and two liquids. The beaker, A (Fig. 151), contains strong nitric acid, in which is placed a porous pot, B, containing a solution of caustic potash. If the ends of two platinum wires are immersed respectively in the acid and the potash, and their free ends connected with a simple galvanometer, the needle will be deflected in a direction which proves that the current flows through the wire from the acid to the alkali.

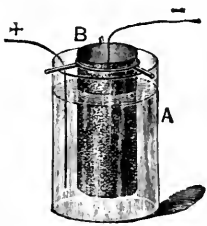


FIG. 151.

Local Action.—The simple cell obtained by placing zinc and copper plates in dilute sulphuric acid has two principal defects, to which *all* cells are more or less subject. The first is known as **local action**. This term signifies the useless consumption of zinc, when the cell is on "open circuit." We have already shown that this defect can be remedied by amalgamating the zinc. At the same time, this is only a partial remedy, and in any case frequent re-amalgamation is necessary. To explain this action, we must refer to preceding experiments, in which it was shown that *pure* zinc is not attacked by dilute sulphuric acid, unless it is touched by a copper[1] plate or wire either *under* or *outside* the liquid. From such experiments, it may be inferred that the solution of ordinary zinc in dilute sulphuric acid is voltaic in nature. Such zinc contains a certain proportion of other substances as impurities, *e.g.* particles of iron, carbon, arsenic, which behave to it in much the same way as copper.

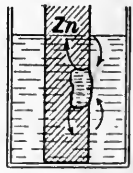


FIG. 152.

In Fig. 152, such a particle is shown embedded in the zinc surface. The combination forms a minute voltaic cell, short circuited on itself, and acting like the zinc and copper plates in Experiment 129, when they are made to touch beneath the surface of the liquid. The current direction, in that case, is shown in Fig. 153, *i.e.* the current passes from zinc to copper *inside* the liquid, and from copper to zinc where

[1] As we have seen, other substances, *e.g.* platinum, silver, gold, or carbon, would serve for this purpose.

these make contact, whether that be inside or outside (compare Fig. 147). Hence, the direction of the current set up by the zinc and the impurity will be as shown in Fig. 152, and *hydrogen gas will be liberated, not at the zinc surface, but at the surface of the impurity.* As these impurities are exceedingly small and numerous, the observed effect is the evolution of gas at the zinc surface as a whole. The effect of amalgamating the zinc is not easy to explain completely; roughly, we may regard the mercury as dissolving some zinc but not the impurities, and thus covering the whole surface with a thin, semi-fluid layer of zinc amalgam, which coats over the impurities and presents a uniform surface to the acid. The efficiency of the amalgam diminishes as the mercury soaks into the zinc, so that the operation must be repeated, in fact, new zincs require very frequent amalgamation. After a time, owing to their gradual absorption of a considerable amount of mercury, they work with much less attention.

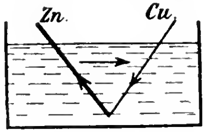


FIG. 153.

Polarisation.—The second defect has long been known as **polarisation**, a term to which no special meaning should be attached, except as signifying the phenomenon about to be described. This is best realised by a simple experiment.

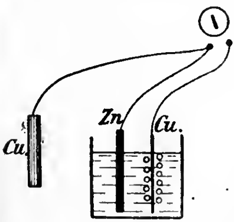


FIG. 154.

Exp. 136. Connect a simple cell to a galvanometer, and notice the direction of deflection. Connect a second copper plate, exactly like the first, to the galvanometer as shown in Fig. 154. After the cell has worked until there is a perceptible accumulation of hydrogen, take out the zinc and replace it by the copper. Notice that a slight deflection, which rapidly decreases in amount, is produced in the *opposite* direction.

From this experiment, we learn that a copper plate covered with hydrogen can form a voltaic cell in conjunction with a second copper plate—the hydrogen acting in the same way as the zinc.

Now, this tendency to generate a current in the reverse direction must be in action during the normal working of the cell, and, although it cannot actually send such a current, it must tend to weaken the main copper-to-zinc current.

Later, we shall learn that it is preferable to regard the hydrogen, not as setting up a reverse *current* but as giving a reverse *electromotive force.* To prevent misunderstanding, we must mention that the presence of this reverse electromotive force would, for many purposes, be immaterial, if only it remained constant in amount. This, however, is not the case, for we see that it will be zero at the instant the circuit is closed, and that it will gradually increase to a maximum as the gas accumulates; every disturbance, which causes the gas to escape, necessarily altering its value. The slight variations in current strength thereby produced give rise to difficulties, especially in electrical measurements.

Besides the action illustrated in the last experiment, the hydrogen has also a mechanical effect, inasmuch as its presence decreases the available surface of the copper plate.

Hence, to prevent polarisation we must get rid of the hydrogen. This was first successfully accomplished by Daniell, whose cell is still largely used.

Daniell's Cell is constructed in a variety of forms. Its essential parts are a zinc rod or plate immersed in dilute sulphuric acid, separated by a porous cell from a solution of copper sulphate, in which a copper rod or plate is immersed.

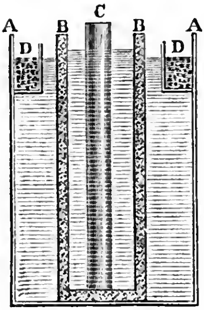


FIG. 155.

It is often constructed as shown in Fig. 155, so that the outer vessel, A, is entirely of copper, and contains a strong solution of copper sulphate. In order to keep the strength of the solution constant, crystals of the substance are placed on a perforated shelf, D, with which the copper vessel is generally provided. Inside this, is a cylindrical porous cell, B, containing dilute sulphuric acid and a rod of amalgamated zinc, C.

Theory of Voltaic Cells.—It may be pointed out in passing, that a voltaic cell is really a device which enables us to control the evolution of the energy due to chemical action, and to apply it, readily and conveniently, to various useful purposes. The solution of zinc in dilute sulphuric acid is an action whose nature is essentially the same as that of the burning of coal. In the latter case, all the energy is liberated as heat, and this heat must be utilised at the place where it is produced. Zinc *might* be burnt in an open fire, and every pound of it would then give out a definite amount of heat, subject to the same restriction, although it would obviously be a more expensive fuel than coal. When zinc dissolves in sulphuric acid to form zinc sulphate, the reaction is not quite the same, although here again a definite amount of energy is given out per pound weight. As a general rule, this energy is liberated as heat, which merely raises the temperature of the liquid and its surroundings, but when the zinc forms part of a voltaic cell, other possibilities arise. It is still possible to liberate all the energy as heat, but that heat is not now necessarily produced at the spot where the zinc is consumed. It *may* be all produced in the liquid if we wish, and in any case some heat *must* be liberated there, but this may be as small in amount as we please, and the remainder can be produced at any part of the circuit, no matter how large that circuit may be. Further, the energy is not *necessarily* liberated as heat; it may be employed to do useful mechanical work at any point in the circuit. It is this flexibility of application which makes an electric current useful for so many purposes. In itself it does not create energy: that must always be obtained from some source of power—coal, a waterfall, zinc, &c.—but it serves to distribute such energy, to transmit it without serious loss over great distances, and to apply it at any given point to some required purpose in an efficient and an easily controllable manner.

Evidently zinc is not the only substance that could serve as fuel in a cell; it is, on the whole, simply the most convenient one. Unfortunately, there is little prospect of utilising the oxidation of carbon itself in a voltaic cell, although much work has been done with that end in view. Again, many inventors have claimed that they have made cells capable of working vigorously without any serious consumption of zinc, but it is obvious from the above statements that a certain definite relation, which is independent of the particular kind of cell used, must exist between the weight of zinc consumed and the amount of electrical energy produced. Further information will be given later upon this subject.

The question naturally arises: Whence come the charges, positive and negative, in the case of a voltaic cell? Of course, it is possible to study the properties of cells and currents without attempting to give an answer, but it is almost impossible to think correctly about the matter, unless we have some working theory as a guide. The following brief sketch is, therefore, given, although admittedly imperfect and open to criticism in many respects.

When a simple salt, *e.g.* common salt, or an acid (which for our purpose may be regarded as a salt of hydrogen) is dissolved in water, there are good reasons for believing that part of the molecules become dissociated—the extent of dissociation increasing with the dilution of the solution. That is, instead of NaCl, we have Na and Cl; and instead of H_2SO_4, we have H, H, and SO_4. These particles are not in their ordinary state, and do not display their ordinary chemical properties; for each carries an *electric charge*—the metals and hydrogen (in simple salts) always carrying a positive charge, and the acid component a negative charge. Sometimes one kind of atom carries more charge than another kind, but the greater charge is always some simple multiple of the smaller one. It is probably the electrical forces due to these charges which constitute chemical affinity.

Such free *charged* atoms or groups of atoms are known generally as **ions**. If the charge be removed by any means, the particle or group is no longer an ion, and it at once displays its ordinary chemical behaviour.

According to this theory, dilute sulphuric acid contains a number of free hydrogen ions, each carrying a certain positive charge, and half that number of SO_4 groups, each carrying a double negative charge. When a plate of zinc and a plate of copper (or carbon, platinum, &c.) are placed in this solution, there is immediately a tendency (due to chemical affinity) for the zinc to form zinc sulphate, which really means that zinc tends to form ions passing into the solution with a positive charge, and thereby leaving the zinc plate, as a whole, slightly negatively charged. This negative charge will cause any further action of the kind to cease almost immediately, and thus we say that *pure* zinc is insoluble in dilute acid. During this action, the copper plate has been practically inert, having less tendency to ionise into the solution, but when the zinc and copper plates are connected by a wire, the negative charge on the zinc can spread over the wire and copper. This produces two effects: (*a*) by reducing the charge on the zinc, it permits further chemical action to take place; (*b*) the presence of a negative charge on the copper now makes it attract the wandering hydrogen ions in its vicinity. These move up to it, and (1) neutralise some of the negative with their own positive, (2) cease to be ions, immediately acquiring their own chemical properties and coming off as a gas. Evidently these various actions may go on continuously, the nett result being the replacement of hydrogen ions by zinc ions. The effect known as *polarisation* is partly a consequence of a certain difficulty experienced by the hydrogen ions in transferring their charge to the copper plate. This difficulty would be removed if they could combine chemically with the copper.

Let us now apply this theory to a Daniell's cell. Here, we have exactly similar actions occurring at the zinc plate, but the copper plate, when it is joined up, attracts *copper* ions, there being no hydrogen ions in its vicinity. The copper ions move up to it and give up their charge; polarisation, moreover (in its ordinary sense), ceases because the copper plate merely becomes coated with fresh copper.[1]

If the action goes on long enough without renewing the copper sulphate, evidently hydrogen ions may be liberated as before, for there is a steady passage of zinc ions into solution, and of hydrogen ions through the porous pot towards the copper plate.

When such a cell is required to work for a long time without attention, the dilute sulphuric acid may be replaced by a saturated solution of zinc sulphate, which completely prevents local action. At first sight, it seems absurd to think that the zinc will dissolve under such circumstances, but the previous explanations show that there is no real difficulty in the process, because as the zinc ions and copper ions in solution both move slowly towards the copper plate, new zinc ions will pass into solution from the zinc plate, the nett result being

[1] It is usual to express these results by writing:—

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$
$$H_2 + CuSO_4 = H_2SO_4 + Cu.$$

But such equations are apt to be misleading as to the real nature of the actions.

the replacement of copper ions by zinc ions in what was originally copper sulphate solution.

Gravitation Cells.—It is worthy of mention that the use of porous vessels has several objections, so that cells have been devised in which the liquids are separated merely by a difference in density. Two such cells will now be described, both of which have an action similar to that of the Daniell's cell just described.

Callaud's Cell (which appears to be chiefly used in America) is represented in section in Fig. 156. V is a glass or earthenware vessel; C, a copper plate soldered to a guttapercha-covered wire; Z, a zinc cylinder. A layer of crystals of copper sulphate ($CuSO_4$) is placed on the copper plate, and the cell is then filled up with water. These crystals dissolve in the water, forming a solution which gradually diffuses, and, as it comes in contact with the zinc, forms zinc sulphate ($ZnSO_4$), which, owing to its lower density, floats on the solution of copper sulphate.

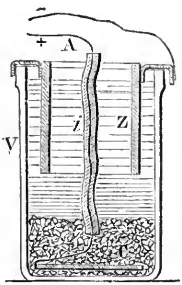


FIG. 156.

Minotto's Cell (used in India) has a layer of sand or sawdust instead of a porous cell. The arrangement of the cell (Fig. 157) is as follows: At the bottom of an earthenware vessel, V, a layer of crystals of copper sulphate, *ab*, is placed; and on this a copper plate, C, having attached to it an insulated wire, *i*. Above this is placed a layer of sand or sawdust, *bc*, having a cylinder of zinc, Z, resting upon it.

The vessel is filled with water, the same action occurring as we have described in Callaud's cell.

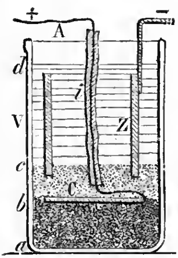


FIG. 157.

The Electric Circuit.—We have seen that the continuous flow of a current necessarily takes place in *a circuit*, i.e. a conducting path extending from one terminal of the current generator to the other, and completed through the generator itself. The *magnitude* of the current depends upon two factors: (1) the nature of the conducting path, and (2) the electromotive force of the generator. Its *strength* is found to be the same at every part of the circuit.

Electromotive Force.—This term, which has been already referred to, is best defined as being the P.D. between the terminals of any current generator *when no current is allowed to flow*.

The P.D.'s we are concerned with in voltaic experiments, although they are usually of much smaller magnitude, are identical in nature with those discussed previously, and all that has been said in

Chapter IV. still applies. Hence, it seems natural to measure E.M.F. in terms of the same unit (see p. 34), but the student will find that it is customary to use one set of units in dealing with problems in statics—known as *static units*—and another wholly distinct set in voltaic work—known as *electromagnetic units.* Why these units came into existence, and how they are related to each other, will be discussed in Chapter XXXIV. The fundamental or absolute electromagnetic unit of potential cannot be defined at the present stage (see p. 350); it is, however, much too small for ordinary purposes, and a multiple of it, called the **volt**, is known as the *practical unit.* It is one hundred million times larger than the absolute unit—*i.e.* 1 volt $= 10^8$ absolute units of potential—and is very nearly equal to the E.M.F. of a Daniell's cell.

To give some idea of relative magnitudes, it may be remarked that the static unit of potential, defined on p. 34, is equal to 300 volts. A proof will be found on p. 582.

Resistance.—The student must, however, remember that the strength of the current does not depend merely upon the E.M.F. of the cell or battery. Another important factor has to be brought into consideration, viz. the *resistance* which the current has to overcome both in the cells themselves and in the external circuit. The greater the resistance, the smaller the current which a given E.M.F. will produce; *e.g.* if the cells have a dense porous partition, or if the liquids are separated by sand or sawdust (as in the Minotto cell), and if the external wires be very long and very thin, the strength of the current is small, although the E.M.F. may be large.

The practical unit of resistance is called the **ohm**. It is 10^9 times the absolute unit of resistance (see p. 587), and is the resistance of a mercury column, 1 square millimetre in section, and 106·3 centimetres long, at 0° C. To give a more definite idea of the value of an ohm, we may mention that about 50 yards of 20 gauge copper wire has 1 ohm resistance, and the same size and length of iron wire would have about 6 ohms resistance. The fuller study of resistance and its laws will be found in Chapter XVIII., where it is shown that the resistance of a conductor of uniform section is directly proportional to its length, and inversely proportional to its sectional area.

Current Strength.—The meaning of this term has already been explained. We applied it to the flow of charge, which takes place when two points, between which a P.D. exists, are connected by a conductor. Such a flow always involves a passage of positive electrification in one direction—which we arbitrarily call *the direction* of the current—and an equal flow of negative electrification in the opposite direction. Hence, *current strength is naturally measured by the quantity of charge which flows past any one point in the circuit in unit time.* Although strictly correct, this is not a very helpful definition, and we can define it in a more convenient way by the aid of Ohm's law.

Ohm's Law.—Ohm discovered, long before any electrical units existed, that the strength of current in any circuit varied directly as the E.M.F. in that circuit, and inversely as its total resistance. This statement is known as Ohm's law. It is also true of any *portion* of a circuit, the current in that portion being directly proportional to the P.D. between its extremities, and inversely proportional to its resistance. Hence, we have $C \propto \frac{E}{R}$, which, by the choice of suitable units, may be expressed as an equality, and it becomes natural to define the *absolute* unit of current as the current which flows through one absolute unit of resistance when the P.D. between the ends of that resistance is one absolute unit of E.M.F.

The practical unit of current—called the ampere—is that which flows through a resistance of 1 ohm when the P.D. across it is 1 volt. As 1 volt $= 10^8$ absolute units of E.M.F., and 1 ohm $= 10^9$ absolute units of resistance, it follows that 1 ampere is $\frac{1}{10}$ the absolute unit of current.

It may be pointed out that the relation

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Resistance}}$$

holds good numerically for *either* practical units *or* absolute units, but care must be taken not to confuse them in the same equation. Hence, it is desirable to have some means of indicating which kind of unit is being used, and for this purpose we shall in future reserve the letter C for current strength in *amperes*, and the letter i for current strength in *absolute units*. The occurrence of i in any expression will then indicate that absolute units are being used throughout, whereas C will mean that practical units are being employed.

Two other important practical units will be required later, and for convenience we may now mention them. These are (1) the "coulomb," the practical unit of quantity, which is defined as the quantity conveyed by 1 ampere flowing for 1 second; (2) the "farad," the practical unit of capacity, which will be understood if we remember that the fundamental equation used in connection with statics, $Q = VC$, applies generally, and that a condenser is said to have a capacity of 1 farad when a charge of 1 coulomb produces a P.D. of 1 volt between its coatings.

Verification of Ohm's Law.—Most of the simple experiments performed by students with the object of verifying Ohm's law, really beg the question at issue. In fact, it is scarcely a suitable task for a beginner. Perhaps the best method of procedure is to maintain a current of *constant* strength (which need not be known) in a long uniform wire of fairly high resistance, and then to measure the P.D.

between points at different distances apart on the wire. It will be found that the P.D. is directly proportional to the *distance* between the points in question, and, as we know that the resistance of a uniform wire is proportional to its length, it follows that $E \propto R$, when C is constant, which partially verifies the law. But it is necessary to measure the differences of potential by means of some form of electrostatic instrument, such as a quadrant electrometer, which does not take any current and whose theory is quite independent of the matter under consideration. Unless this precaution is taken, it is extremely probable that the reasoning will be invalidated by the fact that the results obtained depend upon the use of instruments and methods based on the assumed correctness of the law.

Other Types of Voltaic Cells.—Although, at the present time, there are many forms of voltaic cells, the only ones of real importance—except the special forms used as standard cells—are

(1) the Daniell's cell and its modifications just described; and

(2) the Leclanché cell and its modification, the so-called dry cell.

The first group was until recently used exclusively in telegraphy, and, although now largely superseded by accumulators, many thousands of such cells are still in operation for this purpose.

The Leclanché Cells, possessing as they do the power of giving out small and intermittent currents for years without attention, can be applied to many purposes for which accumulators are unsuitable, and they are, therefore, being employed in enormous and increasing quantities.

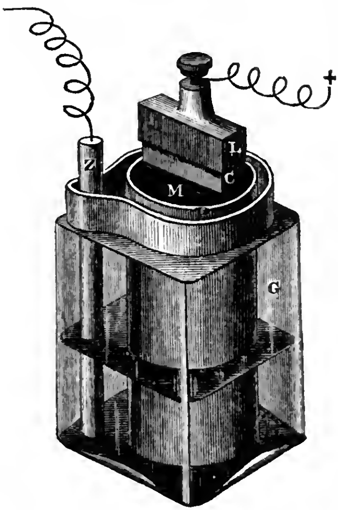


FIG. 158.

The ordinary type, shown in Fig. 158, consists of a rod of gas carbon, C, placed in a porous cell, P, which is then tightly packed round with small pieces of gas carbon and powdered manganese dioxide. These fragments of carbon are then covered by a layer of pitch, M. A piece of lead, L, is soldered to the top of the carbon, to which a binding-screw is fixed. A rod of zinc, Z, is immersed in a solution of ammonium chloride (NH_4Cl),[1] which, when the cell is working, forms zinc chloride ($ZnCl_2$) and liberates hydrogen, which is slowly oxidised by the manganese dioxide.

[1] It is worthy of mention that a cell, which is to work without attention for a long time, *must* be a single fluid cell, and *must* be practically free from local action.

The chemical reactions may be expressed thus—

$$2NH_4Cl + Zn = ZnCl_2 + 2NH_3 + H_2.$$

Ammonia (NH_3) comes off from the cell, and the hydrogen acts on the peroxide of manganese, forming the lower oxide (Mn_2O_3), as shown by—

$$2MnO_2 + H_2 = Mn_2O_3 + H_2O.$$

The depolariser acts too slowly to eliminate polarisation completely, and, therefore, the cell is not suitable for purposes requiring a constant E.M.F. If, however, it is allowed to rest, it regains its original strength, and on this account it is well adapted for ringing electric bells.

The porous pot is used merely to keep the solid depolariser around the carbon, and is a disadvantage, inasmuch as it increases the internal resistance. In the "agglomerate" type its use is avoided, the depolariser being made up into compressed blocks, which are kept in position around the carbon by rubber bands; while in yet another type, the pot is replaced by a canvas bag.

It is a common error to assume that this cell is totally free from local action. This is not the case, as from what has been already said, it is evident that some action will be produced by the impurities in the zinc. In comparison, however, with the amount produced when dilute sulphuric acid is used, it is extremely small; indeed when the zinc is amalgamated, it is practically negligible.

The E.M.F. of a Leclanché cell, when new, is nearly 1·5 volts. The internal resistance depends, of course, upon the size and type. In the ordinary form, it is fairly high and increases with use.

Dry Cell.—The dry cell dates practically from 1884 (although many attempts had previously been made to secure portability by the use of various absorbent materials). In construction and action, it is essentially the same as the Leclanché cell, except that the exciting substance is in the form of paste—sometimes jelly—instead of a solution. Fig. 159 indicates the essential parts of its construction. A zinc cylinder, Z, forms the outer vessel, enclosed in a paper or cardboard case. The carbon rod, C, is surrounded by the depolariser, B—a black paste, composed chiefly of manganese dioxide, powdered carbon, and graphite mixed with glycerine and ammonium chloride solution. This paste is often contained in a porous bag. The zinc cylinder is lined with a white paste, W, made by mixing plaster of Paris with a solution of ammonium chloride; certain other materials, *e.g.* flour and glycerine,

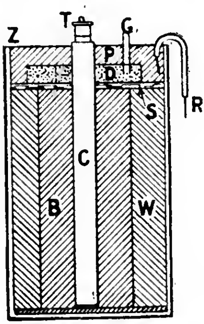


FIG. 159.

being added to improve the consistency. Sand, S, is placed above the paste, and upon this is a layer of sawdust, D, into which a glass tube, G, to carry off the gases, is inserted. The top is then sealed up with bitumen or pitch, P. A wire, R, is connected with the zinc, and the carbon carries the usual binding-screw, T.

Grove and Bunsen Cells.—When a fairly large current for experimental purposes is required for some minutes at a time, the Leclanché cell and its modifications are unsuitable. Wherever possible, accumulators are now employed, but if these are not available, some form of Grove, Bunsen, or chromic acid cell may be used. The first two are chiefly of historical importance. In the Grove's cell the outer vessel is generally a flat cell of glass or earthenware, A (Fig. 160), which contains a strip of amalgamated zinc, B, immersed in dilute sulphuric acid. The zinc is bent round a porous cell, C, in which is placed a piece of platinum foil, D, immersed in strong nitric acid.

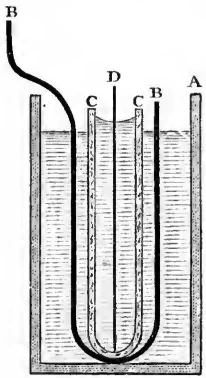


FIG. 160.

The chemical reactions in this cell are represented by the following equations—

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

This hydrogen is liberated at the platinum surface in the presence of nitric acid, upon which it acts to form water and nitrogen peroxide, thus—

$$H_2 + 2HNO_3 = 2H_2O + 2NO_2.$$

The nitrogen peroxide is a dark red gas, which, unlike hydrogen, is incapable of producing polarisation of the platinum plate.

The E.M.F. is from 1·8 to 2 volts, and as the internal resistance is low (on account of the excellent conductivity of nitric acid and the comparatively small distance between the plates), it is capable of giving a very large current on short circuit, but, apart from the fact that—like all double-fluid cells—it must be specially set up and taken to pieces again every time it is used, the initial cost of platinum is prohibitive, nitric acid is somewhat expensive, and the nitrogen peroxide fumes are injurious to adjacent apparatus. Bunsen's cell is similar to a Grove's in principle and action, and differs in construction only by having a rod of carbon in place of a platinum plate.

If we replace the nitric acid in Bunsen's form by a solution of chromic acid, we obtain a cell which is useful, convenient, and free from fumes.

Chromic Acid or Bichromate Cell.—For many experimental purposes, a single-fluid type of cell is more convenient than a double-fluid one. In the chromic acid (or bichromate) cell, plates of zinc and carbon, attached to a suitable framework, dip into a mixed solution of chromic acid and sulphuric acid, and are lifted out when not in use.

The necessity of setting up the cells every time they are used is thus avoided. The E.M.F., however, remains constant for a short time only, as there is more polarisation than in the double-fluid arrangement. The well-known "bottle bichromate" cell (Fig. 161) is of this type, and consists of a zinc plate, Z, attached to a brass rod, which slides up and down a brass tube passing through an ebonite cover, and by means of which the zinc plate may be removed from the liquid when not in use. The two carbon plates, CC, one on each side of the zinc, are in this case attached to the cover, and remain in the liquid.

The absence of porous pots and the closeness of the plates greatly diminish the internal resistance, and thus enable this cell to give out a relatively large current, when the external resistance is low. The E.M.F. is nearly 2 volts.

When the circuit is completed (*a*) the sulphuric acid acts on the zinc, forming zinc sulphate, (*b*) the hydrogen reduces the chromic anhydride, according to the equation

$$3H_2 + 2CrO_3 = Cr_2O_3 + 3OH_2.$$

The chromic oxide thus formed then dissolves in the sulphuric acid.

$$Cr_2O_3 + 3H_2SO_4 = Cr_2(SO_4)_3 + 3OH_2.$$

There is no reason to suppose that the reaction actually occurs in two stages, and the two equations may be combined by uniting

FIG. 161.

$$2CrO_3 + 3H_2 + 3H_2SO_4 = Cr_2(SO_4)_3 + 6OH_2.$$

The student will notice from these equations that the effective depolariser is chromic anhydride (CrO_3). Years ago, this substance was not on the market, and it is very troublesome to prepare on a small scale. A solution in water (known as chromic acid) is, however, readily obtained by adding sulphuric acid to a solution of potassium bichromate—

$$K_2Cr_2O_7 + H_2SO_4 + H_2O = 2H_2CrO_4 + K_2SO_4.$$

For a long time this method was employed, and hence such cells are termed *bichromate cells*. The presence of potassium sulphate is immaterial.

Now that chromic anhydride can be bought cheaply, its solution is much better for use, and less expensive to make, than that of the bichromate.[1]

Edison-Lalande Cell.—This cell is largely used in America for special purposes (*e.g.* operating railway signals), which require fairly strong currents without risk of failure. It is a single-fluid cell consisting of plates of compressed copper oxide and of zinc in a solution of sodium hydrate. It is free from local action and polarisation; also, as no porous pot is required, the internal resistance is low. Hence, it is capable of giving out relatively large currents for a long period without attention, and it is very reliable in operation. Its greatest disadvantage is the lowness of the E.M.F., which is about ·7 volt per cell.

The reactions may be written—

$$Zn + 2NaOH = ZnNa_2O_2 + H_2.$$

The hydrogen combines with the oxygen of the copper-oxide plate, and does not appear as gas.

$$H_2 + CuO = Cu + OH_2.$$

When the copper-oxide is exhausted, it is readily replaced by a new plate.

We take the following data from information supplied by the Edison Manufacturing Co.: A medium size of cell is contained in an outer jar $7\frac{1}{8}$ in. in diameter and $8\frac{3}{4}$ in. high. The internal resistance is ·043 ohm, and hence on short circuit the maximum current is 15·5 amperes, although such a current could not be long maintained. It is rated to give a continuous current of 4 amperes and to possess a capacity of 300 ampere-hours, *i.e.* it can yield a current of 4 amperes for 75 hours, 2 amperes for 150 hours, and so on, before the plates require renewal.

The Benkö Cell.—This is the most recent innovation in primary batteries, and it possesses certain novel features, which may cause it to become very advantageous in practice. It is a carbon-zinc single-fluid cell (using some form of chromic acid solution), the chief point of interest being the method adopted for eliminating polarisation. This consists in keeping up a slow flow

[1] *Single-fluid Solution*—

1 lb. CrO_3 (red powder).
6 lbs. water (nearly 5 pints).
1 lb. strong H_2SO_4 (added last with constant stirring).
Allow the solution to cool before using.

Depolariser for Double-fluid Cells—

3 lbs. CrO_3.
4 lbs. water.
3 lbs. strong H_2SO_4.

of exciting liquid, so that the layers in contact with the carbon are being continually removed. The carbon, which is specially prepared to make it porous, is in the form of a hollow, flattened cylinder with open ends. The lower end of the carbon is closed by a leaden plate, through which passes a leaden pipe (carried up outside and bent over like a siphon to form the waste-pipe mentioned below), and the upper end is fitted with a leaden ring, thus making a vessel open at the top. Inside this vessel is placed the zinc plate. Surrounding the carbon is an outer jacket of sheet-lead, fused to the lead plate below and to the lead ring above, so as to form a narrow, closed chamber outside the carbon. This chamber is also provided with a pipe, through which the exciting fluid, contained in a vessel placed about five feet above the cell, is delivered into the chamber, under the pressure due to this height. It then slowly percolates through the walls of the carbon to the zinc, whence it is led off by the waste-pipe, either to be thrown away or, if not exhausted, to be pumped up again to the supply vessel.

Evidently the arrangement is not convenient for single cells; at present the standard form is a seven-cell battery.

Excellent results have been obtained, and it seems possible that this battery may to some extent answer the purpose of accumulators in places where there are no conveniences for charging the latter.

Standard Cells.—The simplest method of measuring a difference of potential or an E.M.F. (and thereby indirectly a current) is to compare it with a known E.M.F.; and such an E.M.F. is best obtained by means of a voltaic cell.

A cell, which is to serve as a standard of E.M.F., must satisfy very stringent conditions. It must be made up from materials readily obtainable and easily purified, so that cells prepared by various observers in all parts of the world may not differ in E.M.F. by any appreciable amount, and it must last indefinitely, without attention, when once set up. It is also very desirable that the E.M.F. should not vary to any material extent with change of temperature, *i.e.* it should have a small temperature coefficient. On the other hand, as it is not required to give a current, its size may be very small (and consequently its internal resistance very large) without affecting its usefulness.

For many years the legal standard of E.M.F. in this country has been Clark's cell, but the International Conference on Electric Units (held in London in 1908) recommended the adoption of the Weston cadmium cell in its stead, and hence it is desirable to describe both these cells. It will be understood that their actual E.M.F. at various temperatures has been determined with the utmost care by the most skilled observers, who have made absolute measurements by methods so refined as to be unsuitable for ordinary purposes.

Clark's Standard Cell.—The metals used are zinc and mercury; the exciting fluid is a solution of zinc sulphate saturated at 30° C.; and the depolariser, mercurous sulphate (which is an insoluble powder). This substance is reduced to metallic mercury by the zinc ions travelling towards the negative plate (*i.e.* the mercury)—an action which may be written $Zn + ZnSO_4 + Hg_2SO_4 = 2ZnSO_4 + 2Hg$. The materials are purified, and the cell is set up in accordance with a very elaborate specification drawn up by the Board of Trade. One of its forms is shown—full size—in Fig. 162. The containing vessel is a small glass tube, the mercury being at the bottom. Connection is made to it either by means of a platinum wire sealed through the glass, or, as in the figure, by a platinum wire protected by a glass tube. The E.M.F. of this cell is usually taken as 1·434 volts at 15° C., but at the 1908 International Conference this was altered to 1·4326 volts at 15° C. The cell has the disadvantage of possessing a rather high temperature coefficient, its E.M.F. falling by ·00115 volt for each centigrade degree rise of temperature above 15° C. The practical disadvantage is not so much due to any uncertainty as to the value of the E.M.F. at any given temperature, as to the difficulty of ascertaining the *exact* temperature of the cell and of keeping it constant.

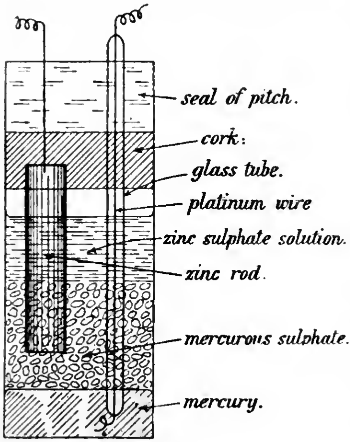


FIG. 162.

Weston Cadmium Cell.—In this cell, cadmium in the form of amalgam is used instead of zinc; and cadmium sulphate instead of zinc sulphate.

The pattern as improved and adopted by the National Physical Laboratory is shown in Fig. 163, for which we are indebted to Mr. H. Tinsley. The tubes are hermetically sealed, and the slight constriction at their lower parts results in the formation of a taper plug of crystals, which holds everything in place, and which makes the cell much more portable and safe during transit.

The E.M.F. was formerly taken as 1·0195 volts at 17° C., but very careful measurements, carried out recently, have shown that its true value is 1·0183 (International)[1] volts at 17° C. Its great advantage is due to its small temperature coefficient, which is about

[1] The meaning of the term "International" will be understood after reading Chapter XXXIV.

·000036 volt per degree centigrade. The vertical tubes are about

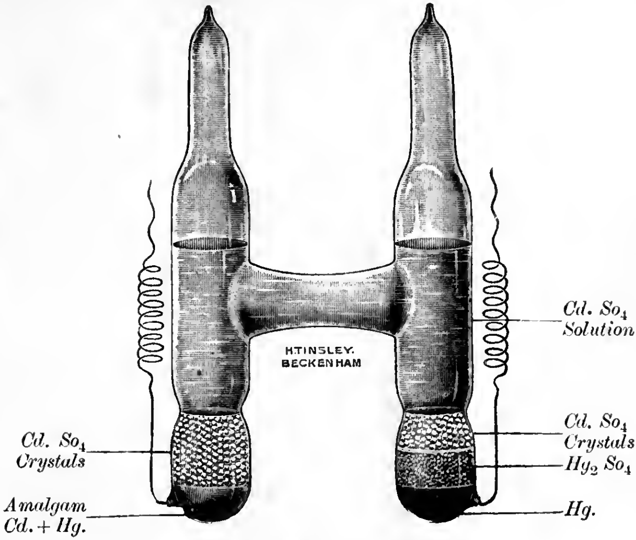


FIG. 163.

3 inches long by $\frac{1}{2}$ inch diameter, and the internal resistance (for this size) is about 900 ohms.

EXERCISE X

1. When zinc and copper are placed in contact in dilute sulphuric acid, hydrogen bubbles (produced by the action of the sulphuric acid on the zinc) are given off at the copper. How is this explained?

2. What do you understand by the term *electric current?*

3. What is meant by polarisation of the electro-negative plate? How can you show its effects on the current?

4. Each terminal of a battery of, say, 100 cells is connected with a separate delicate gold-leaf electroscope. State and explain the effect on each electroscope produced by connecting with the earth: (i.) the middle of the battery; (ii.) the electroscope connected with the platinum end of the battery; (iii.) the electroscope connected with the zinc end of the battery.

5. A plate of zinc and a plate of copper are in contact at one end. Their other ends are connected (i.) by a piece of platinum, (ii.) by a drop of dilute acid. Why does a current flow round the circuit in the second case, and not in the first?

6. A Leclanché cell is connected by long thin wires to a galvanometer, the needle of which is deflected. The poles of the cell are then bridged across for a short time by a piece of thick copper wire. After the removal of the thick wire, the galvanometer deflection is much less than before, but gradually rises to its former value. Explain this. (B. of E., 1898.)

7. Describe one form of primary battery suitable for electric bells, and give a general explanation of its action.

CHAPTER XV

OHM'S LAW APPLIED TO BATTERY CIRCUITS

LET us consider a Daniell's cell having its terminals joined by a wire. The E.M.F. is about 1·1 volts, and *is independent of its size* (see p. 199). Let this be denoted by E. Then, if R be the *total* resistance of the circuit, we have $C=\frac{E}{R}$

It is important to notice that R is made up of two distinct parts: (1) the *external* resistance, which can be varied at will; and (2) the *internal* resistance, which depends upon the size of the plates, their distance apart, and the nature of the liquids used, and which cannot be altered after the cell is set up.

The internal resistance of a Daniell's cell of about one pint capacity will usually be from $\frac{1}{2}$ to 1 ohm.

It is convenient to retain R for the total resistance, and to indicate its components by small letters. Thus, if $r_{\times}$ * indicate the external resistance, and r_b the internal or battery resistance, we have

$$R=r_{\times}+r_b$$

$$\text{and } C=\frac{E}{r_{\times}+r_b}$$

The greatest current the cell can give out will be obtained when $r_{\times}=0$, *i.e.* when the terminals are joined by a thick wire of negligible resistance, and its value is evidently $\frac{E}{r_b}$, which (with the cell in question) is probably less than about 2 amperes. In this case the cell is said to be "short-circuited."

Grouping of Cells.—(*a*) *In series.* In this arrangement, the copper of one cell is connected with the zinc of the next, and so on (Fig. 164).

When similar cells are arranged "in series," the difference of potential, or as we may call it, the E.M.F., of *n* cells is *n* times that of one cell.

Let us consider two Daniell's cells, A and B.

There is a certain difference of potential between the zinc and

* In this notation, the symbol × is a suffix representing external; the form *x* should not be used, as it is apt to be regarded as an algebraic quantity.

the copper of A, and an equal difference between the zinc and the copper of B, but when the zinc of A and the copper of B are joined, their potentials are equalised; whence the difference of potential between two cells arranged "in series" is twice that of one.

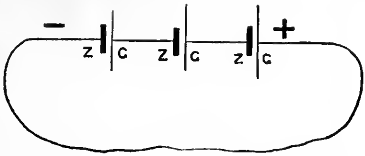


FIG. 164.

The resistance, however, of n cells is n times that of one, since the length of the liquid traversed is n times that of one cell. Ohm's law, therefore, becomes, with n cells arranged "in series,"

$$C = \frac{nE}{r_{\times} + nr_b} \qquad \text{(i.)}$$

(1) If the external resistance $r_{\times}$ is *very large* compared with nr_b, this equation becomes $C = \frac{nE}{r_{\times}}$, *i.e.* the current is n times that of one cell, acting through the same resistance.

(2) If the external resistance is *very small* compared with r_b, we have $C = \frac{nE}{nr_b} = \frac{E}{r_b}$, hence, in this case, the current can never be greater than that given by one cell on short circuit, and can only reach that value when the battery is short circuited.

(*b*) *In parallel*, i.e. all the copper terminals are connected with one another, and all the zinc terminals with one another (Fig. 165). In this arrangement the zincs, being connected together, form, as it were, one large zinc plate; similarly the coppers (in a Daniell's battery) form one large copper plate. Now, as we have pointed out (and as Volta demonstrated long ago), the difference of potential between two metals in a given liquid does not depend upon their size but merely upon the kind of metal employed. Whence, in parallel, the E.M.F. is the same as that of a single cell. The resistance, however, of n cells is $\frac{1}{n}$ that of one cell, because the sectional area of the column of liquid traversed is n times that of one cell. Whence, for this arrangement with n cells, Ohm's law becomes

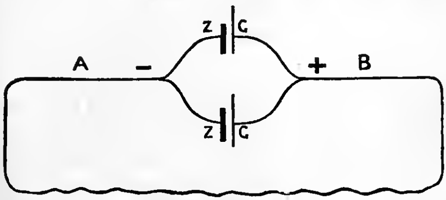


FIG. 165.

$$C = \frac{E}{\frac{r_b}{n} + r_{\times}} = \frac{nE}{r_b + nr_{\times}} \qquad \text{(ii.)}$$

(1) If, therefore, the external resistance becomes *very large*, so that r_b is very small compared with $nr_\times$, we have

$$C = \frac{nE}{nr_\times} = \frac{E}{r_\times}$$

i.e. the current is the same as that of one cell.

(2) If the external resistance is *very small*, so that $r_\times$ becomes practically zero, we have $C = \frac{nE}{r_b}$

i.e. the current is n times that of one cell.

We thus learn that, if we wish to obtain a current strength directly proportional to the number of cells, we must use the *in series* arrangement when we have a *large* external resistance, and the *in parallel* arrangement when we have a *small* external resistance.

(*c*) *Mixed circuit* is a combination of (*a*) and (*b*). In Fig. 166,

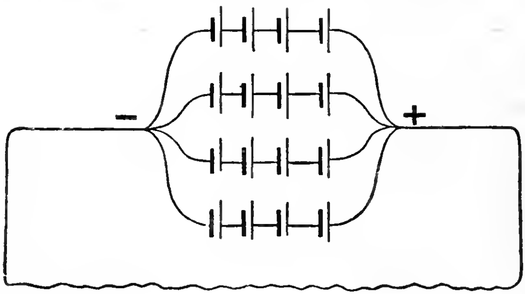


FIG. 166.

each set of four cells is joined in series, while the four at each end are joined in parallel.

Suppose that we have m rows of n cells arranged in series. Each row of n cells will have an E.M.F. of nE, and a resistance of nr_b, but as we have m rows arranged in parallel, the internal resistance will be $\frac{1}{m}$ of one row, *i.e.* $\frac{1}{m}$ of nr_b, or $\frac{nr_b}{m}$, whence, the current strength is given by

$$C = \frac{nE}{r_\times + \frac{nr_b}{m}} \qquad \text{(iii.)}$$

Best Grouping of Cells.—Sometimes it is required to find how a *given* number of cells is to be arranged in order to send the maximum current through a *given* external resistance (although the problem has little practical value). We have, therefore, to find the values of n and m which will make C a maximum, when $mn =$ a constant = the total number of cells.

Dividing equation (iii.) by n, we obtain

$$C = \frac{E}{\frac{r_\times}{n} + \frac{r_b}{m}}$$

Now, the denominator is the sum of two terms, whose product is constant for all values of n and m, and by a well-known theorem, this sum is least when the terms are equal.

Therefore, the denominator will be least, and the current greatest, when

$$\frac{r_\times}{n} = \frac{r_b}{m}$$

$$\text{i.e. } r_\times = \frac{nr_b}{m}$$

Thus, it is evident that we must choose n and m so that the internal resistance of the battery is as nearly as possible equal to the external resistance. (See remark on p. 234.)

P.D. between the Terminals of a Cell.—In applying Ohm's law to actual circuits, students are liable to some confusion of thought, and as we have previously mentioned, it is advantageous to use a fixed notation, especially at first. To that given on p. 216, we may now add: $e_\times$ to represent the P.D. between terminals when a current is flowing; e_b, that portion of the E.M.F. required to send the current through the internal resistance of the battery; where E is the E.M.F. of the battery (or cell).

Here, we regard E as a definite quantity for a given cell, which is not altered by the passage of a current, *i.e.* we assume freedom from polarisation. On *open circuit*, i.e. when no current is flowing, the P.D. between the terminals is identical with E, hence E may be measured by connecting a *suitable* voltmeter[1] to the terminals of the cell. When *a current flows*, we may regard E as split up into two components, one of which ($e_\times$) can be measured by a voltmeter, and denotes that portion of E which is actually useful in sending the current through the external resistance, $r_\times$; while the other (e_b), which apparently disappears and cannot be directly measured, represents the portion of E required to send the current through the cell itself. There is, of course, only one current, which is the same all round the circuit.

[1] For the time being, we shall assume that certain well-known instruments are available, e.g. (*a*) *voltmeters*, which measure directly, with moderate accuracy, the P.D. between two points to which they are connected; (*b*) *ampere meters or ammeters*, which record directly the current flowing through them when introduced into a circuit. In Chapter XXXIII., their principle and construction will be more fully discussed.

Hence, although $e_{\times}$ and e_b vary with the current flowing, we have under all circumstances

$$E = e_{\times} + e_b$$

$$\text{also} \quad e_{\times} = Cr_x \text{ and } e_b = Cr_b$$

$$\text{Whence, we have} \quad e_{\times} + e_b = C(r_{\times} + r_b)$$

$$\text{or} \quad E = CR$$

These results may be demonstrated as follows, assuming that we have (1) a voltmeter of fairly high resistance and of suitable range, (2) a suitable ammeter of very low resistance, and (3) a variable external resistance, whose value need not be known.

Exp. 137. Connect up, as shown in Fig. 167, a Daniell's cell, a voltmeter, V, an ammeter, A, a plug key, K, and an adjustable resistance, $r_{\times}$. (The voltmeter connections are shown by dotted lines to help the student remember that the voltmeter should be regarded, not as forming a parallel circuit, but as a mere accessory, which neither disturbs nor alters the state of affairs in the real or current circuit.)

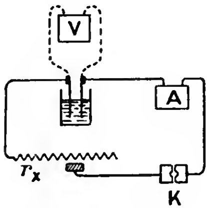


Fig. 167.

The cell should be set up a short time before use, or it should be short-circuited for two or three minutes,[1] in order to bring it into a steady state, otherwise, as the liquids soak through the porous pot, the internal resistance may alter during the experiment. A plug key is used—not a tapper key, as this is apt to introduce a variable resistance (depending on the pressure applied) which would make the readings unsteady.

(1) Read the voltmeter with key open, *i.e.* while no current is flowing. Suppose that the reading is 1·05 volts. Then 1·05 volts is the P.D. between the terminals on *open circuit*, and is, therefore, identical with the E.M.F.

Whence E = 1·05 volts.

Now close the key, having first introduced the maximum resistance, so that only a small current flows. Notice that the voltmeter reads *less* than before. Open the key, and notice that the voltmeter reading *instantly* returns to 1·05 volts. Close the key and gradually reduce the resistance. Notice that as the current increases, the voltmeter reading steadily decreases, but that it always returns to its old value when the circuit is opened.

This effect is not to be confused with that of polarisation (which would reveal itself if the experiment were performed with a Leclanché cell by the reading not returning *instantly* to its old value when the circuit is opened), for it is inherent in generators of all kinds, and depends upon their possession of an internal resistance.

Let us suppose that the voltmeter reads ·75 volt when the ammeter reads ·6 ampere. Then the P.D. at the terminals is ·75 volt, which is the portion of the E.M.F. sending the current through the external resistance, $r_{\times}$.

[1] See the note at the end of Experiment 138 on short-circuiting a Daniell's cell.

i.e. in our notation, $e_{\times} = \cdot 75$ volt when $C = \cdot 6$ ampere,

$$\text{but} \quad e_{\times} = Cr_{\times}$$
$$\cdot 75 = \cdot 6 r_{\times}$$
$$\therefore \quad r_{\times} = \frac{\cdot 75}{\cdot 6} = 1\cdot 25 \text{ ohms}$$

Hence this measurement gives us the value of the external resistance (including, of course, that of the ammeter and of the connecting wires).

Again, $1\cdot 05 - \cdot 75$ volt or $\cdot 3$ volt has apparently disappeared. This portion of the E.M.F. has been lost through the internal resistance, and is denoted by e_b.

$$\text{Now} \quad e_b = Cr_b$$
$$\therefore \quad \cdot 3 = \cdot 6 r_b$$
$$\text{whence} \quad r_b = \frac{\cdot 3}{\cdot 6} = \cdot 5 \text{ ohm}$$

Whence we see that by taking readings (*a*) on open circuit and (*b*) when the cell is sending a current, we obtain data, which determine both the internal and the external resistance.

(2) Alter the current by varying $r_{\times}$, and take another pair of readings. Suppose the voltmeter reads $\cdot 15$ volt when ammeter reads $1\cdot 8$ amperes. Then we know that in this case

$$e_{\times} = \cdot 15 \text{ volts, and } e_b = E - e_{\times} = 1\cdot 05 - \cdot 15 = \cdot 9 \text{ volt}$$

$$\text{but} \quad e_{\times} = Cr_{\times} \qquad \text{and} \qquad e_b = Cr_b$$
$$\text{i.e.} \quad \cdot 15 = 1\cdot 8 r_{\times} \qquad\qquad \cdot 9 = 1\cdot 8 r_b$$
$$\therefore \quad r_{\times} = \frac{\cdot 15}{1\cdot 8} = \tfrac{1}{12} \text{ ohm} \qquad \therefore \quad r_b = \frac{\cdot 9}{1\cdot 8} = \cdot 5 \text{ ohm}$$

Here we notice that the internal resistance, r_b, is a property of the cell, which is not affected by the passage of a current. Strictly, this is not quite true, for the resistance of the liquids in the cell depends on the strength of the solutions and the extent to which they are mixed, and these factors are slightly altered by a current; but within reasonable limits our statement is correct.

Simple values have been taken in the above example for the sake of clearness; in practice, the accuracy of the results depends upon the suitability of the instruments, and, for reasons which will be apparent later, the voltmeter reading will be usually slightly less than the true E.M.F.

It follows from our results that the cell in question cannot, under any circumstances, give out a larger current than $\frac{E}{r_b}$ or $\frac{1\cdot 05}{\cdot 5} = 2\cdot 1$ amperes, a fact that may be shown by experiment.

Exp. 138. Short-circuit the Daniell's cell, by connecting it up with thick wires to an ammeter of very low resistance, and observe that the current is approximately of the above value.

(A Daniell's cell is one that may be short-circuited without injury, but this experiment should not be done with other cells without due consideration, as both cell and instrument may be seriously damaged.)

Exp. 139. Repeat Experiment 137, using a cell of low internal resistance—an accumulator answers well, or failing that, a double-fluid chromic acid cell.

If an accumulator is used we may obtain, on open circuit, a reading of 2 volts (or slightly less),

$$\therefore \ E = 2 \text{ volts.}$$

Insert the maximum resistance, and notice, on closing the key, that a much larger current is obtained. (The experimenter must be careful to insert the resistance *before* completing the circuit, or he may easily injure the ammeter. With a cell of fairly high internal resistance, such as that used in the previous experiments, there is little danger of this kind.) Gradually reduce the resistance, and notice particularly that the voltmeter reading decreases remarkably little, even for quite large currents.

For instance, it may be found that $e_\times = 1{\cdot}9$ volts when $C = 5$ amperes

This means that $e_b = {\cdot}1$ volt, and as

$$e_b = Cr_b$$
$${\cdot}1 = 5r_b$$
$$\therefore \ r_b = \tfrac{1}{50} \text{ ohm.}$$

Again, by altering $r_\times$, we may find that $e_\times = 1{\cdot}8$ volts when $C = 10$ amperes, which again gives $r_b = \frac{1}{50}$ ohm.

An accumulator has such a remarkably small internal resistance because its plates are large in area and are placed close together, and we see, from the above values, that low internal resistance is a very desirable feature when large currents are required.

We also learn that on short circuit, the current should be $2 \div \frac{1}{50} = 100$ amperes. No doubt it might be something like this for the first instant, but such a heavy discharge would, by chemical changes, immediately increase r_b; in any case, the cell would be ruined, and students must be especially careful to avoid short circuits when working with accumulators.

On account of the importance of this subject, we will now work a few numerical examples suggested by the above-mentioned facts.

Example 1.—Six cells are arranged in series and connected to an external resistance of 3·6 ohms. Each cell has an E.M.F. of 1·8 volts and an internal resistance of ·3 ohm. Find the P.D. between the terminals of any one of the cells.

$$\text{Now } C = \frac{6E}{r_\times + 6r_b} = \frac{10{\cdot}8}{3{\cdot}6 + 1{\cdot}8} = \frac{10{\cdot}8}{5{\cdot}4} = 2 \text{ amperes.}$$

We may arrive at the required result by several independent lines of thought, and it is useful for a student to check his work in this way. For instance, (*a*) we may argue that if 2 amperes flow through an external resistance of 3·6 ohms, then $2 \times 3{\cdot}6 = 7{\cdot}2$ volts must be the P.D. across $r_\times$, *i.e.* across the terminals of the battery, or in other words, $e_\times = 7{\cdot}2$ volts.

Now, if the P.D. across 6 cells is 7·2 volts, it is $\frac{7{\cdot}2}{6} = 1{\cdot}2$ volts across one cell, which is the answer required.

(*b*) We may consider each cell individually instead of the whole battery. We know that 2 amperes flow through each cell. in which the resistance is ·3 ohm. Therefore, $2 \times {\cdot}3$ or ·6 volt will be required inside the cell, and this portion apparently disappears, whence each cell will lose ·6 volt out of the 1·8 volts it possesses, and the P.D. across it will be 1·2 volts.

Example 2.—Using the same data, repeat the calculation when the six cells are in parallel.

In this case $C = \dfrac{E}{r_\times + \dfrac{r_b}{6}} = \dfrac{1{\cdot}8}{3{\cdot}6 + \dfrac{{\cdot}3}{6}} = \dfrac{1{\cdot}8}{3{\cdot}65} = {\cdot}49$ ampere.

(*a*) Now if ·49 ampere flows through 3·6 ohms, the P.D. required is $\cdot49 \times 3{\cdot}6$ volts or 1·76 volts, and this must be the P.D. between the terminals of the battery. As the cells are in parallel, this must also be the P.D. between the terminals of any one of the cells.

(*b*) Considering one cell only, we notice that it must give out $\frac{\cdot49}{6} = {\cdot}081$ ampere, and if ·081 ampere flows through an internal resistance of ·3 ohm, the volts lost will be $\cdot081 \times \cdot3 = \cdot0243$ volt, and the cell will have left $1{\cdot}8 - {\cdot}0243 = 1{\cdot}77$ volts, which will be the P.D. at the terminals.

Example 3.—The P.D. between the terminals of a battery of twelve cells in series is found to be 16 volts when the current is 4 amperes. When the external resistance is reduced until the current is 6 amperes, it falls to 12 volts. Find the E.M.F. and the internal resistance of each cell, and the two values of the external resistance.

The external resistances are found at once, for as $e_\times = Cr_\times$, we have in the first case $16 = 4r_\times$, i.e. $r_\times = 4$ ohms; and in the second case $12 = 6r_\times$, i.e. $r_\times = 2$ ohms.

Again, let E = E.M.F. of the whole battery, and

r_b = internal resistance of the whole battery,

then we have $E - e_\times = e_b$, where $e_b = Cr_b$

$\therefore \quad E - e_\times = Cr_b$

so that we obtain the simultaneous equations

$$E - 16 = 4r_b$$
$$E - 12 = 6r_b$$

from which $r_b = 2$ ohms (for the twelve cells in series)

and E = 24 volts.

Hence, E.M.F. of each cell is 2 volts, and its internal resistance is $\frac{2}{12}=\frac{1}{6}$ ohm.

Example 4.—Three cells, each of E.M.F. 1 volt and internal resistance 1 ohm, are connected in series by wires of negligible resistance, but one cell is reversed so that it acts in opposition to the others (Fig. 168). Find the P.D. between the terminals of this cell.

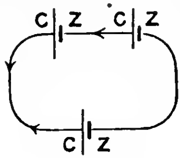


FIG. 168.

The current in the circuit is evidently

$$C=\frac{2E-E}{3r_b}=\frac{1}{3}\text{ ampere.}$$

Now, to send $\frac{1}{3}$ ampere through any one of the cells requires $\frac{1}{3}\times 1=\frac{1}{3}$ volt. But the current has to flow through the reversed cell *against* its own E.M.F., and it will be necessary to apply an equal and opposite P.D. of 1 volt to balance that: hence the P.D. across the cell is $1\frac{1}{3}$ volts.

We can check this result by the following method. As the connecting wires are of negligible resistance, the P.D. across this cell will be identical with the P.D. between the terminals of the other two cells. Now each of these cells loses $\frac{1}{3}$ volt through internal resistance, and has $\frac{2}{3}$ volt remaining. The P.D. across them will, therefore, be $\frac{4}{3}$ volt, a reading which a voltmeter connected across the reversed cell would give.

Example 5.—A battery of E.M.F. 16 volts and internal resistance 3 ohms is joined up in parallel with another battery of E.M.F. 12 volts and internal resistance 2 ohms, the circuit being completed by a wire of 6 ohms resistance (Fig. 169). Determine what occurs.

Let E = E.M.F. of larger battery = 16 volts.
b = its internal resistance = 3 ohms.
C = current given out by it.
E_1 = E.M.F. of smaller battery = 12 volts.
b_1 = its internal resistance = 2 ohms.
C_1 = current *through* it.
and $r_\times$ = external resistance = 6 ohms.

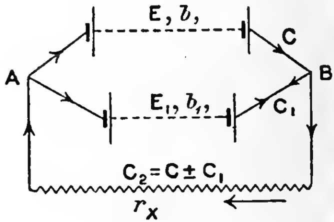


FIG. 169.

(We have written b and b_1 instead of r_b and r_{b1} to simplify the notation.)

Notice that, although the more powerful battery must give out a current, there are three possible cases as regards the other.

(1) It *may* give out some current C_1, so that the total current through $r_\times$ is $C+C_1$ amperes.

(2) The current C may divide and a portion C_1 may flow through the weaker battery in opposition to its E.M.F. (E_1).

(3) It may neither give out nor receive a current, *i.e.* C_1 may be zero, and then it might be disconnected without in any way altering the current through the other battery or through $r_\times$.

The determining condition is the P.D. between the points A and B. This must necessarily be less than E. If it is greater than E_1, then a current flows backwards against E_1; if it is less than E_1, both batteries generate current; if it is equal to E_1, then no current flows in either direction through the smaller battery.

Let $e_\times$ = P.D. between the points A and B.

Now the volts lost in larger battery $= C \times b$.

$$\therefore \; E - Cb = e_\times \qquad (1)$$

Considering the smaller battery, if it generates a current C_1 we must have

$$E_1 - C_1 b_1 = e_\times$$

If C_1 flows backwards through it, we have (see last example)

$$E_1 + C_1 b_1 = e_\times$$

$$\text{or generally, } E_1 \mp C_1 b_1 = e_\times \qquad (2)$$

Again, if C_2 = current through $r_\times$, we have

$$C_2 r_\times = e_\times$$

$$\text{or } (C \pm C_1) r_\times = e_\times \qquad (3)$$

In applying these equations to numerical calculations, remember that either the two upper signs or the two lower hold good in (2) and (3). The beginner will usually find it best to take one pair of values and work out the result. If one of the currents becomes a minus quantity, it is evident that he has chosen the wrong case, and the calculation can be repeated with the signs changed. For instance, in our problem we have, using the upper signs—

$$E - Cb = E_1 - C_1 b_1 \quad \text{and} \quad E - Cb = (C + C_1) r_\times$$

$$16 - 3C = 12 - 2C_1 \qquad 16 - 3C = (C + C_1)6$$

$$\text{or } 3C - 2C_1 = 4 \qquad \text{or} \quad 9C + 6C_1 = 16$$

$$\text{whence} \quad C = \tfrac{14}{9} \text{ amperes}$$

$$\text{and} \quad C_1 = \tfrac{1}{3} \text{ ampere}$$

$$\therefore \quad C_2 = \tfrac{17}{9} \text{ amperes.}$$

If the lower signs had been selected, we should have obtained $C_1 = -\frac{1}{3}$, which is absurd.

We can check the result by noticing that $e_\times = \frac{17}{9} \times 6 = \frac{34}{3} = 11\frac{1}{3}$ volts, which is less than E_1, and therefore this battery acts as a generator.

To find the condition, mentioned in (3) above, which makes $C_1 = 0$, we put

$$e_\times = E_1$$
$$\text{or } E - Cb = E_1$$
$$16 - 3C = 12$$
$$\therefore\ C = \tfrac{4}{3} \text{ ampere.}$$

$\therefore$ $r_\times$ must be such that a P.D. of 12 volts sends $\frac{4}{3}$ ampere through it,

$$i.e.\quad \tfrac{4}{3}r_\times = 12$$
$$\text{or}\quad r_\times = 9 \text{ ohms.}$$

For this value of $r_\times$ there is only one current, which is the same whether the smaller battery is present or not. If we make $r_\times$ greater than 9, then $e_\times$ is greater than E_1 and a current flows backwards against E_1.

Example 6.—The terminals of a battery of E.M.F. 4 volts and resistance $1\frac{1}{2}$ ohms are connected to those of a battery of E.M.F. 3 volts and resistance $\frac{1}{4}$ ohm by wires of resistances 1 ohm and 6 ohms respectively, so that both batteries act in the same direction. Show that, if a third wire be placed so as to join the middle points of these wires, no current passes through it.

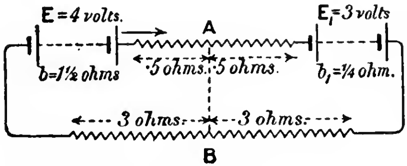


FIG. 170.

We have to show that the points A and B (Fig. 170) are at the same potential, and we notice that when that condition is fulfilled, there is only *one* current in the circuit whether AB is joined by a wire or not.

Let E and E_1 = the two E.M.F.'s,
b and b_1 = the two internal resistances,
and e = the P.D. between A and B.

Suppose that A and B are *not* connected, and let C be the current in the circuit. Then considering the portion of the diagram to the left of AB, we see that e can be obtained by deducting from 4 volts the volts lost in the battery and the volts required to send C through 3·5 ohms. Exactly the same argument can be applied to the portion of the diagram to the right of AB, but whereas the effect of the former part is to make the potential of A higher than that of B, the effect of the latter part is to make the potential of B higher than that of A.

To satisfy the required condition, e must be zero.

$$\therefore \quad e = E - Cb - 3{\cdot}5C = 0$$

$$\text{and} \quad e = E_1 - Cb_1 - 3{\cdot}5C = 0$$

$$\therefore \quad E = C(b + 3{\cdot}5)$$

$$\text{and} \quad E_1 = C(b_1 + 3{\cdot}5)$$

$$\text{i.e.} \quad \frac{E}{E_1} = \frac{b + 3{\cdot}5}{b_1 + 3{\cdot}5}$$

On substituting values, it will be found that this condition is fulfilled, and hence the P.D. between A and B is zero.

This result has several useful applications; see pp. 306 and 457.

Exercise XI

1. What is *Ohm's Law?*
A cell of electromotive force of 2 volts and internal resistance ½ ohm is used to send a current through a wire of 11½ ohms resistance. Find the strength of the current. (Oxford Local, Senior, 1899.)

2. The terminals of a battery, of E.M.F. 4 volts and resistance 3 ohms, are connected by a wire of resistance 9 ohms. By how much is the difference of potential altered thereby? (B. of E., 1900.)

3. Describe the action of a Daniell cell.
The E.M.F. between the poles of a battery is 12 volts when the circuit is "open," and 10 volts when it is closed by a resistance such that a current of 6 amperes is passing. Find the resistance of the battery. (Camb. Local, Senior, 1895.)

4. State the relation between the E.M.F. of a battery, the resistance of a circuit, and the strength of the current produced. The E.M.F. of a battery being 12, and its resistance 8, find the strength of the current generated by it when its poles are connected (1) by a wire whose resistance is 16, and (2) by a wire whose resistance is 40.

5. If an increase of the resistance of a circuit by 10 ohms causes the strength of the current to decrease from 5 to 2, find the total resistance of the circuit after the change.

6. Ten voltaic cells, each of internal resistance 3 and E.M.F. 2, are connected—
(*a*) in a single series;
(*b*) in two series of five each, the similar ends of each series being connected together.
If the terminals are in each case connected by a wire of resistance 20, show what is the strength of the current in each case.

7. You have two voltaic cells each having a resistance of 3 units (ohms) and an E.M.F. of 1·1 unit (volt). Show what is the strength of the current which would be produced in a wire of resistance 9·5 units (ohms)—
(i.) By one of the cells alone.
(ii.) By both cells connected in series.
(iii.) By both cells connected abreast.

8. The terminals of a battery of five Grove's cells, the total E.M.F. of which is 9 volts, are connected by three wires, the resistance of each of which is 9 ohms. The current through each wire is $\frac{2}{7}$ of an ampere. What is the internal resistance of each cell?

9. Five cells are arranged in series, with a line of 100 ohms resistance. The resistance of each cell being 7·3 ohms, and its E.M.F. being 1·5 volts, find the current strength.

10. How would you connect two equal constant cells of internal resistance

5 ohms each, if you wished to deposit copper as rapidly as possible in a voltameter of 7 ohms resistance? (B. of E., 1906.)

11. Four cells, each of 1 ohm internal resistance, are coupled up in series with an external resistance of 8 ohms. Two similar cells are coupled up in parallel with an external resistance of 5·5 ohms. Compare the currents in the two circuits. (Oxford Local, 1901.)

12. A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. The E.M.F. of each cell is 1 volt, and its internal resistance 5 ohms. Determine the difference of potential between the positive and negative poles of any one of the cells. (B. of E., 1894.)

13. A circuit is formed of five similar cells in series and a coil of 80 ohms resistance. The resistance of the connecting wires is 1 ohm. The E.M.F. of each cell is two volts and its internal resistance ·4 ohm. Calculate the difference of potential between the positive and negative poles of any one of the cells. (Camb. Local, 1906.)

14. Six cells arranged in series, each having an internal resistance of ·4 ohm, are connected by a wire of 1·6 ohms. If each cell has an electromotive force of 1 volt, what is the potential difference between the positive pole of the battery and the point of junction of the third and fourth cells? (B. of E., 1899.)

15. A metal ball, connected by a fine wire with the copper terminal of a battery of 120 cells in series, the zinc terminal of which is connected to earth, acquires a charge of 100. Show what charge the ball will acquire if the twenty-fifth, sixty-first, or ninety-first zinc plate, always counting from the zinc end of the battery, be connected to earth.

16. Show that if the external resistance is equal to the resistance of a cell, the same current strength is produced whether the battery is coupled in series or in parallel. (Camb. Local, 1907.)

17. Define *electromotive force between two points of a circuit.*

The two poles of a battery are connected to the terminals of an electrometer, and an E.M.F. of 4 volts is indicated. The poles of the battery are then connected through a resistance of 10 ohms, and the electrometer indicates an E.M.F. of 3·5 volts. Find (*a*) the current flowing round the closed circuit, (*b*) the E.M.F. of the battery, (*c*) the resistance of the battery. (Oxford Local, 1900.)

18. A zinc-and-copper couple is joined up with another couple similar in all respects except that the plates are twice as large. Describe the effect when they are joined up in series with a galvanometer; and, secondly, joined in series so that their action opposes each other. (Coll. of Preceptors, 1897.)

19. (*a*) What is meant by "polarisation" in a cell? Explain how the difficulty is met with in the arrangement of a Daniell cell. (*b*) Three Leclanché cells are joined in series, forming a battery. Three more form a precisely similar battery. The two batteries are joined in parallel circuit, forming a composite battery. The resistance of each cell is 2 ohms, its E.M.F. is 1·4 volt. Find what current will flow round a wire of 20 ohms resistance, which joins the poles of the composite battery. (Oxford Local, 1903.)

20. A battery of 10 volts and internal resistance ·5 ohm is connected in parallel with one of 12 volts and internal resistance ·8 ohm. The poles are connected to an external resistance of 20 ohms. Find the current in each branch. (London Univ. B.Sc., Internal, 1907.)

21. The electrodes of a quadrant electrometer are joined to the terminals of a battery of five cells in series. In what ratio will the deflection of the needle be altered if the electrodes are *also* joined to the terminals of a battery of three cells in series similarly arranged, all the cells being alike, and the connecting wires thick? (B. of E., 1899.)

22. The terminals of a battery formed of seven Daniell's cells in series are joined by a wire 35 feet long. One binding-screw of a galvanometer is joined by

a wire to the copper of the third cell (reckoned from the copper end). With what point of the 35 feet wire can the other screw of the galvanometer be connected so that the needle shall not be deflected?

23. Six similar cells are arranged in series, and the circuit completed through a coil of wire and a galvanometer. The resistances of the battery, coil, and galvanometer are 10, 50, and 20 ohms respectively. If the difference of potential between the terminals of the galvanometer be 2 volts, what is the E.M.F. of each cell of the battery?

24. You have a battery of twelve similar cells connected in series; each has an E.M.F. force = 1·1, and an internal resistance = 3. If the poles of the battery are connected by a wire whose resistance = 240, what will be the strength of the current? What will be the effect on the strength of the current of removing from the battery three of the cells, and replacing them with their poles inverted?

25. Two cells, A and B (E.M.F. and internal resistance of each are 1 volt and 1 ohm respectively), are arranged in series. The positive and negative poles of this battery are connected with the positive and negative poles respectively of a third cell, C, exactly like A and B, the connecting wires having negligible resistance. What is the current in the circuit, and what is the potential difference between the positive and the negative poles of the cell C?

26. Find the best arrangements of twenty-four cells having an external resistance of 3 ohms, and each cell having an internal resistance of 2 ohms.

27. Four points, A, B, C, D, are connected together as follows: A to B, B to C, C to D, D to A, each by a wire of 1 ohm resistance; A to C, B to D, each by a cell of 1 volt E.M.F. and 2 ohms resistance. Determine the current flowing through each of the cells. (B. of E., 1900.)

28. A battery is formed of four Grove's cells in series, and its poles are joined by a wire. If one electrode of a Thomson's quadrant electrometer is connected to the middle point of this wire, and the other electrode to the platinum of each cell in turn, describe the indications of the electrometer.

29. A battery of twelve equal cells, in series, screwed up in a box, being suspected of having some of the cells wrongly connected, is put into circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged, so as to work with or against the battery. What is the state of the battery? Give reasons for your answer. (B. of E., 1895.)

30. How would you arrange a given number of cells, which are to form a battery sending a current through a given external resistance, so that this current shall be greatest?

Prove that an electric lamp, whose resistance is 20 ohms, and which requires a current of ·6 ampere, cannot be lighted with a battery of fifty cells each having a resistance of 2·5 ohms and a voltage of 1·4, if the cells are placed in series, but that it can if they are arranged in two parallel rows of twenty-five. (Oxford Local, 1908.)

31. Give full practical definitions of the ampere, the ohm, and the volt.

A battery of fifty-four cells, each of E.M.F. 2 volts and resistance ·005 ohm, is employed with a number of 100-volt glow lamps all in parallel, each lamp requiring ·6 ampere; what is the maximum number of lamps which can be used so that the voltage at their terminals shall not fall below 100 volts? (Oxford Local, 1907.)

32. An electric light installation consists of a group of lamps in parallel arc between the ends of leads. The leads have total resistance ·4 ohm, and bring current from sixty accumulators, each with E.M.F. 2 volts and resistance ·01 ohm. When twenty-five lamps are switched on, each takes ·4 ampere. Find the resistance of a lamp, and the watts used in each part of the circuit. (London Univ., Inter. Hon., 1894.)

CHAPTER XVI

ENERGY EXPENDED IN A CURRENT CIRCUIT—HEATING EFFECT

It has been shown in Chapter V. that if a quantity of charge Q flows from one point to another, between which the difference of potential is V (its being understood that V is not altered by the passage of the charge), the work done *on*, or done *by* the charge is QV ergs, where Q and V are expressed in static units.

In a voltaic circuit the facts are essentially similar, although the units employed are different. Let E be the E.M.F. *in absolute units* (see p. 206), and let a current of i absolute units flow round the circuit for t seconds. Then a "quantity" Q, such that $Q = i.t$ has passed through a difference of potential E, and the units have been so chosen that the energy expended is EQ, or Eit *ergs*. When the circuit contains merely ordinary conductors, *e.g.* those dealt with in all the cases up to the present, this energy is wholly converted into heat, part of which appears in the external resistance, and part in the generator itself.

Let us now suppose that practical units are used, and that a current of C amperes flows for t seconds in a circuit in which the E.M.F. is E volts. The quantity passing round the circuit is Ct ampere-seconds, and, as *one* ampere-second is the practical unit of quantity, known as a "coulomb," the quantity is, therefore, Ct coulombs. From the above statement, the energy expended is evidently measured by ECt, and will be in ergs, if we express the volts and amperes in absolute units. Now it has already been mentioned (see p. 206) that 1 volt $= 10^8$ absolute units of E.M.F., and 1 ampere $= \frac{1}{10}$ of an absolute unit of current, and hence the energy expended as heat is $E \times 10^8 \times C \times \frac{1}{10} \times t$ ergs

$$= ECt \times 10^7 \text{ ergs, or } EC \times 10^7 \text{ ergs } \textit{per second.}$$

From this, it follows that, if a current of 1 ampere flows through a P.D. of 1 volt, energy is expended at the rate of 10^7 ergs per second during the whole time of flow.

Electrical Power.—The *rate* at which work is done is the measure of the "power" expended in the circuit; and we see that this rate is proportional to the product of current and E.M.F., and is independent of the time during which the current flows.

Definition.—The power corresponding to the product of 1 volt × 1 ampere is called a *watt*. Hence, 1 watt $= 10^7$ ergs per second.

Relation between Horse-power and Watts.—On p. 577, it is shown that 1 H.P. $= 746 \times 10^7$ ergs per second. Therefore, 1 watt $= \frac{1}{746}$ of an H.P.

Again, we know that 1 H.P. $= 33{,}000$ foot-pounds per minute, from which it appears that 1 watt $= \frac{33{,}000}{746} = 44\frac{1}{4}$ foot-pounds per minute.

Example.—Twelve cells are connected in series, and joined up to an external resistance of 5 ohms. If the E.M.F. of each cell is 2 volts, and its internal resistance is ·25 ohm, find the total power expended in the circuit.

Let C = current in amperes,

$$\text{then } C = \frac{12 \times 2}{5 + (12 \times \cdot 25)} = \frac{24}{8} = 3 \text{ amperes}$$

The total power expended in the circuit is EC watts, or $24 \times 3 = 72$ watts, which is $\frac{72}{746}$ H.P.

Now the P.D. between the terminals of the battery is $e_\times$,

$$\text{where } e_\times = Cr_\times$$

$$\therefore e_\times = 3 \times 5 = 15 \text{ volts}$$

$$\text{and } e_b = 24 - 15 = 9 \text{ volts}$$

∴ 15×3, or 45 watts, is expended in the external resistance, and 9×3, or 27 watts, in the battery itself. It will be observed that these values do not depend upon the time the current flows. On the other hand, if it were required to find the energy expended, it would be necessary to take into account the time of flow.

It should be noticed that the argument may be applied to the whole circuit, or to any part of that circuit.

Example.—Find the horse-power required to send 9 amperes through 10 ohms.

In this case, the 10 ohms may be regarded as a portion of a circuit through which 9 amperes are flowing. By Ohm's law, the P.D. across the 10 ohms must be 90 volts, and the power expended is $90 \times 9 = 810$ watts $= \frac{810}{746} = 1{\cdot}09$ H.P.

Equivalent Expressions for Power.—

$$\text{We have Power} = \text{EC watts} \qquad (1)$$

$$\text{also } E = CR, \text{ and } C = \frac{E}{R}, \text{ hence}$$

$$\text{Power} = \text{CR} \times \text{C} = \text{C}^2\text{R watts} \qquad (2)$$

$$\text{or Power} = \text{E} \times \frac{\text{E}}{\text{R}} = \frac{\text{E}^2}{\text{R}} \text{ watts} \qquad (3)$$

The three expressions are numerically identical, and we may use the form which is most convenient. For instance, in the last example, as current and resistance are given it would have been simpler to write

$$\text{Power} = \text{C}^2\text{R watts}$$

$$= 9^2 \times 10 = 810 \text{ watts}$$

In saying that the three expressions are numerically identical, it must be understood that E is the voltage which sends the current C through the resistance R. There is no difficulty in such cases as we have already considered, but it may be remarked that if a current C flows between two points in a circuit, the P.D. between these being e volts, then in *all* cases, the power expended in that portion of the circuit is eC watts. On the other hand, if r is the resistance between the points in question, we cannot say that the power expended is C^2r watts. That *will* be the case if the points are joined by an ordinary resistance, but if, for example, a motor is included between them, the product C^2r merely gives the power which is converted into heat, the *difference* being the power which is converted into work of some kind. These matters will become clearer as we proceed; in the meantime we may sum up by saying that eC *always* gives the power supplied to a given portion of a circuit, and C^2r *always* gives that portion of the power which is converted into heat. Very frequently the latter expression is a measure of the wasted power, although this is not always the case, *e.g.* in an electric lamp or oven it is useful power.

Efficiency of a Cell or Battery.—The term "efficiency," when applied to any mechanism whatever, denotes the value of the ratio $\frac{\text{useful power}}{\text{total power}}$, and is very frequently expressed as a percentage.

In the case of a cell or battery, some convention must be made regarding the meaning of the expression "useful power." Evidently the power expended *inside* the cell is totally wasted, whereas that expended *outside* may be usefully applied. Hence, it is convenient to regard the power expended in the external circuit as useful power. From this it follows that, if $e_\times$ is the P.D. at the terminals when a current C is flowing, we have

$$\text{Efficiency} = \frac{\text{useful watts}}{\text{total watts}} = \frac{e_\times \text{C}}{\text{EC}} = \frac{e_\times}{\text{E}}; \text{ (or } \frac{e_\times}{\text{E}} \times 100 \text{ per cent.)}$$

It is instructive to write this in another form :—

Putting $e_\times = Cr_\times$ and $E = C(r_\times + r_b)$, we have

$$\text{Efficiency} = \frac{Cr_\times}{C(r_\times + r_b)} = \frac{r_\times}{r_\times + r_b}$$

This expression tells us that the smaller the internal resistance, the greater the efficiency, and that it can only reach 100 per cent. when $r_b = 0$. It also tells us that, if we consider any given cell for which r_b has a constant value—large or small as the case may be—the efficiency may be made as nearly 100 per cent. as we please, provided we make $r_\times$ sufficiently large in comparison. On the other hand, if $r_\times$ be small, the efficiency is low, and becomes zero on short circuit.

Maximum Rate of Working.—It is, therefore, evident that maximum efficiency is not the same thing as maximum power. When, for example, $r_\times$ is great, the current is small, and the power expended is small also.

In fact, it will be seen that the efficiency reaches 100 per cent. in the limiting case when $r_\times = \infty$, that is, when no work is done.

The power expended in the external circuit is, therefore, zero when the current is zero, and also zero when the current is greatest, *i.e.* on a short circuit. Hence, for some intermediate strength of current, it will have a maximum value. The same inference may be drawn by noticing that the useful power is $e_\times C$, from which it will be seen that $e_\times$ decreases as C increases (see Experiment 137). We have, therefore, to find the condition for which $e_\times C$ shall have its greatest value.

Now, we know that $e_b = Cr_b$, or $C = \frac{e_b}{r_b}$

$\therefore$ useful power may be written $\frac{e_\times \times e_b}{r_b}$

Now, the denominator of this expression is constant, and the numerator is the product of two quantities whose sum is a constant (for $e_\times + e_b = E$). Hence, the numerator (and, therefore, the fraction itself) will be greatest when $e_\times = e_b$, i.e. when the external resistance is equal to the internal resistance.

From the relation already given, it follows that the efficiency in this case is $\frac{1}{2}$, or 50 per cent.

The preceding argument is applicable to a dynamo or any form of current generator. The main point to grasp is the vital distinction between "maximum efficiency" and "maximum output." In practice, the distinction is important, for a dynamo is never worked with a view to maximum possible output (and, as a rule, it would be seriously injured if the attempt were made), but under conditions which enable its efficiency to reach a reasonably high value.

It may be mentioned that on p. 219, we found that the maximum current through a given resistance, using a given number of cells, was obtained by grouping them, so that the internal resistance of the battery became equal to the external resistance. In this case also the efficiency is only 50 per cent., and from an economical standpoint, it is in no sense a *best* grouping of the cells.

Electrical Energy.—The energy corresponding to a power of 1 watt acting for 1 second is called a "Joule." Evidently 1 joule $= 10^7$ ergs. It may be called a "watt-second."

For commercial purposes, the unit of energy is the kilowatt-hour, also known as the "Board of Trade unit." It will be seen that 1 kilowatt-hour $= 1000$ watt-hours $= 1000 \times 60 \times 60$ watt-seconds or joules.

Example.—Two hundred lamps, each taking ·5 ampere at a pressure of 100 volts, are run for 24 hours. What is the cost, at 4d. per Board of Trade unit?

$$\text{The current will be } 200 \times \cdot 5 = 100 \text{ amperes, and}$$

$$\text{Watts} = \text{EC} = 100 \times 100 = 10{,}000$$

$$\text{Watt-hours} = 10{,}000 \times 24 = 240{,}000$$

$$\text{Kilowatt-hours} = \frac{240{,}000}{1000} = 240$$

$$\therefore \text{ cost} = 240 \times 4\text{d.} = £4$$

Heating Effect of a Current.—The fact that a conductor carrying a current is heated thereby, is now a matter of common observation, on account of its application in electric lamps. It is, however, necessary to examine experimentally the laws of such heat production.

Exp. 140. Connect a piece of thin platinum wire in circuit with a battery of a few chromic acid cells. Probably it will become sensibly hot to the touch, but not sufficiently hot to be luminous. Gradually shorten the wire, and notice that it becomes red hot, then white hot, and finally fuses.

This experiment demonstrates the existence of a heating effect, but it is also apt to suggest that the effect depends in some way on the length of the wire. This will be shown later to be a misconception; in fact, shortening the wire in the above experiment was merely a convenient method of reducing the resistance of the circuit, and thereby increasing the current strength.

The following well-known experiment may be used to show that, when the current is constant, the heating effect depends upon the resistance of the conductor.

Exp. 141. Make a chain of alternate links of platinum and silver wire (32 gauge) by cutting the wire into inch lengths, and then bending them into loops. Pass a current from a 6-cell chromic acid battery through the chain. Notice that the platinum links become white hot, while the silver links remain comparatively cool.

This result, however, does not depend only upon the difference in the resisting powers of the two metals, but also upon their specific heats. Although the specific resistance of platinum is nearly six times that of silver, its specific heat is about half as great; so that the increase in temperature would be nearly twelve times as great in platinum as in silver, if the thickness, current strength, and loss by radiation remained constant. As a matter of fact, the platinum wire will lose much more heat by radiation than the silver, and hence the rise in temperature will be correspondingly reduced.

Exp. 142. Connect up an ammeter and an adjustable resistance in circuit with a battery capable of giving out a fairly strong current (or to electric light mains). Include in the circuit a pair of terminals, between which lengths of fine iron (or tin) wire may be conveniently connected (platinum wire is too expensive for use in such experiments).

(*a*) Keeping the diameter of the wire constant, determine the fusing current for various lengths, between say $\frac{1}{2}$ inch and 6 inches. It will be found that the fusing current is abnormally great for very short lengths, but that it falls rapidly as the length is increased, and soon becomes constant for *all* lengths greater than a certain value. The large fusing current required for short lengths is due to the cooling action of the terminals, which rapidly carry off heat by conduction. When the wire is sufficiently long, this action is negligible, and then we find that the fusing current for a wire of given diameter is *independent* of the length.

(*b*) Select a length great enough to give the true fusing current, and keep this constant whilst using wires of the same material but of different diameters. Determine the fusing current for as many gauges as possible. We shall then find, as would naturally be expected, that the fusing current increases with the diameter, and we might reasonably infer that it would be proportional to the area of cross section, *i.e.* to the square of the diameter, for evidently *two* similar wires side by side would require *twice* the fusing current taken by one of them.

Plot a graph, taking current strengths as ordinates, and the squares of the diameters as abscissæ. Notice that the graph is not straight, but that it bends in a direction which indicates that the fusing current for the thicker wires is rather less than would be anticipated. This is because the surface area of a thick wire does not increase at so great a rate as its cross section, and hence it does not lose proportionally the same amount of heat by radiation. For example, if we arrange, say, 6 separate wires side by side, the total fusing current will be 6 times that required for a single wire, but if we imagine that they are rolled up into a single circular wire of 6 times the sectional area, then the surface of the single wire is considerably less than the total surface of the separate wires, consequently it loses less heat by radiation, and its temperature is higher for a given current. When, however, we are dealing with wires of nearly the same area, we may sum up our two experimental results by saying that the fusing current is independent of the length, and practically proportional to the area of its cross section.

To give some idea of the magnitude of a current to fuse a size of wire in ordinary use, we take the following figures from a student's note-book :—

20 gauge copper wire became red hot with	55 amperes	and fused at	68 amperes.
20 ,, iron ,, , ,, ,,	15 ,,	,,	25 ,,

These metals oxidise rapidly at high temperatures, which also influences the result. It may also be pointed out that these figures do not help in determining the greatest current which may be carried under ordinary conditions, because that depends almost entirely upon the risk of injury to the insulation. For instance, the "safe" current to be carried indefinitely by a large coil of double cotton-covered 20 gauge copper wire would be from 1 to 2 amperes.

Relation between Heat Production, Current, and Resistance. Joule's Law.—**Exp. 143.** Support three stout copper wires on a light frame. Wind exactly equal lengths (two or three yards) of silk-covered platinoid wire or manganin wire (30 gauge) into two open spirals, and solder them to the copper wires as shown at **A**, **B**, and C, Fig. 171. Immerse them in distilled water contained in a glass beaker (or any other convenient vessel), which may with advantage be surrounded by cotton wool. Join up to a battery through a plug key or switch, and include in the circuit an ammeter and an adjustable resistance.

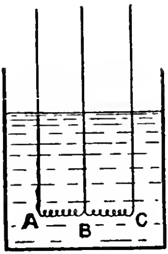


FIG. 171.

(*a*) *To find the relation between heat production and resistance, when the current is constant.*

Connect up *one of the coils*, adjust the current to some convenient value, switch off the current, stir up the water, and take its temperature by means of a thermometer, and then at some noted instant of time, again switch on the current and let it flow steadily for, say, exactly one minute (or more). Keep stirring gently, and at the end of the time, break the circuit, stir up well, and again note the temperature, thus determining its rise in a given time.

Next, place *both* coils in series in the circuit, alter the adjustable resistance until the current is the same as before, then switch off, and take the temperature. Switch on again at a given instant, and let the current flow for exactly the same time as before. Then note temperature and determine its rise.

It will be found that the rise in the second case is approximately twice as great as at first, thus showing that *when the current is constant, doubling the resistance doubles the heat produced.*

(*b*) *To find the relation between heat production and current, when the resistance is constant.*

Use *either* of the two resistances, or put them in parallel, which is sometimes convenient, *i.e.* connect A and C together, and make AC and B the terminals. In either case do not alter the arrangement during the experiment. Adjust the current by means of the resistance to a convenient value, and determine the rise of temperature in a given time exactly as before. Then alter the current until it is exactly twice its previous strength, and repeat the operations.

It will be found that the rise in temperature is now approximately four times as great as before, thus showing that when the resistance is constant, doubling the current gives four times the amount of heat.

A series of values may be taken if desired, but the results are sufficient to verify the law that when resistance and current both vary, the amount of heat produced is proportional to the square of the current, to the resistance, and to the time for which the current flows,

$$\text{or Heat} \propto C^2rt.$$

This law was established experimentally by Joule, and is known by his name.

From what has been already said (see p. 232), it follows that the rate of heat production in any given conductor is proportional to the power in watts expended in it, and that instead of C^2r we may write eC or $\frac{e^2}{r}$ as may be most convenient.

In order to evaluate the amount of heat in terms of some heat unit, we require a knowledge of the *mechanical equivalent of heat*. For instance, it is known that 1 gram-centigrade-degree heat unit (the heat required to raise 1 gram of water through 1° C.) is produced by $4{\cdot}18 \times 10^7$ ergs, and we also know that 1 watt represents 10^7 ergs per second. Therefore, if W = power in watts, and t = the time in seconds during which the current flows, we have energy converted into heat $= W \times t \times 10^7$ ergs, and as $4{\cdot}18 \times 10^7$ ergs produce 1 heat unit, we have

$$\text{Heat units} = \frac{W \times t \times 10^7}{4{\cdot}18 \times 10^7} = \frac{Wt}{4{\cdot}18}$$

This may be written, Heat units $= {\cdot}24Wt$, from which it appears that a "watt-second" or "joule" corresponds to ·24 of a heat unit.

It is necessary to distinguish carefully between the *amount of heat* produced in a conductor and its *rise of temperature*. The former is a definite quantity, and as we have seen, depends only upon the watts expended on it. The latter, however, can be estimated only in simple cases. It depends upon the specific heat and upon the mass of the conductor, and also upon the nature of its surface and the surrounding conditions, for obviously, the temperature will rise until the heat lost by radiation, &c., per second is equal to the number of heat units produced per second.

Evidently, if we measure both the watts expended in a conductor and the number of units of heat produced, we can determine the value of the mechanical equivalent of heat. Such a determination can be made with the arrangement used in Experiment 143, but it is better to use a continuous flow of water, as the "water equivalent" of the apparatus is thereby eliminated. For example, in a certain experiment a spiral of platinoid wire was enclosed in a glass tube, and a steady flow of water maintained through it. A current was sent through the spiral, the voltage across it and the current through it being measured by a very accurate voltmeter and ammeter respectively. Two thermometers indicated the temperatures at which the water entered and left the tube, and the amount of water passing per second was found by weighing. It was arranged that the water should enter at a temperature a little less than that of the room and leave at a temperature somewhat higher, thus, to a great extent, automatically compensating for loss of heat by radiation. The following readings were obtained :—

Current = 3·37 amperes ; P.D. across spiral = 10·60 volts.

The current was allowed to pass for 3 minutes ; during which time, 182 grams of water passed through the apparatus, entering at a temperature of 16° C., and leaving at 24·3° C.

$\therefore$ Power expended in coil $= eC = 10{\cdot}6 \times 3{\cdot}37 = 35{\cdot}7$ watts.

Energy converted into heat in 3 minutes $= 35{\cdot}7 \times 180 \times 10^7$ ergs.

Heat units produced $= 182 \times (24{\cdot}3 - 16) = 1511$ gram-centigrade-degrees.

$\therefore$ 1 heat unit is produced by $\dfrac{35{\cdot}7 \times 180 \times 10^7}{1511} = 4{\cdot}25 \times 10^7$ ergs.

Exercise XII

1. Two Grove's cells, alike in all respects except that in one the plates are twice as far apart as in the other, are arranged in series, and the poles of the battery so constituted are united by a copper wire. The liquid in both cells becomes heated. In which is the rise in temperature the greater, and why?

2. A thick copper wire and a thin copper wire, of such lengths as to have the same resistances, are joined end to end and used to connect the terminals of a battery, so that the same current flows through them both. Explain why the thin wire becomes hotter than the thick one.

3. The poles of a cell are joined by two wires similar in all respects, except that one is longer than the other. In which is the greatest amount of heat produced, and why?

4. The E.M.F. of a battery is 18 volts, and its internal resistance 3 ohms. The difference of potential between its poles, when they are connected by a wire A, is 15 volts, and falls to 12 volts when A is replaced by another wire B. Compare the amounts of heat developed in A and B in equal times.

5. A current of 1 ampere passes through a coil whose resistance is 2 ohms. What amount of heat is developed in the coil in 5 seconds?

6. A current of 10 amperes passes through a wire whose resistance is ·9 ohm for 5 seconds. What amount of heat is developed?

7. The resistance of two wires made of the same metal, a and b, are as 2 : 3. What are the relative amounts of heat developed in the wires—(1) when they are fastened end to end, and the same current passes through them ; (2) when they are arranged in "multiple arc," that is, when each connects the ends of the same battery, so that the battery current is divided between them?

8. State the laws relating to the production of heat by an electric current. What will be the ratio of the currents which will produce in 1 second the same amount of heat in two wires of the same material and length, if the radius of one wire is twice that of the other? (B. of E., 1902.)

9. A current of 5 amperes flows for 3 minutes through a wire whose resistance is 2 ohms ; given that 1 water-gram-degree = 4·2 joules, find the amount of heat, in water-gram-degrees, generated in the wire.

(Oxford Local, Senior, 1908.)

10. A current of 1 ampere, flowing for 1 second through a resistance of

1 ohm, produces ·239 gram-centigrade units of heat. What current would have to flow for an hour through a resistance of 41·84 ohms in order that the heat produced might suffice to raise a kilogram of water from 0° C. to the boiling-point? (B. of E., 1896.)

11. An electric battery of constant E.M.F., having an internal resistance of 5 ohms, is connected to resistance coils of 10 ohms and 20 ohms respectively, arranged (1) in series, (2) in parallel. Neglecting the resistance of the connecting wires, compare the amounts of heat produced in the two cases (*a*) in the whole circuit, (*b*) in the two coils. (B. of E., 1904.)

12. Two circuits, whose resistances are respectively 1 ohm and 10 ohms, are arranged in parallel. Compare the amount of current passing through each of these circuits with that through the battery. Compare also the amount of heat developed in the same time in the two circuits. (B. of E., 1901.)

13. Explain how the mechanical equivalent of heat may be determined by measuring the electric energy spent in heating a resistance. What instrument would you require, and how would you perform the experiment? (B. of E., 1906.)

14. A current of 10 amperes is sent through a platinum wire, the resistance of which is 2 ohms. Find the mechanical equivalent in ergs of the heat generated per second. (B. of E., 1905.)

15. It is required to generate 10 kilograms of steam per hour with power developed from a 110-volt circuit. What resistance should the heating coil have in order to do this, supposing loss from radiation negligible? (B. of E., 1907.)

16. A current of 5 amperes is passed through a wire, and therein produces 500 calories per second. If the current were increased to 7 amperes, what number of grams of water would it heat in 1 hour to 100° C.? Assume that the resistance of the wire does not change, and that the initial temperature of the water is 15° C. (Lond. Univ. Matric., 1901.)

17. Describe some method you have employed for measuring the strength of a current. A lamp, the voltage between the terminals of which is 100, is placed in a calorimeter, which is immersed in 400 grams of water, the water-equivalent of the calorimeter, &c., being 40; the temperature is found to rise 2·5° C. per minute. Find the current in the lamp. (Camb. Local, Senior, 1901.)

18. The reactions within a cell generate electrical energy at the rate of 1 watt per ampere; a current of 10 amperes is being generated, with the result that energy is dissipated within the cell in the form of heat at the rate of 1 watt. What is the difference of potential between the terminals of the cell; also what is the internal resistance of the cell? (Lond. Univ., B.Sc. Internal, 1909.)

19. What would it cost to raise 10 kilograms of water from 20° to 100° C., with an electric supply at 5d. per unit? The mechanical equivalent of heat may be taken as 427 kilogram-metres per kilogram-calorie.

20. A coil of platinum wire of 9·45 ohms resistance is immersed in paraffin-oil in a vessel surrounded by non-conductors of heat; paraffin-oil is kept flowing through the vessel at the rate of 8·1 grams per minute; and a current is kept flowing in the wire of such strength that the paraffin-oil leaving the vessel is 5° C. warmer than it is on entering. Calculate the *strength of current* in the wire. Specific heat of paraffin-oil = ·474.

The above current is to be maintained by Grove's cells, each of electromotive force 2 volts and resistance ·12 ohm: show how many cells will be required, and what must be the resistance of the connecting wires.

CHAPTER XVII

MAGNETIC PROPERTIES OF A CURRENT

Magnetic Field due to a Current.—**Exp. 144.** Send a strong current—say from a 6-cell battery—through a copper wire. Immerse the wire in iron filings, and on withdrawal, notice that they cling to it. On breaking contact, the filings fall off.

This experiment suggests that the wire has acquired magnetic properties, but as we know that copper is not capable of being magnetised, we are led to explore the region surrounding the wire.

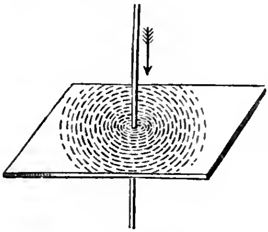


FIG. 172.

Exp. 145. Pass a copper wire vertically through a hole in a sheet of cardboard (Fig. 172). Connect the ends with a 6-cell chromic-acid battery. While the current is flowing (*a*) shake iron filings from a muslin bag upon the cardboard, and at the same time tap it gently. Observe that the filings tend to arrange themselves in concentric circles round the wire. (*b*) Place a few small toy compasses on the cardboard, and notice that they tend to set themselves crosswise to the conductor, thus making a similar circular arrangement (except in so far as they are disturbed by the magnetic field of the earth).

We learn from this experiment that lines of magnetic force, identical to those of a steel magnet, surround a conductor whilst it is carrying a current, and that, when the conductor is straight and at a distance from other conductors, these lines of forces are practically circles. This constitutes a profound difference between the flow of a current and the flow of a liquid like water; and although we can draw many useful analogies between them, the flow of water produces no effect whatever outside the pipe containing it, whereas the flow of a current disturbs the whole of the surrounding space. This disturbance is due to the fact that, when a current is started, it creates a magnetic field, which dies away when the current is stopped. The field thus formed extends without definite limit throughout space, although it gradually becomes weaker and weaker as the distance from the conductor increases.

We may also mention that these lines of force differ from those of a steel magnet, inasmuch as they exist entirely in air, or in whatever medium may be outside the conductor,[1] *i.e.* there are no poles

[1] We need not consider here what occurs *inside* the conductor.

in the ordinary sense of the term, for a magnetic pole is the region where lines of force emerge from a magnet into air. If, for example, we thread a ring of soft iron on the wire, some of the lines would pass through the ring, and it would be magnetised, although we should find no trace of that fact, because in no part of it are lines of force emerging. The condition would, however, become apparent if a piece were cut out of the ring, for we should find at the surfaces of the gap well-defined poles like those of a horse-shoe magnet.

We are now in a position to understand why the conductor attracts iron filings. It is a special case of property 5, p. 131—the filings tending to place themselves tangential to the lines of force in the strongest part of the field, *i.e.* at right angles to the conductor and as near to it as possible.

If we regard a compass-needle as a large pivoted filing, we can partly anticipate the results of the following experiments:—

Oersted's Experiment.—**Exp. 146.** (*a*) Hold one of the wires from a voltaic cell immediately above a compass-needle, in the direction of its length, and complete the circuit by touching the end of this wire with the wire attached to the other terminal of the cell. Observe that the needle is deflected, tending to set itself at right angles to the current.

(*b*) Repeat this with the wire below the needle. Observe that the deflection is in the opposite direction.

(*c*) Repeat (*a*) and (*b*) with the direction of the current reversed, and notice that the direction of deflection is reversed.

This action of a current on a magnetic needle was noticed by Oersted in 1820, and is historically interesting as being the first discovered relation between electricity and magnetism.

The positions taken up by a small magnet placed near a vertical wire carrying a current are shown in Fig. 173. In these diagrams, we adopt the useful convention, introduced by Professor Silvanus Thompson, that a current flowing outwards is signified by a dot, and a current flowing inwards by a cross. These may be remembered by thinking of them as arrows used to denote current direction, the dot being the point coming outwards and the cross representing the tail-feathers going inwards. In accordance with p. 131, we shall also denote the direction of the magnetic field by arrows, but it must be very carefully borne in mind that such marks on a line of force are not intended to suggest motion of any kind; they simply indicate the direction in which a free north pole would tend to move.

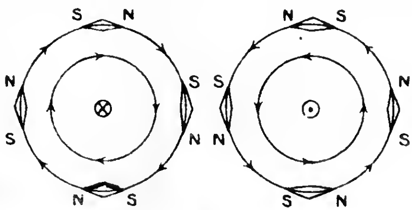


FIG. 173.

In the diagrams (Fig. 173) only two lines of force are shown, and the action of the earth's magnetic field is neglected.

Ampère's Rule.—These experimental results are conveniently expressed by Ampère's rule. *Let the observer imagine that he is swimming in the wire in the direction of the current with his face turned towards the particular magnet under consideration and with arms outstretched, then the N pole will be on his left hand.*

These diagrams are very important, and will be frequently used. Evidently, if we know the direction of the current, we can mark the direction of the lines of force, and *vice versâ*.

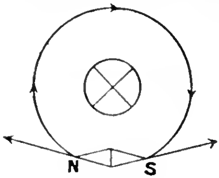


FIG. 174.

Again, these figures suggest that there is a force acting on the N pole, which tends to make it rotate continuously around the wire in a certain direction, and an equal force tending to make the S pole rotate in an opposite direction. It is evident that, although the magnet can place itself only tangentially to the line of force, these two forces (Fig. 174) must have a resultant directed *towards* the conductor, and it is this resultant force which causes the iron filings to cling to the wire in Experiment 144.

It is not difficult to show by experiment that the suggestion mentioned above is a real and undoubted fact. It is, of course, impossible to make a magnet with only *one* pole, but it is possible to arrange that one of its poles becomes inoperative. A simple form of apparatus for this purpose is shown in Fig. 175.

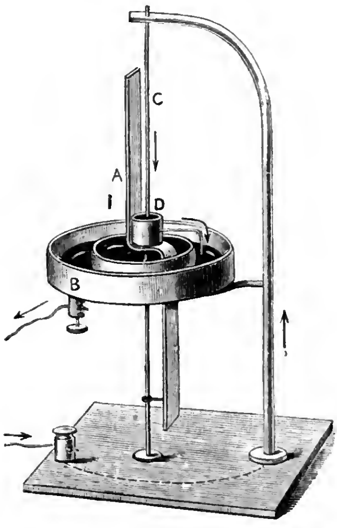


FIG. 175.

Exp. 147. A magnet, A, is bent, and then suspended on a finely pointed wire, the current being brought by a wire, C, to a mercury cup, D, supported on the magnet, and carried away by a bent wire which dips into an annular cup of mercury, B. The other end of the battery

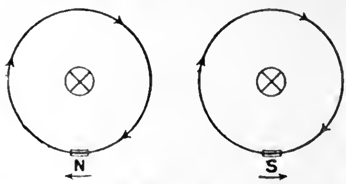


FIG. 176.

wire is attached to the binding-screw in B, which is in metallic connection with the mercury. The current from six chromic acid cells, arranged in series, will cause the magnet pole to rotate. If the current passes *down* the wire, as shown in the diagram, and the uppermost pole be N, it moves in a clockwise direction. If the S pole is uppermost, it rotates in an anti-clockwise direction. If the current passes *up* the wire, the direction is reversed.

The movements will be easily understood by reference to Fig. 176, which represents the state of affairs when the apparatus is looked at from above.

Again, from Fig. 175, we infer that if the magnet *were flexible*, it would tend to wrap itself round the conductor. A sufficiently flexible magnet is unattainable, but the converse experiment can easily be performed.

Exp. 148. Select a rather long steel bar magnet and clamp it in a vertical position (Fig. 177). Suspend loosely by the side of the magnet, with a fair amount of slack, a length of tinsel strip (such as is obtained from toyshops) and connect this to a current reverser (see p. 253) and a battery. Observe that, when a current is sent through the strip, it wraps itself round the magnet, and when the current is reversed, it unwinds itself and wraps around again in the opposite direction. It will be found that the current always arranges itself so that its influence strengthens the existing magnetism of the bar. An explanation of this action will be found on p. 486.

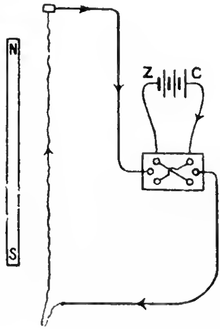


FIG. 177.

Attractions and Repulsions between Conductors carrying Currents.—As these forces are somewhat feeble with currents of ordinary strength, it is necessary, in order to demonstrate their existence, to devise some simple form of apparatus, in which a portion of the circuit may be freely movable throughout a small range. Such an arrangement is shown in Fig. 178.

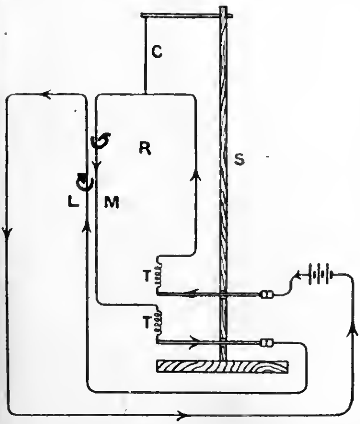


FIG. 178.

Exp. 149. Bend a piece of copper or brass wire into a rectangle, R, about 5 inches wide and 9 inches long. Shape the ends as shown in the figure, so that one is vertically above the other. Suspend it by a cotton thread, C, and connect it by two tinsel strips, TT, to fixed conductors, which may conveniently be fastened to a wooden stand, S. (These strips are shown as spirals for clearness, but straight pieces are really used. It is desirable to place three or four strips in parallel and twist the ends together, as a single connection may become too hot and break.) Send a current through the rectangle by means of a battery, and intercalcate in the circuit a convenient length of wire, L, which can be held close to the side, M, of the rectangle. It is really to be held in front or behind the side, M, so as to produce a rotation, and not as shown in the figure. It will be found that when the currents in L and M are in *opposite* directions, there is a distinct though feeble *repulsion* between them, which becomes an *attraction* when L is turned round so that the currents are in the *same* direction.[1]

The actions will be understood by drawing figures showing the magnetic fields around the wires. When the currents are in the *same* direction (Fig. 179) the two sets of lines are either both clock-

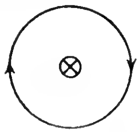


FIG. 179.

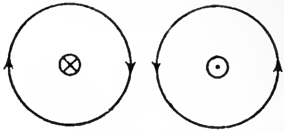


FIG. 180.

wise or both anti-clockwise, and there is a tendency for them to merge into one resultant field surrounding *both* conductors.

When the currents are in *opposite* directions (Fig. 180) we see that the lines of force between them are in the same direction, and, as we already know, repulsion will occur.

Adjacent conductors, which carry currents and which are free to move, will always tend to set themselves so that the currents are in the same direction and as close together as possible. It is, however, important to bear in mind that the forces acting are between two sets of lines of force and not between the conductors directly. Such forces are usefully applied to purposes of measurement in the various forms of electro-dynamometers, described in Chapter XXXIII.

Magnetic Field due to a Current in a Circular Wire.— **Exp. 150.** Pass the two ends of a piece of stout copper wire through two holes (about ten inches apart) in a piece of cardboard. Make the wire into circular form (Fig. 181), one half being above, and the other half below the cardboard. Bend the free ends, as shown in the diagram, and connect them with the terminals of a 6-cell chromic acid battery. While the current is flowing through the wire, scatter iron filings over the paper. From this graphic representation, observe (1) that the lines of force are circular near the wires, (2) that at the centre, they are normal to the plane of the circle.

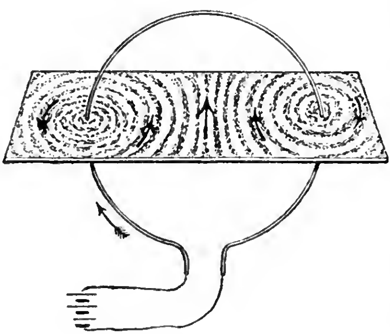


FIG. 181.

[1] It is quite easy to get greater sensitiveness by making a rectangle of, say, four to twelve turns of insulated copper wire bound together here and there with cotton thread.

This is an important result in studying the tangent galvanometer (see p. 284).

Magnetic Field due to a Solenoid.—The word solenoid means "tube-like," and is conveniently used to denote a hollow helix of wire, when its length is considerably greater than its diameter.

Exp. 151. Place a solenoid in a piece of cardboard, so that its axis is in the plane of the board (Fig. 182). This is best done by cutting three sides of a rectangle in the middle of the board, and then passing the free end of the strip through the solenoid. Attach the free ends of the solenoid to a battery, and then sprinkle iron filings over the cardboard, gently tapping it as they fall. Observe that (1) the lines of force *outside* are similar to those of a bar magnet, and (2) *inside* they lie crowded together in a direction parallel to the length of the solenoid.

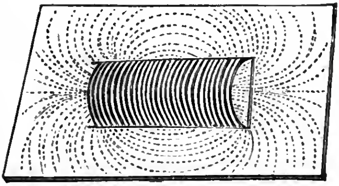


FIG. 182.

It follows from this experiment that a solenoid carrying a current behaves in many ways exactly like a weak bar magnet.

Exp. 152. Hold a solenoid in the hand, and whilst a current is passing, bring each end in turn near an ordinary compass-needle. Notice that it possesses polarity. Determine the N and S poles, and observe that they are reversed by reversing the direction of the current. Ascertain the relation between polarity and current direction, and verify the fact that it is in accordance with Fig. 183.

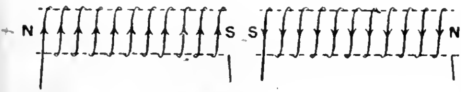


FIG. 183.

It will be noticed that the polarity can be ascertained by using Ampère's rule, if we modify it by saying that when it is applied to a coil of any kind, the observer must face towards the *inside* of the coil.

These conclusions are not limited to a solenoid. They apply to a coil of any shape or size, *e.g.* a short, ring-shaped coil, or to a single turn of wire, which behaves as if the whole of one face were a N pole, and the whole of the other, a S pole. A very old method of illustrating these facts is given in Experiment 153, by means of a "floating battery," as the apparatus is commonly called.

Exp. 153. Fit a beaker in a cork or in a wooden tray (Fig. 184). Fasten strips of copper and zinc (C and Z) side by side to a cross piece of wood, A. Bend silk-covered copper wire into a coil (say, about 20 turns) and solder the ends to the strips. On filling the beaker with dilute sulphuric acid, a current will pass round the coil. Bring the poles of a bar magnet up to each face of the coil in turn. It will be found that the coil acts like a very short magnet, the whole of one side having N polarity and the other side, S polarity. Notice the direction of the current in the coil, and apply the swimming rule to determine its polarity. The results will agree with those found in the experiment.

It will be noticed that the coil tends to thread itself on the bar magnet, and to move up to the middle of the latter. If the magnet be put through the coil in the wrong direction, the coil will

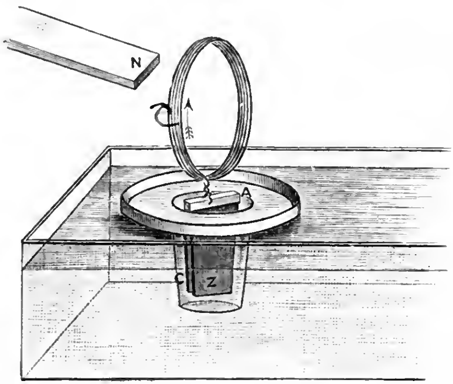


Fig. 184.

move away, turn round, and then return to thread itself on the magnet in the right direction.

Determine the relation between the polarity of the magnet and the polarity of the coil in the latter position.

A solenoid differs from a bar magnet in having a penetrable interior, and thus it can produce effects which cannot be obtained from a bar magnet.

Exp. 154. While a current is flowing, hold two or three knitting-needles, or straight pieces of iron wire, with their ends just inside the solenoid. Notice that they will be sucked into the coil, owing to the tendency to move into the strongest part of the field (see p. 131). This effect has numerous practical applications in arc lamps and in other mechanisms.

At first sight there appears to be no relation between the distribution of the field around a solenoid and the circular distribution around a straight conductor. To show that such a relation really does exist, let us consider Fig. 185, which shows a section of a solenoid of one layer deep. If we think of the lines originating in circles as shown, and then consider their mutual influence, it will be evident that between any two adjacent conductors, they will

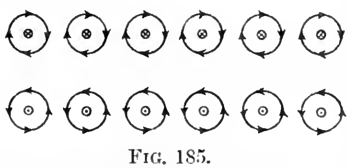


Fig. 185.

neutralise each other, for they are in opposite directions, whereas above and below they will merge into continuous lines, as shown in Fig. 186. If we also remember that the lines are very numerous, and that they *repel each other laterally*, we shall see that, outside the solenoid, this repulsion must make them bulge out into the shapes found experimentally. The student should also notice that, as the lines tend to contract in length, they must tend to squeeze the turns together like a compressed spring.

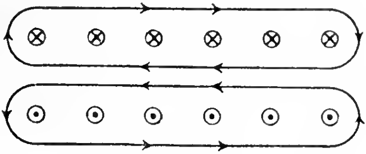


FIG. 186.

As the current in any one turn is in the *same* direction as that in the adjacent turn, we may express the same action by saying that the attraction is due to the currents flowing in the same direction. This action is very easily verified by means of Roget's vibrating spiral, which may be made of twenty or thirty turns of moderately thin copper wire, suspended from a suitable support. The lower end just dips into a mercury cup cut in the base of the support (Fig. 187).

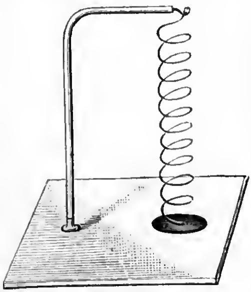


FIG. 187.

Exp. 155. Connect one wire from a battery of six cells to the top of the coil, and place the other wire in the mercury. A current, therefore, flows through the coil. Observe that attraction takes place between the wires, so that the lower end of the coil is lifted out of the mercury. By this action the circuit is broken, and the spiral drops back to its first position. Attraction again takes place, and so on, thus giving an up-and-down motion to the coil. Fix a soft iron rod inside the spiral, and observe that the effect is intensified.

Effect of an Iron Core.—**Exp. 156.** Place a knitting-needle inside the solenoid used in Experiments 151, 152, and after passing a current, show that the needle is magnetised. Verify the fact that its polarity is the same as that of the solenoid itself.

Exp. 157. Wind 20 or 22 gauge cotton-covered copper wire on a glass tube about a quarter-inch diameter, to form a solenoid. A single layer of wire will be sufficient, but instead of winding it uniformly in the ordinary way, reverse the direction of winding in two or more places, as shown in Fig. 188. Place within it an unmagnetised knitting-needle, and after passing a current, remove the needle and roll it in iron filings. Also test it by means of a compass-needle. It will be found to possess *consecutive poles*, which were produced where the direction of the winding was changed.

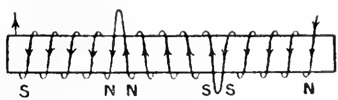


FIG. 188.

Exp. 158. Repeat Experiments 156, 157, using a rod or wire of soft iron. Show that it behaves like the knitting-needle, except that the effect is temporary —only a slight residual magnetism being left when the current is stopped.

These experiments show that the presence of an iron core intensifies enormously the magnetic effect of the coil, without altering its polarity, the difference in the behaviour of soft iron and steel being exactly what would be expected from experiments in Chapter X.

It is evident that each particle of iron in the core tends to behave like an iron filing placed in the solenoid as in Experiment 151.

Electromagnets.—A coil of wire on a soft iron core constitutes an electromagnet. Their shapes and applications are innumerable. Their enormous practical importance depends on the fact that a mechanical force, completely under control, can be produced at any distance from the operator.

The general theory of the magnetisation of iron is discussed in Chapter XXV.; here, it is only necessary to remark that the magnetising power of a coil of given dimensions is proportional to the number of *ampere-turns*, i.e. to the product of the current flowing and the number of turns. Hence, exactly the same effect may be produced by a small current through many turns of fine wire as by a large current flowing through fewer turns of thick wire. On the other hand, the magnetism produced in a soft iron core by a *given number of ampere-turns* depends on the shape and quality of the iron, the more nearly the former approaches a closed ring the better. Hence, a U-shaped core is more strongly magnetised than a straight one, and the effect is again greatly increased by putting a soft iron keeper across the poles (see Experiment 238). It must also be pointed out that the magnetisation of the iron is *not* proportional to the ampere-turns (except, roughly, when weakly magnetised), for the iron eventually reaches a condition when any further increase in the ampere-turns has no practical effect. The iron is then said to be *saturated*.

Ampère's Theory of Magnetism.—From the fact that a solenoid acts in every respect like a magnet, Ampère propounded a theory that magnetism is due to current circulation. He considered that every molecule of a magnet has closed currents circulating round it. Before magnetisation the molecules, and hence the currents, move irregularly; during magnetisation they assume parallel directions, and the more perfect the magnetisation, the more parallel they become. The separate currents, moving round the various molecules, may be considered as equivalent to one resultant current flowing round the whole magnet (Fig. 189). It will be seen that the direction of the current therein agrees with the results found experimentally and also with the

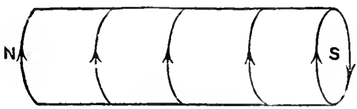


Fig. 189.

swimming rule. Looking at a S pole, it is in a clock-wise direction, and *vice versâ*.

Recent discoveries have thrown considerable light upon the true nature of these molecular currents, and at the same time strongly support the essential correctness of Ampère's hypothesis (see p. 422).

A very useful form of electromagnet, which will be frequently referred to in connection with subsequent experiments, is shown in Fig. 190 (for which we are indebted to Messrs. Pye & Co., of Cambridge). The soft iron cores should be about $1\frac{1}{2}$ inches in

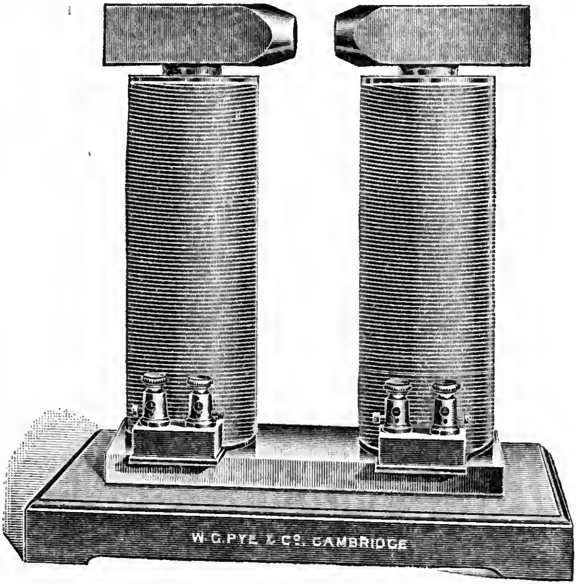


FIG. 190.

diameter and 8 inches long, screwing into the massive iron base and readily removable. They are provided with soft iron pole-pieces, which may be placed in any position, and which are useful in certain experiments for concentrating the field. The coils are wound on wooden bobbins, and are also removable.

Exp. 159. Connect the electromagnet to a battery of a few cells in series. Place a piece of cardboard on the poles and pour iron filings upon it. Notice how their arrangement indicates the distribution of the lines of force between the poles.

Exp. 160. Place a piece of iron on either pole, switch on a current, and test the pull. Then break the circuit and test the pull again. Notice that the magnetism has almost completely and instantly disappeared, leaving merely a feeble "residual" magnetism.

Place a flat iron bar, with an ordinary rough surface, across the poles like a keeper. Switch the current on and off. Whilst the current is flowing the bar is, of course, held on firmly. When it is interrupted, measure, by means of a spring balance, the pull required to detach the bar. It will be found that the magnetism has not completely disappeared, a fairly strong pull being required to detach the bar, but when *once detached*, it is not held on again when replaced.

Repeat the experiment, using a bar which has a very smooth surface. It will be found to adhere very strongly *after* the circuit is broken, most probably too firmly to be pulled off with the balance. But when it is pulled off, this residual magnetism instantly disappears.

Repeat the experiment with the same bar, placing between it and the poles sheets of paper or cards of varying thickness. It will be found that even a very thin sheet of non-magnetic material greatly reduces the amount of residual magnetism when the current is switched off.

These experiments illustrate what has been said on p. 117, with reference to the relative power of open and closed chains of particles to maintain their orderly arrangement. They, also, show that when, in any mechanism (*e.g.* an electric bell), an electromagnet is required to gain or lose its power rapidly, the armature it may have to attract should never actually touch the poles, or there will be a strong tendency for it to adhere after the circuit is broken.

Winding Electromagnets.—Any given electromagnet may be regarded as requiring a certain number of ampere-turns to develop its desired strength. The gauge of wire to be selected for the coils then depends upon the working conditions. If it is known that a current of fair strength is available, then evidently comparatively few turns of thick wire may be used, but if it is to be placed in a circuit of high resistance in which the current will necessarily be small, it must be wound with many turns of fine wire.

It often occurs in practical applications that an electromagnet is to be excited by a constant voltage maintained between the terminals of the winding. In this case, it is interesting to notice that the number of ampere-turns produced depends only upon the *diameter* of the wire used, and (neglecting the effect of the thickness of insulation) is independent of the actual number of turns.

Let E be the constant voltage, r the resistance of *one* turn of mean length, and N the number of turns. Then the resistance of the winding is Nr.

$$\text{Hence, } C = \frac{E}{Nr} \text{ or } CN = \text{ampere-turns} = \frac{E}{r} = \text{a constant.}$$

For instance, if there is one turn, $C = \frac{E}{r}$

$$\text{and ampere-turn} = \frac{E}{r} \times 1 = \frac{E}{r}$$

If there are 1000 turns, $C_1 = \frac{E}{1000r}$

$$\text{and ampere-turns} = \frac{E}{1000r} \times 1000 = \frac{E}{r} \text{ as before.}$$

The question therefore arises: How many turns shall be used? In answering it, there are two points to be kept in mind—

(1) The particular gauge of wire in question can carry only a certain current without getting hot enough to damage the insulation, and therefore sufficient wire must be used to bring the current down to a safe value.

(2) The more wire put on, the *less* becomes the expenditure of energy in the coil when in use (although the greater becomes the first cost of winding), and in practice, therefore, sufficient wire is used to effect a fair compromise between these two conditions.

These matters will be understood better, if we work out the values in a particular case.

Example.—An electromagnet requires 4300 ampere-turns, and is to be excited at a constant pressure of 50 volts. What gauge of wire must be used, if the average length of one turn is 18 inches?

$$\text{We have from above} \qquad 4300 = \frac{E}{r} = \frac{50}{r}$$

$$\therefore\ r = \frac{50}{4300} = \cdot 0116 \text{ ohm.}$$

We have, therefore, to find a size of wire such that a length of 18 inches has a resistance of ·0116 ohm. For this purpose we must anticipate a result arrived at in Chapter XVIII., where it is shown that

$$r = \frac{\text{Length in inches}}{\text{Area in square inches}} \times \frac{\cdot 65}{10^6}$$

$$\therefore\ \cdot 0116 = \frac{18 \times \cdot 65}{\text{Area} \times 10^6}$$

$$\textit{i.e.}\ \text{Area} = \frac{18 \times \cdot 65}{\cdot 0116 \times 10^6} = \cdot 001 \text{ sq. in.}$$

On referring to wire tables, this is found to correspond to 20 gauge, and it is also stated that this particular size of wire will carry a maximum current of 1·5 amperes without undue heating. (It would carry a larger current if freely exposed to the air, but we have to remember that the heat cannot readily escape from the inner turns of the winding.) Hence, the total resistance of the winding must be such that 50 volts send 1·5 amperes through it, or $R = \frac{50}{1 \cdot 5} = 33 \cdot 3$ ohms.

As each turn has an average resistance of ·0116 ohm, we shall need at least $\frac{33\cdot3}{\cdot0116}$, or 2870 turns.

The only loss of energy in the coil will be that due to heat production, and this will be at the rate of C^2R watts. In this case, it will be more convenient to write it in the form $\frac{e^2}{R}$, which gives $\frac{50^2}{33\cdot3} = 75$ watts (or about $\frac{1}{10}$ H.P.).

From this expression, we see that, as e is constant, the greater we make R the smaller will become the waste of energy, as previously stated. Of course, if we push the argument to its logical conclusion, it follows that it is theoretically possible to obtain an electromagnet of any strength whatever, without appreciable expenditure of energy, and it will be seen later that this result embodies a very important truth.

Electric Bell.—The simplest and most familiar electro-magnetic device is found in the ordinary electric bell. It consists of an electromagnet, whose base is prolonged to carry a steel spring, S (Fig. 191), to which is screwed a soft iron armature, A, and which is finally bent outwards, as shown, to make contact with an adjustable screw carried by an insulated brass pillar, P. At the point of contact, a small disc of platinum is attached to the spring, and a short piece of platinum wire to the screw, in order to prevent oxidation due to sparking, which would soon cause bad contact and stop the action. One of the ends of the electromagnet-winding is connected to one of the terminals, T, and the other end is connected to the iron frame by means of a screw, F; the second terminal of the bell being joined up to the insulated pillar, P. The two wires from the battery (which usually consists of one or two Leclanché cells) are connected to the terminals TT. In one of these wires is placed a key—commonly called the "push"—by means of which the circuit is made or broken.

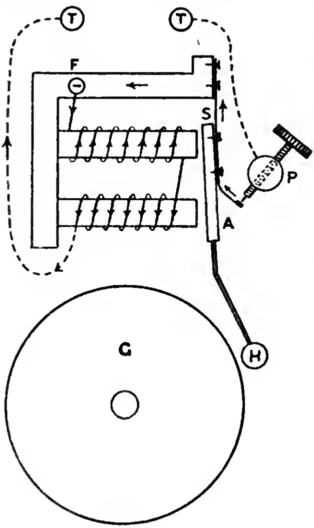


Fig. 191.

When the push is pressed, the circuit is completed, and a current flows round the electromagnet, causing the armature to be attracted. When this occurs, the hammer, H, being carried by the armature, strikes a gong, G; and at the same time, owing to contact being broken between A and the screw in P, the current ceases, and the iron core loses its magnetism. The spring, S, now comes into play, bringing the armature back to the screw in P, thus completing the circuit again. This cycle of operations is repeated as long as the push is held down.

Current Reverser.—A very convenient form of current reverser, suitable for use in Experiment 148, and in many later experiments, is shown in Figs. 192, 193, for the former of which we are indebted to Messrs. J. J. Griffin & Co. A wooden or ebonite base contains six mercury cups, arranged as shown, each of which is provided with a binding-screw, making contact with the mercury. C is joined to F,

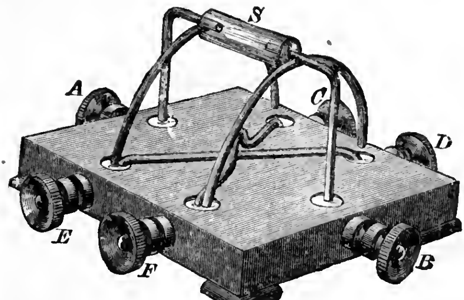


FIG. 192.

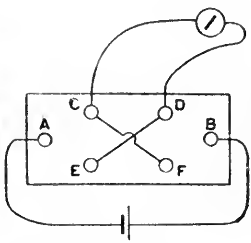


FIG. 193.

and D to E, by means of copper strips (which should be readily removable).

The "rocker" consists of an ebonite handle carrying at *each* end three bent copper rods in metallic connection, but insulated by the handle from the similar group at the other end. This enables A and B to be connected with either CD or EF as required.

For use as a current reverser, only four terminals are required, which must be AB and *either* CD *or* EF. If the galvanometer (or portion of the circuit in which the current is to be reversed) is joined up to, say, CD, and the battery to AB, it will be seen from the figure that the current is reversed when the rocker is moved over, and evidently the galvanometer and battery connections may be interchanged without affecting the result.

It is, however, usual to provide six terminals, because then by removing the diagonal connections CF and DE, the arrangement is converted into a useful "change-over" switch, referred to in various subsequent experiments, *e.g.* see Experiments 178, 198.

Exercise XIII

1. What would be the magnetic effect produced on a straight steel tube by the passage of a strong current through a straight wire placed along the axis of the tube? And how would you prove your statement? (B. of E., 1898.)

2. A wire is stretched from east to west (magnetic). How, without breaking it, can you test whether, and in what direction, an electric current is passing through it? (B. of E., 1896.)

3. A road in the northern hemisphere runs magnetic north and south. At one point an insulated conductor passes beneath it in which an electric current flows from east to west. How will the indications of a dip circle be affected at points near to the conductor? (B. of E., 1894.)

4. A metal ring, through which a current circulates, can move horizontally, its plane remaining always vertical. Describe and explain what happens when one pole or the other of a bar magnet is presented to the ring.

5. Describe and explain any arrangement for causing a magnet to rotate continuously about a wire through which a current is passing, and show the relation between the direction of the current and the direction of rotation.

6. Two parallel covered wires are traversed by equal currents in the same direction: what is the joint effect of the currents upon a bar of soft iron (*a*) laid across the two wires, on the same side of both; (*b*) held between the wires at the same distance from each.

7. A square of guttapercha-covered copper wire is suspended from one arm of a balance, so that an electric current can be passed through it, and it is counterpoised when the lower side of the square is immersed in a trough containing copper sulphate. If an independent current is passed through the liquid, what effect will be produced on the equilibrium of the balance?

8. A wire, through which a current is passing, is stretched vertically from floor to ceiling of a room. A small magnet, suspended by a silk fibre so that it can turn freely in a horizontal plane, but cannot turn vertically, is brought near the wire on the east (magnetic) side, and the time is measured in which the magnet makes, say, 20 oscillations. The magnet is then placed at the same distance west (magnetic) of the wire, and the time in which it makes the same number of oscillations is observed. How could you tell from these experiments whether the current in the wire is flowing upwards or downwards?

CHAPTER XVIII

RESISTANCE AND ITS MEASUREMENT

Exp. 161. If electric lighting mains are available, connect them to an ordinary incandescent lamp, putting in circuit with it a suitable ammeter to measure the current flowing. Observe the value of the current. Add another similar lamp *in series* with the first, and observe that the current is *half* its previous value. If a third lamp be added, the current is one-third of its original value, and so on.

As the voltage across the mains is constant, this experiment shows that the resistance of a conductor is directly proportional to its length.

Exp. 162. Join up two lamps *in parallel*, and notice that the total current is twice as great as that with one lamp. With three lamps in parallel, it will be three times as great, and so on.

Hence, the resistance of a conductor varies inversely as its area of cross section.

We may, therefore, write, for any conductor—

$$\text{Resistance} \propto \frac{\text{length}}{\text{area}} \text{ or } R \propto \frac{l}{A}$$

Specific Resistance.—We may express the above relation as an equality by multiplying by a constant, thus—

$R = \frac{l}{A} \times s$, where s is a number which is constant for any given material, but which varies for different materials.

If $l = 1$ and $A = 1$, then $R = s$, i.e. s is the resistance of a portion of the material of unit length and unit sectional area. It is called the **specific resistance** of that material, and its numerical value will depend upon the units of length and of resistance chosen. If, for example, we take the centimetre as the unit of length, and consequently the square centimetre as the unit of area, we might state the value of s for any substance in *ohms per centimetre-cube*, i.e. as the resistance in ohms of a portion 1 centimetre in length and 1 square centimetre in area. For the majority of metals, this would be a very small fraction, and it is more conveniently expressed in *absolute units of resistance per centimetre-cube* by multiplying by 10^9.

For many practical purposes, the value of s is expressed in *microhms per inch-cube*, a microhm being the one-millionth of an ohm.

The specific resistance of a very poor conductor may be so enormous that its value is better expressed in *megohms*, a megohm being one million ohms.

The following table shows the specific resistance of a few materials :—

	Absolute Units per Centimetre-Cube.	Microhms per Centimetre-Cube.	Microhms per Inch-Cube.
Silver annealed	1,468	1·468	0·578
Copper annealed	1,562	1·562	0·614
Aluminium	2,665	2·665	1·049
Zinc	5,751	5·751	2·225
Iron	9,065	9·065	3·569
Platinum	10,917	10·917	4·298
Platinoid (alloy)	41,000	41·0	16·14
German silver (alloy)[1]	20,243	20·243	7·97
Mercury	94,070	94·07	37·03
Manganin (alloy)	46,000	46·0	18·54

An examination of the above table shows that the specific resistance of alloys is very much greater than that of the pure metals (excepting mercury). This is a characteristic property of alloys, which is taken advantage of in the preparation of wires of high specific resistance. Even a slight trace of another metal, which by itself may be a good conductor, has an enormous effect on the resistance, and hence copper used for electrical purposes has to be exceptionally pure.

Example.—A copper wire 150 centimetres long and ·04 centimetre in diameter is found to have a resistance of ·196 ohm. Find the specific resistance of copper, (*a*) in ohms per centimetre-cube, (*b*) in absolute units of resistance per centimetre-cube, (*c*) in microhms per inch-cube.

$$\text{As} \quad R = \frac{l}{A} \times s$$

$$\text{we have} \quad s = \frac{R \times A}{l}$$

$$\therefore \quad \text{in case } (a) \qquad s = \frac{\cdot 196 \times \pi \times (\cdot 02)^2}{150}$$

$$\text{or} \quad s = \cdot 000001643 \text{ ohms per centimetre-cube.}$$

For (*b*), put R in absolute units, *i.e.* multiply by 10^9; then $s = 1643$ absolute units per centimetre-cube.

[1] German silver contains copper, nickel, and zinc. Platinoid is German silver with the addition of tungsten. Manganin contains copper, manganese, and nickel.

For (*c*), we must first express the dimensions in inches, knowing that 2·54 centimetres = 1 inch.

$$\therefore \text{ length of wire} = \frac{150}{2{\cdot}54} \text{ inches}$$

$$\text{and radius of wire} = \frac{{\cdot}02}{2{\cdot}54} \text{ inch}$$

$$\therefore s = \frac{{\cdot}196 \times \pi \times \left(\frac{{\cdot}02}{2{\cdot}54}\right)^2}{\frac{150}{2{\cdot}54}}$$

$$i.e.\ s = \frac{{\cdot}196 \times \pi \times ({\cdot}02)^2}{150 \times 2{\cdot}54} = {\cdot}00000065 \text{ ohms per inch-cube}$$

$$\text{or } s = {\cdot}65 \text{ microhms per inch-cube}$$ [1]

Effect of Change of Temperature on Resistance.— **Exp. 163.** Wind a yard of iron wire (about 20 gauge) into a rough open spiral. Connect it in series with an ammeter and a single cell of fairly low internal resistance, *e.g.* a double-fluid chromic acid cell. Read the current. Now heat the spiral in a Bunsen flame, and notice that the current falls in strength, but that it returns practically to its old value when the flame is removed.

This experiment shows that the resistance of iron increases with the temperature.

Exp. 164. Connect two small spirals of thin platinum wire *in series* with a battery of a few cells, so that both spirals are heated to dull redness. Roll up a paper tube, and blow through it on one spiral, and notice that, whilst this spiral becomes cooled, the other glows brighter. Again, heat one spiral in a Bunsen flame, and notice that the other becomes cooler.

This is a very old experiment, but it illustrates in a striking manner that the resistance of platinum increases as the temperature rises, and *vice versâ;* for as one of the spirals has increased in temperature, this can be due only to the fact that the current through it (and therefore through the *whole* circuit) has increased. Such an increase in current means that the resistance of the circuit has decreased, which must have been caused by the cooling of the other spiral.

It must be pointed out that in order to perform Experiments 163 and 164 successfully, it is necessary for the iron wire in the former and for the platinum in the latter to be practically the only resistance in the circuit, so that slight variations in their resistances may alter the currents by a perceptible amount. Hence, regulating resistances should, if possible, be avoided, and the cell or battery should have a fairly low internal resistance.

[1] Notice that the expression "cubic centimetre," or cubic inch," though occasionally used, would be absurd, for the resistance of a cubic centimetre of any material may have any value between 0 and ∞.

The resistance of all bodies varies with temperature. If they vary directly, *i.e.* if the resistance increases as the temperature increases, the bodies are said to have a *positive* temperature coefficient. This is characteristic of all pure metals and of most alloys.

If they vary inversely, *i.e.* if the resistance decreases as the temperature increases, the bodies are said to have a *negative* temperature coefficient. In the latter class, we have the non-metallic conductors in general—carbon, conducting solutions and liquids; and insulators like glass, marble, slate, &c., which become partial conductors at high temperatures.

It has just been discovered that the non-metallic element boron possesses an abnormally high negative temperature coefficient.

Until recently, it was known only in the form of loose powder, but Dr. Weintraub has now succeeded in preparing it in the fused state. In this condition, a certain portion had a resistance of 5,620,000 ohms at 27° C., which fell to 5 ohms at a dull red heat.

We have said that pure metals and most alloys are characterised by having a positive temperature coefficient, but it is important to remember that this coefficient is much smaller in alloys than in pure metals, a property which gives alloys their great value in the preparation of standard resistances. The majority of *pure* metals behave with curious uniformity with regard to their changes in resistance during variations in temperature, *e.g.* the resistance of wires made from them increases by nearly 40 per cent. between 0° C. and 100° C., and, moreover, this increase is nearly uniform through quite a wide range of temperature. To this general rule, iron is a marked exception: not only is the increase about 60 per cent. between these temperatures, but there is a remarkable and unusual increase near a red heat—a property of iron advantageously utilised in the steadying resistance of a Nernst lamp (see Chapter XXXII.).

The effect of a change of temperature on the resistance of carbon and metals may be illustrated as follows:—

Exp. 165. (*a*) Connect an ordinary incandescent lamp *with carbon filament* to supply mains, and put an ammeter in series with it and a voltmeter across its terminals. Also arrange in the circuit an adjustable resistance, or other convenient method of varying the voltage on the lamp, and take a series of readings spread over as wide a range as possible. From these readings, calculate the resistance of the lamp filament in each case. It will be found that the resistance decreases as the current increases, *i.e.* as the temperature of the filament rises.

(*b*) Repeat the observations, using a metallic filament lamp, and show that the converse is the case.

The following numbers are given as an illustration. They were obtained in an ordinary class experiment, and have no special value in themselves. Each lamp was made for working at 100 volts; the filaments were, therefore, non-luminous at the lowest voltage, and abnormally bright at the highest:—

Volts.	Carbon Lamp.		Tantalum Lamp.	
	Current in Amperes.	Resistance in Ohms (Calculated).	Current in Amperes.	Resistance in Ohms (Calculated).
22	·08	275	·14	157
49	·22	223	·23	213
73·5	·37	198	·32	229
102·5	·53	191	·40	256
124	·67	185	·46	269
152	·89	170	·53	286

A more exact method of measuring the change of resistance with temperature is described on p. 264, where the subject is further discussed.

Measurement of Resistance: By using a Voltmeter and Ammeter.—This method is correct in principle, and is the only one available under certain conditions (*e.g.* for measuring the resistance of a lamp whilst working, as used in the last experiment). The results, however, are for many purposes not sufficiently exact; indeed, the method must be regarded as a rough and ready one, which is very useful when only approximate accuracy is required.

By comparison with a known Resistance.—The independent or absolute measurement of a resistance is a matter of considerable difficulty, and is dealt with later (p. 587). On the other hand, it is easy to *compare* two resistances. All ordinary measurements are of this nature, and involve the use of a standard resistance, or more conveniently, an adjustable set of such standards, known as a **resistance box.**

Method of Substitution.—One of the earliest and most obvious methods of comparison is that of substitution, which is of such limited application that it requires only a brief explanation. It assumes that resistances of known value (*e.g.* a resistance box) are available.

Exp. 166. Connect a Daniell's cell in series with the wire or coil whose resistance is to be measured and with a galvanometer, adjusting the sensitiveness of the latter until a convenient deflection is obtained. Read this deflection, and then remove the wire, substituting for it the adjustable known resistance. Vary the latter until the deflection of the galvanometer is the same as before. The known resistance required for this purpose is evidently equal to that of the wire to be measured.

This method, under ordinary conditions, gives only approximate values, and is, as a rule, quite inapplicable for practical purposes, although the principle will be applied later to a special method of dealing with very high resistances.

Construction of Resistance Boxes.—Fig. 194 shows the arrangement of a resistance box of the ordinary pattern. Other varieties are used, but it is unnecessary to describe them here. A thick plate of ebonite forms the top of the box, and on the upper surface of this is screwed a series of brass blocks, which can be connected together by well-fitting brass plugs. It is usual to cut away the blocks a little at the base in order to facilitate cleaning. The various coils are wound on wooden bobbins, and are supported by means of brass pins screwed into the ebonite. The coils are wound "non-inductively," *i.e.* the wire is doubled back upon itself before winding. If the coils were wound in the usual way, each, when carrying a current, would act like a weak magnet, and there would be other difficulties due to "self-induction" (see p. 360), which would greatly limit the usefulness of the box. The two ends of each coil are soldered to thick copper bars firmly screwed into the brass blocks,

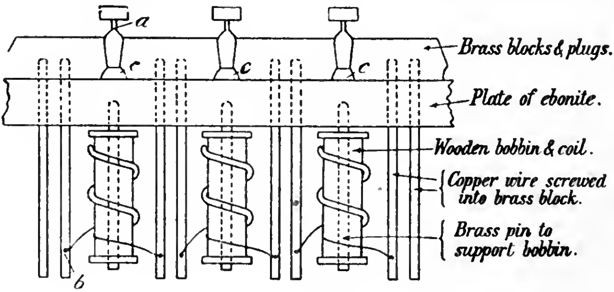


FIG. 194.

the final adjustment of value being made with great care when in position.

It will be seen that the removal of a plug throws the corresponding coil into the circuit. Of course the brass blocks and plugs have some resistance also, but for many purposes this is small enough to be negligible.

As the amount of space available is the same for each coil, whatever its resistance may be, it is necessary to use finer wires for the larger resistances. The wire, which is always silk-covered in order to save space, should be of some material having a high specific resistance, so that any given value can be made up without using either an abnormally great length or a dangerously thin section, and it should also have as small a temperature coefficient as possible. These requirements are best satisfied by alloys — platinoid or manganin being largely employed.

The values of the coils are determined by the condition that it should be possible to make up any integral value within the range of the box with the fewest number of coils. For instance, any value from 1 to 10,000 ohms can be obtained from 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, 1000, 1000, 2000, 5000. One of the gaps, marked "Infinity," has no coil attached to it, and therefore its plug serves as a key for opening and closing the circuit.

The general appearance of such a box is shown in Fig. 200.

Comparison of Resistances by the Wheatstone Bridge.—This is the most usual method of comparing resistances. The principle involved is very simple, and will be easily understood from Fig. 195, which shows a current dividing into two branches at the points A and B. If we take any point, P, in one branch, the potential at that point will be less than the potential at A, and greater than the potential at B (assuming the current to flow from A to B), and there is necessarily some point in the other branch, which is at the same potential. This can be found experimentally by connecting one terminal of a galvanometer to P, and moving the wire attached to the other terminal along the branch AQB, until some point Q is found for which there is no deflection. We then know that

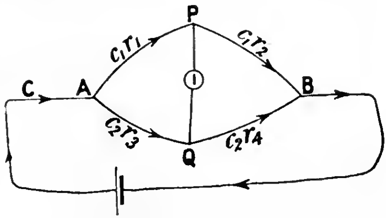


FIG. 195.

P.D. between A and P = P.D. between A and Q, and that
P.D. „ P and B = P.D. „ Q and B.

If r_1, r_2, r_3, r_4, be the resistances of the four arms of the bridge when the above condition is satisfied, and if c_1 and c_2 be the currents in the upper and lower branches respectively, as marked in the figure,

then P.D. between A and P $= c_1r_1$
„ „ A „ Q $= c_2r_3$
„ „ P „ B $= c_1r_2$
„ „ Q „ B $= c_2r_4$

$$\therefore \quad c_1r_1 = c_2r_3$$

$$\text{and} \quad c_1r_2 = c_2r_4$$

$$i.e. \quad \frac{r_1}{r_2} = \frac{r_3}{r_4}$$

This is the law of the Wheatstone bridge. It will be seen that if we know the value of *one* resistance and the *ratio* of two others, the fourth can be found by calculation. It is also obvious that the above expression may be written in the form $\frac{r_1}{r_3} = \frac{r_2}{r_4}$, which indicates that if the positions of the galvanometer and of the battery were interchanged, there would still be no deflection.

The Slide Wire Bridge.—One of the best known forms of bridge, used in practice, is known as the *slide wire bridge*, or, as the wire is generally 1 metre long, as the *metre bridge*, which is shown diagrammatically in Fig. 196. A series of thick brass or copper bars, fitted with the necessary terminals, is carried by a wooden base. Gaps between the bars occur at P, x, R, Q. In the simple forms of the instrument, the gaps at P and Q are often omitted, and even when present they are fitted with stout copper wires or bars until required. Some of their uses will be explained later: for the present the bridge may be considered as having only two gaps at x and R. A uniform wire of platinoid or other high resistance alloy, L, is soldered to the two end pieces. Near this wire is attached a scale, usually divided into millimetres, on which the position of a movable contact piece may be read off. When connected up as shown, R and x being the known and the unknown resistance respectively, it will be seen that the current divides, as indicated by arrows, part flowing through x and R and the other part through the slide wire. The position of the galvanometer contact is then adjusted on this wire, until there is no deflection, say at d divisions from the left-hand side; then if L is the total length in scale divisions, the other portion of the wire is of length $L-d$, and we have

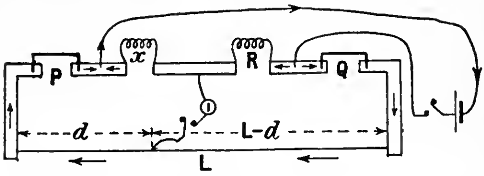


FIG. 196.

$$\frac{x}{d}=\frac{R}{L-d}$$

from which x is found at once.

In practice, a number of precautions are necessary. In the first place, however carefully the slide wire may be soldered to the end pieces, it is difficult to avoid introducing some unknown resistance at these points, which of course is equivalent to a slight extra length of wire at each end. If n and n_1 represent these extra lengths in scale divisions, the true proportion is

$$\frac{x}{d+n}=\frac{R}{L-d+n_1}$$

and in exact work, the values of n and n_1 are found experimentally to begin with. This can be easily done, but here it is merely necessary to point out the existence of such a source of error, which for the present may be taken as negligible.

Secondly, it is impossible to be certain that the scale is accurately placed with respect to the bridge wire, but it is easy to eliminate the errors due to this source by taking double readings. The scale is always numbered both ways, and after taking one position of balance as explained above, it is usual to interchange x and R, and again obtain balance. The reading is now taken from the *other* end of the scale, and the number obtained should be in close agreement with the first. If the discrepancy is considerable, the first observations must be repeated. Let d and d_1 be the two readings thus obtained, then the mean value $\frac{d+d_1}{2}$ represents the true reading corrected for scale error, and we have

$$\frac{x}{\frac{d+d_1}{2}}=\frac{\mathrm{R}}{\mathrm{L}-\frac{d+d_1}{2}}$$

Again, it is important to acquire the habit of closing the battery circuit *before* depressing the galvanometer key, so that the current is established before the galvanometer is in circuit. Otherwise, if the resistance to be measured is *inductive*, e.g. that of a coil of wire or of an electromagnet, a momentary kick or throw will be produced even if exact balance exists, and, in some cases, it may be great enough to injure the galvanometer.

As it is always possible to find a position of balance, whatever be the relative values of x and R, it may be thought that a single known resistance is sufficient for all purposes. In one sense this is true, but, if we consider the influence of unavoidable experimental errors on the final result, it appears that these have the least effect when the position of balance is near the middle of the wire, *i.e.* when x and R are approximately equal. When balance is obtained near one end of the wire, the effect of such experimental errors becomes very great, and renders the results unreliable. It is unnecessary here to discuss the question in detail, but its nature may be illustrated by taking an extreme case. Let us suppose that the wire is divided into 500 equal parts, and that the position of balance, as found by experiment, is liable to a possible error of one division either way. Let $\mathrm{R}=1$ ohm, and let x be such that the *true* position of balance is at 246 divisions (measured from the end under x),

$$\text{then } \frac{x}{246}=\frac{1}{254}, \text{ i.e. } x=\frac{246}{254} \text{ ohms.}$$

If the reading, as found by the experiment, is 245 divisions or 247 divisions, we obtain $x=\frac{245}{255}$ or $x=\frac{247}{253}$, neither of which differs very much from the true value.

But if x be such that the balance ought to be obtained at 499 divisions, and working with the same accuracy as before, we actually get 498 divisions, then the true value is

$$x=\frac{499}{1} \text{ ohms, whereas we find it to be}$$

$$x=\frac{\cdot 498}{2} \text{ ohms, the error being 100 per cent.}$$

Hence, it is desirable to have a range of known resistance, *i.e.* a resistance box, so that we may always choose a suitable value for R.

Evidently, this is scarcely possible with extremely large resistances, and, on the other hand, there is some uncertainty in dealing with extremely *small* resistances, because the resistance of the bars of the bridge is not always negligible in such measurements. Special methods, which are described later, are required for both cases.

Experiments with Slide Wire Bridge.—Exp. 167, *to measure the specific resistance of a material in the form of wire.* It will be convenient at first to choose one of the high resistance alloy wires (such as platinoid), because a suitable resistance can be obtained with a moderate length: afterwards the method may be extended to wires of other metals and alloys.

Assuming that a standard 1-ohm coil is available, cut off a length of the given wire estimated to have a resistance of *about* 1 ohm. This can be found from wire tables, or by means of a rough preliminary measurement. Very carefully solder thick copper wires to the ends so that the specimen has a definite length between these ends. Measure its resistance, taking double readings, as already explained. Measure with great accuracy the length of the wire, up to the commencement of the soldered portion, straightening it if necessary. Also measure the diameter in several places by means of a micrometer gauge, and take the mean value. If such a gauge is not available, the area of section can be found from the formula for the volume of a cylinder—

$$\text{Volume} = \text{length} \times \text{area of section,}$$

the volume being determined by weighing a measured length in air and in water. Express the results in absolute units per centimetre-cube, in microhms per centimetre-cube, and in microhms per inch-cube.

This experiment should be repeated with other metals.

Exp. 168, *to find the temperature coefficient of a metal.* Copper will be the most convenient metal to use. Measure off roughly a length of cotton-covered wire having a resistance of about 1 ohm. To avoid an inconvenient length, a small size should be selected—say 30 gauge, of which 5 yards will be required. Select a test-tube of about 1 inch in diameter and 6 or 7 inches long. Fit it with a good cork having two holes bored through it, one to carry a thermometer, and the other a piece of narrow glass tubing, 4 or 5 inches long, which must fit tightly and must only just pass through the cork. This tube is used merely to allow for the expansion of the oil (see below). Another small cork may conveniently be placed at the top of this tube to afford a firm grip for a clamp by which the whole arrangement can be supported. Wind the specimen of wire on a short piece of glass tubing, tie it down with thread, and solder the ends to two pieces of 20 gauge copper wire. Fill the tube with colza oil, and then insert the coil, bringing out the ends of the connecting wires through small cuts in the cork. Then place the whole arrangement in a large beaker.

Fill the beaker with crushed ice or snow, and wait until the temperature of the oil and wire has become uniform. Measure the resistance very carefully by the slide wire bridge. Now melt the ice, or, more conveniently, substitute cold water, and raise the temperature, by means of a Bunsen flame, a few degrees above 0° C. Remove the burner, stir up the bath, and again measure the resistance, noting also the temperature as given by the thermometer in the oil. Again raise the temperature and repeat the observations, taking ten or twelve measurements between 0° C. and 100° C.

Plot the values thus obtained on squared paper, taking resistances as ordinates and temperatures as abscissæ. The graph should be a straight line, showing that the rate of increase of resistance with temperature is uniform over the range of temperature in question.

Calculate the value of the temperature coefficient from the definition

$$\text{Temperature coefficient} = \frac{\text{Increase of resistance for a rise of } 1^\circ \text{ C.}}{\text{Resistance at } 0^\circ \text{ C.}}$$

Notice that, if we put R_{100}, R_0 for resistances at 0° C. and 100° C. respectively, and a for the temperature coefficient, this becomes

$$a = \frac{\frac{1}{100}(R_{100} - R_0)}{R_0} = \frac{R_{100} - R_0}{100 \times R_0}$$

For copper, $a = {\cdot}00428$

Again, if we write R_t for the resistance at any temperature t° C., we have generally

$$a = \frac{R_t - R_0}{t \times R_0}$$

$$\text{or } R_t = R_0(1 + at)$$

From which the resistance at any temperature, t° C., may be calculated, if a is known; or conversely, t° may be calculated if R_t is measured by experiment. This is the basis of a very exact method of measuring temperatures, using platinum instead of copper.

But although the simple expression given above holds good with considerable exactness between 0° and 100° C., it breaks down when extremely great ranges of temperature are dealt with. In fact the graph is not really a straight line, and is better represented by a parabolic curve, *i.e.* by an equation of the form $R_t = R_0\,(1 + at + bt^2)$, in which b is an additional constant, small compared with a.

Effect of very Low Temperatures.—It is evident that metals become better conductors as the temperature is lowered, and it has been a question of great interest as to whether their resistance would actually vanish at the absolute zero of temperature (−273° C.) or not. The researches of Dewar and others, first at the temperature of liquid air, and later at that of liquid hydrogen, have proved that the law changes rapidly near absolute zero.

This investigation has been carried still further by Professor Kamerlingh Onnes, who has just made known preliminary results of the highest importance. Using boiling helium, he reached a temperature of 1·5° absolute, or −271·5° C. (as measured on a krypton thermometer), and found that the resistance of gold and of mercury dropped rather suddenly to a value too small to be measurable.

For instance, in a certain case the mercury employed had a resistance of 172·7 ohms as a liquid at 0° C. (The change from liquid to solid produces an effect of its own, which may be allowed for by saying that if *solid* at 0° C., the resistance would have been 39·7 ohms). At 4·3° *absolute*, this had fallen to ·084 ohm, and at 3° absolute it was below 3×10^{-6} ohm, *i.e.* less than one ten-millionth of its value at the freezing-point of water. When the temperature was raised, the resistance first became measurable at about 4·2° absolute. In striking contrast to this was the behaviour of an alloy (Eureka), whose resistance remained nearly constant throughout the same range of temperature.

If it should ever become possible to apply these facts to practical purposes, far-reaching consequences of the greatest importance may be anticipated, for conductors of very small section will be able to carry heavy currents without serious heating and consequent loss of energy.

Another Method of applying the Bridge Principle.—Suppose that we arrange the apparatus as shown in Fig. 197, in which A, B, C, D are thick metal plates of negligible resistance fitted with terminals to which the various wires can be attached, x is the resistance to be measured, R is a resistance box, and r, r are two *exactly equal* resistances, preferably of the same order of magnitude as x, but whose values need not be known. We begin by adjusting the resistance in R until a balance is obtained. Then obviously $x = R$; but as it is impossible to change R by less than 1 ohm steps, exact balance will rarely be obtained, and we can say only that x is *between*, say, 24 and 25 ohms, and *nearer* 24 than 25. Such a measurement is quickly made, and is sufficiently exact for many purposes; on the other hand, resistances greater than the total value of R cannot be estimated, and it is rather difficult to be certain that the resistances r, r are exactly equal.

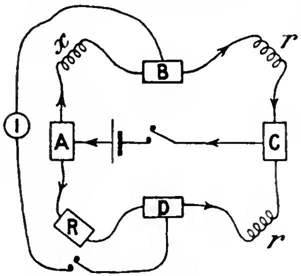


FIG. 197.

It is evidently possible to combine the resistance box and the bridge connections to form one compact piece of apparatus. This is done in the *Post Office pattern*, which we have now to describe, but before doing so, it may be pointed out that a simple arrangement of the kind shown in Fig. 197 is especially useful, when it is required to copy a standard coil with great exactness, or to adjust finally the coils in a resistance box. For suppose that we put a standard 1-ohm coil in place of R, and that we balance it against any convenient temporary resistance in place of x. The 1-ohm coil is now removed and another coil (previously wound and adjusted to a value slightly greater than 1 ohm) is substituted for it. It is then finally adjusted until exact balance is again obtained. Obviously it must be exactly equal to the coil it replaced, whether the two resistances, r, r are equal or not.

Post Office Pattern of Wheatstone Bridge.—This will be easily understood from the explanatory diagram, Fig. 198. It differs from the simple arrangement just described only in the fact that, instead of a single pair of coils, r, r, it is provided with three pairs of 1000, 100, and 10 ohms, any or all of which can be thrown into circuit by taking out the corresponding plugs. These are known as the "proportional coils." One advantage, therefore, consists in our ability to choose the pair nearest in value to the resistance to be measured, and another advantage is the extension of range. For, if we put a 10 ohm in the arm CD and 1000 ohm in the arm BC, balance is obtained when R is $\frac{1}{100}$ of x. Hence, if the total value of R be 10,000 ohms, resistances up to 1,000,000 ohms can be measured. The proportional coils can also be used to estimate fractions of ohms. For example, if, when using two 10-ohm coils, a resistance is found to be more than 23 and less than 24 ohms, we may then alter to 100 ohms in DC and 10 ohms in BC, and again obtain balance. Suppose that this requires R to be between 234 and 235, then we know that the true value of x is between 23·4 and 23·5. Obviously a second decimal may be obtained by using the 1000 : 10 ratio, although when such accuracy is required, it is better to use the slide wire bridge, as it is usually impossible to be certain of the exactness of the ratio.

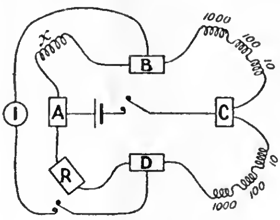


FIG. 198.

The usual arrangement of the box is shown in Fig. 199, lettered to correspond with Fig. 198. The galvanometer and battery connections pass from D and C to the studs under the keys, P and Q, and when these are depressed, it will be seen that the proper connections are made. The actual appearance is shown in Fig. 200 (for which we are indebted to Messrs. Philip Harris & Co). It is a compact and useful instrument for the majority of ordinary purposes.

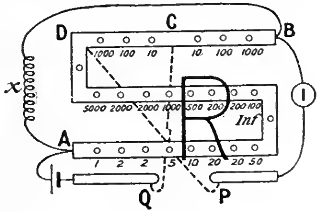


FIG. 199.

Many other forms of bridge have been devised and are in use for special purposes. We have merely described one of the best known.

Carey Foster's Method of comparing Two nearly equal Resistances.—The bridge methods already described are not sufficiently delicate when it is necessary to detect very small differences in resistance, *e.g.* when comparing standard coils with one

FIG. 200.

another. Many modifications of these methods have been devised, of which the following may be taken as an example :—

Exp. 169. An ordinary slide wire bridge may be used, provided with the two additional gaps, mentioned on p. 262, which are closed by thick copper strips when not in use. Two auxiliary adjustable resistances are required, whose values need not be known. Let R, R_1 be the two resistances to be compared; and P, Q the two auxiliary resistances. Connect up the apparatus as shown in Fig. 201.

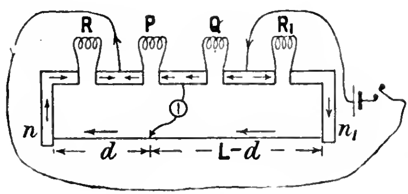


FIG. 201.

(1) Give P and Q any values, which may for convenience be approximately equal to R and R_1, and adjust until balance is obtained at some point reading d divisions on the scale, as in the figure.

(2) Now interchange R and R_1, and again obtain balance at some point reading d_1 divisions *from the same end of the scale.* Let L be the total length of the bridge wire; ρ, the resistance of one scale division of the wire; and n, n_1, the unknown resistance of the joints and metal at each end respectively between the end of the bridge wire and the commencement of R and R_1. (These resistances were assumed to be negligible in the previous experiments.)

Then, noticing that R and R_1 are now really extensions of the bridge wire, we have, in accordance with the simple law of the bridge,

$$\text{from (1)}\quad \frac{P}{Q}=\frac{R+n+d\rho}{R_1+n_1+(L-d)\rho},\quad \text{and}$$

$$\text{from (2)}\quad \frac{P}{Q}=\frac{R_1+n+d_1\rho}{R+n_1+(L-d_1)\rho}$$

$$\therefore\quad \frac{R+n+d\rho}{R_1+n_1+(L-d)\rho}=\frac{R_1+n+d_1\rho}{R+n_1+(L-d_1)\rho}$$

whence, retaining each numerator, and taking the sum of the numerator and denominator of each fraction for a new denominator, we obtain

$$\frac{R+n+d\rho}{R+R_1+n+n_1+d\rho+(L-d)\rho}=\frac{R_1+n+d_1\rho}{R+R_1+n+n_1+d_1\rho+(L-d_1)\rho}$$

and as the denominators are identical, we have

$$R+n+d\rho=R_1+n+d_1\rho$$

$$\text{or}\quad R-R_1=(d_1-d)\rho$$

The "end" corrections have cancelled out, and hence we learn that one of the advantages of this method is that such unknown factors have no influence on the result. From the equation, we also see that the length of the wire, between the two divisions read on the scale, has the same resistance as the difference between the two resistances to be compared, from which it follows that, if this difference is greater than the total resistance of the bridge wire, the method fails, because two points of balance cannot be obtained. By adjusting P and Q, the initial point of balance can be brought to any part of the wire we please.

In order to evaluate the result, ρ must be determined. This may be done by measuring the total resistance of the bridge wire in the ordinary way, using a second bridge; or it can be done more conveniently by a simple application of the preceding method as follows:—

Exp. 170. For R, use a known resistance of a value slightly less than that of the whole length of the bridge wire; and for R_1, use a thick copper strip, the resistance of which may be taken as zero. Repeat the two operations mentioned in Experiment 169, adjusting P and Q until the first point of balance lies near the end of the bridge wire, then the second point will lie near the other end. This gives

$$R-0=(d_1-d)\rho$$

$$\text{or}\quad \rho=\frac{R}{d_1-d}$$

In our treatment of the subject, we have always *assumed* the uniformity of the bridge wire. As a matter of fact, in all exact work, the wire is first carefully calibrated by experiment, but for information concerning these methods, the student must consult some work devoted to electrical measurements.

We may also mention that the previous method is capable of inversion in a way that makes it available for the measurement of very low resistances. For example, it would be practically impossible to measure the resistance of a thick copper wire or bar, say a yard long, by the ordinary use of the bridge. But if the details of the latter are modified, so that the bar can be temporarily but firmly clamped to become the bridge wire itself, we may—using for R and R_1 two known resistances differing very slightly in value—mark off on the bar a length, whose resistance is equal to this difference. Further, by measuring the cross section of the bar, we can deduce the specific resistance of its material.

Measurement of very Low Resistances.—As already indicated, ordinary bridge methods are not well adapted to measure resistances much less than 1 ohm in magnitude, as the resistance of the bridge parts and connections are then no longer negligible, and are difficult to allow for with the necessary accuracy.

Special methods of comparison are then preferable, it being understood that a known resistance of small value is available for the purpose. Such methods usually depend on the measurement or comparison of differences of potential. For example, let R be the known, and x the unknown resistance, and let them be arranged in series with (say) a Daniell's cell, as shown in Fig. 202. If e, e_1 be the P.D.'s between AB and CD respectively, then, as the current is necessarily the same in both, we have

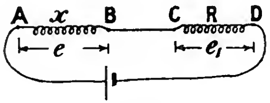


FIG. 202.

$$C = \frac{e}{x} = \frac{e_1}{R}, \text{ from which } \frac{x}{R} = \frac{e}{e_1}$$

Hence, we see that we have to determine the ratio $\frac{e}{e_1}$, which can be done in several ways. The most accurate method is to use a potentiometer (as explained in the next chapter), and by its means to compare the two P.D.'s exactly as if they were due to two cells.

A somewhat simpler plan is to use a sensitive reflecting galvanometer[1] of very high resistance compared with either R or x, and to connect it in turn across AB and CD. It then becomes practically a voltmeter. Let C_1, C_2, be the currents through the galvanometer, producing steady deflections d_1 and d_2 in the two cases, and let g be the resistance of the galvanometer. On account of the high relative resistance of the instrument, C_1 and C_2 are negligible compared with C, the current through R and x, and the values of e and e_1 are not appreciably altered.

$$\text{Now } C_1 = \frac{e}{g} \propto d_1$$

$$\text{and } C_2 = \frac{e_1}{g} \propto d_2$$

$$\therefore \frac{e}{e_1} = \frac{d_1}{d_2}, \text{ and we have } \frac{x}{R} = \frac{e}{e_1}$$

$$\text{whence } x = R \times \frac{d_1}{d_2}$$

[1] It is convenient at this stage to assume that such instruments are available. They are discussed in Chapter XIX.

In order to obtain deflections of convenient magnitude, it is desirable to place a resistance box in the circuit, the actual arrangements being made as shown in Fig. 203. A mercury cup switch (see p. 253) is used to enable the galvanometer connections to be readily interchanged. The connections from C and D are crossed so that *both* deflections may be on the same side of the scale. In order to avoid possible injury to the galvanometer, a large resistance should, at first, be unplugged in the box (RB), and should be gradually reduced until the deflection reaches a suitable value.

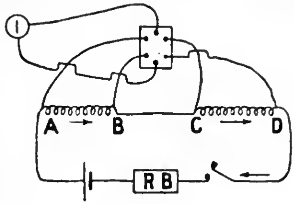


FIG. 203.

Parallel Circuits.—When very large resistances (say, a million ohms or more) have to be measured, the Wheatstone bridge again fails, in this case through lack of sensitiveness, and special methods are required. In order to understand them, it is necessary to study the laws of parallel circuits.

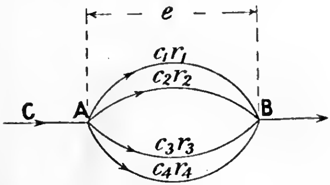


FIG. 204.

Let AB (Fig. 204) be a portion of a circuit, in which the main current C divides up among a number of branches of resistance r_1, r_2, r_3, &c., arranged in parallel.

It is required to find the equivalent resistance R, *i.e.* the value of a *single* resistance, which might replace the branches without altering the current. Let e be the P.D. between A and B. This is common to all the branches, and therefore we have

$$C=\frac{e}{R},\ C_1=\frac{e}{r_1},\ C_2=\frac{e}{r_2},\ \&c.$$

$$\text{Also } C=C_1+C_2+C_3+\&c.$$

$$\therefore \frac{e}{R}=\frac{e}{r_1}+\frac{e}{r_2}+\frac{e}{r_3}+\&c.$$

$$\text{or } \frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\&c.$$

$$i.e.\ R=\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\&c.}$$

Observe that, in the special case when n *equal* resistances are arranged in parallel, this becomes

$$R=\frac{1}{\frac{1}{r}+\frac{1}{r}+\frac{1}{r}+\&c.}=\frac{1}{\frac{n}{r}}=\frac{r}{n}$$

The case of *two* resistances in parallel occurs most frequently in practice; this becomes

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{r_1 r_2}{r_1 + r_2}$$

To ascertain how the current divides up among the various paths, we notice that

$$e = C_1 r_1 = C_2 r_2 = C_3 r_3 = CR$$

$$\therefore\ C_1 = \frac{CR}{r_1},\ C_2 = \frac{CR}{r_2},\ \&c.$$

from which we learn that the currents are inversely proportional to the resistances.

Case of a Shunted Galvanometer.—The easiest way of varying the sensitiveness of a galvanometer is to provide it with a by-pass or "shunt," as shown in Fig. 205.

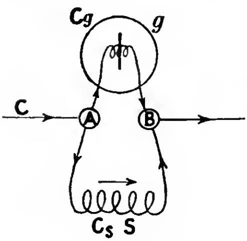


FIG. 205.

For instance, when a delicate reflecting galvanometer is used in connection with "null" methods, *e.g.* in potentiometer or Wheatstone bridge work, it is liable to be seriously damaged by excessive deflections during the initial stages of the measurement whilst the approximate position of balance is being found. It is usual in such cases to connect a resistance box in parallel with the galvanometer, and to begin by unplugging a resistance small in comparison with that of the galvanometer. If the galvanometer has, say, 500 ohms resistance, and if we unplug, say, 1 ohm in the box, then by the above argument only $\frac{1}{501}$ of the total current will pass through the galvanometer, and the deflection will be correspondingly reduced. By unplugging more and more resistance, the galvanometer can be made more and more sensitive, until for the final exact adjustment of the position of balance, the infinity plug can be removed, which is equivalent to disconnecting the shunt and giving the galvanometer its maximum sensitiveness.

On account of the practical importance of this case, it is advisable to discuss it in detail.

Let s be the resistance of the shunt, and g the resistance of the galvanometer, then, by the previous proof, the equivalent resistance of the shunted galvanometer is

$$R = \frac{sg}{s+g}.$$

i.e. the resistance is $\frac{s}{s+g}$ of its old value.

Again, if C_g = current through the galvanometer, C_s = current through the shunt, C = total current, and e = P.D. between galvanometer terminals, then we have

$$e = C_g \times g = C_s \times s$$

$$i.e. \quad \frac{C_g}{C_s} = \frac{s}{g}$$

$$\text{whence} \quad \frac{C_g}{C_g + C_s} = \frac{s}{s+g} \qquad \text{(i.)}$$

$$\text{and} \quad \frac{C_s}{C_g + C_s} = \frac{g}{s+g} \qquad \text{(ii.)}$$

$$\text{but} \quad C_g + C_s = C.$$

whence equation (i.) becomes $C_g = \frac{s}{s+g} . C$

and ,, (ii.) ,, $C_s = \frac{g}{s+g} . C$

Thus, if we wish to make $\frac{1}{10}$ of the total current pass through the galvanometer, then we must give s the value determined by

$$\frac{s}{s+g} = \frac{1}{10}$$

$$i.e. \quad s = \tfrac{1}{9}g$$

Similarly, if we wish $\frac{1}{100}$ or $\frac{1}{1000}$ of the current to pass through the galvanometer, then the resistance of the shunt must be $\frac{1}{99}$ or $\frac{1}{999}$ respectively of the galvanometer resistance.

Sometimes, galvanometers are provided with special "shunt boxes," containing resistances which are usually $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of that of the instrument itself. Their great advantage is that of facilitating calculations—a matter of importance when many measurements have to be made for commercial purposes. On the other hand, there is the disadvantage that such a box can be used only with the instrument for which it was made. The late Professor Ayrton devised a form of shunt box which will serve for any galvanometer, but its action is more complicated, and for the present it will be desirable to limit the argument to the simple form given above.

(Some additional remarks on the effect of shunts are given on p. 299.)

Measurement of very High Resistance.—In practice, such measurements are frequently required, more especially in connection with "insulation resistances." For example, when a building is to be wired for lighting or power, it is necessary to measure the resistance between the positive and negative network of wires before any lamps are connected across them. The measurement is, therefore, a test of the goodness of the insulation, and the value will usually be something in megohms, although it should be remembered that the larger and more extensive the system, the lower the insulation resistance naturally becomes. For this particular purpose, a direct reading instrument—known as an Ohmmeter—is used, as only approximate accuracy is required, and the method must be simple and easily applied.

Makers of the cable used for wiring buildings must also test its insulation resistance before it is sent out from the works, as they guarantee that the insulation resistance shall have a certain number of megohms per mile—from about 300 to 1200, according to quality. This may be regarded as the resistance existing between the copper core and some conductor in contact with the outer surface of the insulation, the second conductor being usually provided by immersing the coil of cable in water, with both ends clear. It will be evident that the resistance, under these circumstances, is *inversely* as the length of the cable, so that, if the value per mile is 600 megohms and a length of $\frac{1}{16}$ mile is under test, the actual resistance to be measured is $16 \times 600 = 9600$ megohms.

Again, the construction of most electrical machinery involves the insulation of copper conductors from a metal framework or core, and it often becomes necessary to test the quality of the insulation, by measuring the resistance between the copper and the metal upon which it is wound.

For such measurements as these, the Wheatstone bridge again fails through lack of sensitiveness, and the most generally useful method for laboratory work is as follows. It involves the comparison of the unknown resistance with a single standard resistance of high value—say 10,000 to 20,000 ohms. As the student is not likely to have a suitably high resistance for measurement, one, having a value of from $\frac{1}{2}$ to 1 megohm, must be extemporised. This may be done by having a glass tube, which is fitted with corks, carrying copper wires, and which is filled with distilled water; or, more conveniently, by making a conducting surface upon a strip of dry wood or ground glass by means of an ordinary lead pencil.

Exp. 171. Select a strip of dry wood, 4 or 5 inches long, about $1\frac{1}{2}$ inches wide, and of such thickness that brass terminal-clamps (such as are used on carbon plates) may be attached at each end. Make a lead pencil track—smooth and uniform, of about $\frac{1}{2}$ to $\frac{3}{4}$ inch wide—from end to end; at each end lay on a bit of clean tinfoil and screw the clamp on it. The soft tin is used merely to assist in making good contact between the graphite and the brass.

Connect this up to a delicate reflecting galvanometer, and to a single cell, Fig. 206 (1), taking care to insulate it and the connecting wires from the bench, because even wood conducts well enough to vitiate the result. (The necessity for care in this respect will be realised if the student tries the effect of holding the terminals of the specimen in his hands, thus providing a shunt path through his body.) If the deflection is inconveniently small, add another cell (or more) in series; if too large, reduce the width of the graphite by scraping the edges with a knife. Carefully note steady deflection (d divisions), then without altering the cells or disturbing the galvanometer, replace the high resistance by a resistance box and connect up another box as a shunt to the galvanometer, Fig. 206 (2). (In such operations, always first remove the infinity plug, which then answers the purpose of a key.) Unplug as high a resistance as possible, using a simple number such as 10,000 or 20,000 ohms, in the box R. See that *all* the plugs, including the infinity plug, are in their places in the box S, and then insert infinity plug in box R. The result will probably be a very small, perhaps scarcely perceptible, deflection (if the shunt were not used, the deflection would be so violent as probably to injure the galvanometer). Gradually unplug resistances in S until a convenient deflection is obtained. Then note the values of S, R, and the deflection of d divisions. The resistance, g, of the galvanometer is also required—if not known it must be measured as a separate experiment, using a Wheatstone's bridge method and another galvanometer as an indicator of balance.

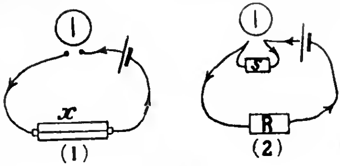


FIG. 206.

Let E = E.M.F of cell or battery; and b, its internal resistance. Then total resistance is $b + R + \frac{sg}{s+g}$ (see p. 272). Now, even if g is great, s is not likely to be very large, and $\frac{sg}{s+g}$ must be less than s, and is probably negligible compared with R. Hence, we may take R as the effective resistance.

If, in the first experiment, C is the current, d the deflection, and x the unknown resistance; and in the second experiment, C_1 is the total current, C_g the current through the galvanometer, and d_1 the deflection, we have

$$C = \frac{E}{x} \propto d \qquad \text{(i.)}$$

$$C_1 = \frac{E}{b + R + \frac{sg}{s+g}} = \frac{E}{R} \text{ (practically)}$$

$$\text{Also from p. 273, } C_g = \frac{s}{s+g} \times C_1$$

$$\text{and} \quad C_g \propto d_1$$

$$\therefore \quad C_g = \frac{s}{s+g} \times \frac{E}{R} \propto d_1 \qquad \text{(ii.)}$$

$$\therefore \quad \frac{\text{ii.}}{\text{i.}} \qquad \frac{s}{s+g} \times \frac{x}{R} = \frac{d_1}{d}$$

$$\textit{i.e.} \quad x = R \times \frac{s+g}{s} \times \frac{d_1}{d}$$

It will be noticed that the factor $\frac{R(s+g)d_1}{s}$ is constant for the given setting up of the apparatus; after it is determined, various unknown resistances can be found without repeating the second portion of the experiment.

Exp. 172. Estimate the resistance of your body, by noting roughly the deflections (probably unsteady) which are obtained by holding in the hands the wires previously attached to x. Observe also the great decrease in resistance when the hands are moistened, or when larger pieces of metal are used as handles.

Exp. 173. Adjust the graphite resistance until it is as near as possible 1 megohm. If the measured value is less than this, calculate the deflection corresponding to 1 megohm, and whilst it is connected in circuit, gently scrape off the graphite at the sides until the deflection has the required value.

Such a resistance is often useful, and should be retained.

Kirchhoff's Laws.—The equivalent resistance, and the currents in the various branches of a divided circuit can be readily calculated by the simple methods already given when there are no cross connections in the network; it can be applied, for instance, in Fig. 207*a*, but the method fails in Fig. 207*b*, which is especially interesting on account of its application to the Wheatstone bridge.

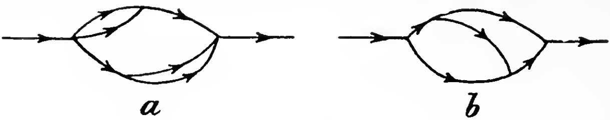


FIG. 207.

Equations for the complete solution of such problems can easily be written down by using Kirchhoff's two laws. These are not "laws" in the same sense as Ohm's law; they contain nothing new to the student, and are merely useful rules which show how Ohm's law is to be applied.

They are: (1) *In any branching network of wires, the algebraical sum of the currents in all the wires meeting at a point is zero.*

The term *algebraical sum* means that if the currents are flowing *from* the point they are taken with a negative sign, and those flowing *to* the point with a positive sign, or *vice versâ.*

(2) *In any closed path in the network, the algebraic sum of the $C \times r$ products is equal to the E.M.F. acting in that path.*

Here clockwise currents are to be reckoned positive and anti-clockwise currents negative, or *vice versâ.*

The first law is obvious. The second will be more easily understood by taking a particular case—as a matter of general interest, that of a Wheatstone bridge.

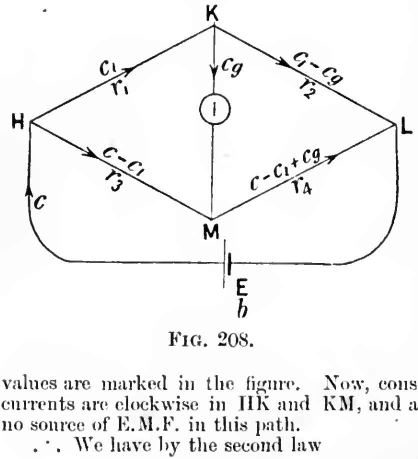


FIG. 208.

Let a cell of E.M.F. = E, and of internal resistance = b, be connected by wires of negligible resistance to the Wheatstone network at H and L, the resistances of whose arms are as marked in Fig. 208.

Then, if we write as usual, C for the total current through the cell, C_g for that in the galvanometer, g for the galvanometer resistance, and if we take C_1 as the current in one of the arms, say HK, we may deduce the currents in the other arms by applying the first law. These values are marked in the figure. Now, consider the closed path HKM. The currents are clockwise in HK and KM, and anti-clockwise in HM; also there is no source of E.M.F. in this path.

∴ We have by the second law

$$C_1 r_1 + C_g g - (C - C_1) r_3 = 0. \qquad \text{(i.)}$$

(This result is more obvious, if we notice that the P.D. between H and K must be C_1r_1, and that between K and M must be $C_g.g$. These are in the same direction. and, therefore, the P.D. between H and M must be $C_1r_1 + C_g.g$. But this latter P.D. must also be $(C - C_1)r_3$, from which the above equation follows.)

Again, considering the closed path KLM, we obtain

$$C_g.g + (C - C_1 + C_g)r_4 - (C_1 - C_g)r_2 = 0 \qquad \text{(ii.)}$$

In the closed path EHML, the currents are all clockwise, and as there is an E.M.F. (E) in this path, we have :—

$$(C - C_1)r_3 + (C - C_1 + C_g)r_4 + Cb = E \qquad \text{(iii.)}$$

and similarly for the path EHKL,

$$C_1r_1 + (C_1 - C_g)r_2 + Cb = E \qquad \text{(iv.)}$$

Taking three of these equations (i. and ii. and either iii. or iv.), we can solve simultaneously for three unknown currents.

The student will perceive that the chief difficulty is not in obtaining the necessary equations, but in solving them afterwards. This is a purely mathematical question, dealt with in more advanced works, and need not be discussed here.

If it is required to find the equivalent resistance of a complex circuit, this really means that we have to determine the P.D. between the common points, as, for instance, H and L in Fig. 208. If this be e, and R the equivalent resistance, then by the definition of R, $e = CR$.

Now, $e = C_1r_1 + (C_1 + C_g)r_2$; or, $(C - C_1)r_3 + (C - C_1 + C_g)r_4$, and can be evaluated when the currents have been determined.

In certain special cases, the value of e can be found much more readily by considerations of symmetry.

Examples.—(1) Find the resistance of a skeleton cube, taken between opposite diagonal corners, if each edge has a resistance of r ohms.

Let C be the total current, and e the P.D. between A and B (Fig. 209). Then by symmetry, the current divides into three equal parts at A, and the three equal parts reunite at B. Hence, currents in AH and KB are each $\frac{C}{3}$. Also at H there are two symmetrical paths for the current $\frac{C}{3}$, and hence, the current in HK must be $\frac{C}{6}$.

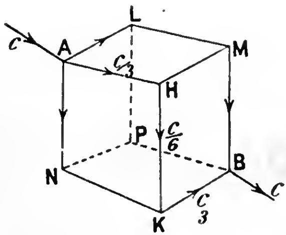


FIG. 209.

Now. P.D. between A and B is the sum of the P.D. along AH, HK, and KB.

$$\therefore \quad e = \left(\frac{C}{3} \times r\right) + \left(\frac{C}{6} \times r\right) + \frac{C}{3}r = \frac{5}{6}Cr$$

but, if R = the equivalent resistance, $e = CR$

$$\therefore \quad CR = \frac{5}{6}Cr$$

$$\text{or} \quad R = \frac{5}{6}r$$

(2) Find the equivalent resistance of the same cube taken between the ends of one of the edges.

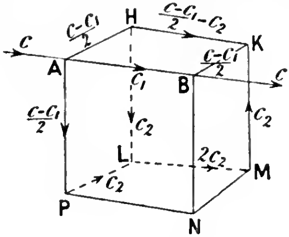


FIG. 210.

Let C be the total current, which divides at A into a component C_1 along AB and into two equal components, each of which may be written $\frac{C-C_1}{2}$ along AH and AP (Fig. 210). These currents, by symmetry, must reunite at B. Then, putting C_2 for the current along HL, all the other components can be written down from the symmetry of the figure, using Kirchhoff's first law.

Again, it is evident that the P.D. (e) between A and B is independent of the path traversed, and hence we have:—

Path AB, $e=C_1r$ (i.)

Path AHKB, $e=\frac{C-C_1}{2}r+\left(\frac{C-C_1}{2}-C_2\right)r+\frac{C-C_1}{2}.r=\frac{3}{2}(C-C_1)r-C_2r$

putting $C_1r=e$, we obtain $e=\frac{3}{5}Cr-\frac{2}{5}C_2r$ (ii.)

Path AHLMKB, $e=2\left(\frac{C-C_1}{2}\right)r+4C_2r$

which becomes $2e=Cr+4C_2r$ (iii.)

From (ii.) and (iii.), $C+4C_2=\frac{6}{5}C-\frac{4}{5}C_2$

or $C_2=\frac{1}{24}C$ (iv.)

Substitute (iv.) in (iii.), then $2e=Cr+\frac{1}{6}Cr$

or $e=\frac{7}{12}Cr$

But $e=CR$

$\therefore$ $R=\frac{7}{12}r$

(3) Find the resistance of a skeleton tetrahedron between any two corners, if each edge has resistance r.

The student will have no difficulty in proving that this is $\frac{r}{2}$, if, on drawing the figure, he notices that the two corners are points, which, by the symmetry of the figure, must be at the same potential. Hence, the resistance is not altered if the points are directly connected, and the figure then reduces to a very simple case.

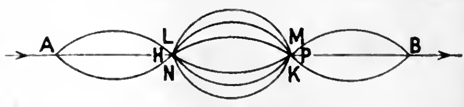


FIG. 211.

The method suggested in this case may also be applied to Examples 1 and 2. For instance, in Fig. 209 it is obvious that L, H, and N must all be at the same potential, and may therefore be directly connected together; the same remark also applies to the points M, H. and K. It is convenient to draw a key diagram as shown in Fig. 211, on which all the letters and sides are indicated.

Regarding LHN and MPK as single points, we join up all the sides in accordance with the lettering of the original figure, and thus obtain an equivalent figure, which can be treated by the ordinary laws of parallel circuits, from which, obviously,

$$R = \frac{r}{3} + \frac{r}{6} + \frac{r}{3} = \frac{5}{6}r$$

The second example is a little more complicated. H and P are at the same potential, and so also are K and N. We, therefore, begin with the four points, A, HP, KN, and B, and join up in accordance with the lettering. The result is as shown in Fig. 212, which, by a repeated application of the laws of parallel circuits, gives

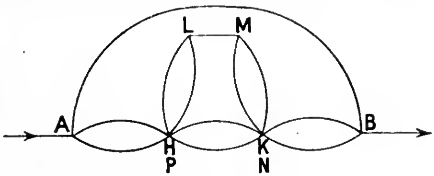


FIG: 212.

$$R = \frac{7}{12}r$$

Lord Kelvin's Method of Measuring the Resistance of a Galvanometer.

—**Exp. 174.** Connect up the galvanometer whose resistance (g) is to be measured in one arm of any form of Wheatstone bridge, omitting the galvanometer ordinarily used to obtain balance, its position being occupied by a simple key, K_1, as shown in Fig. 213.

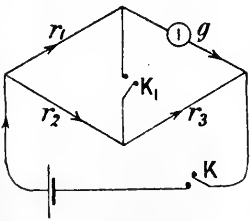


FIG. 213.

If *another* galvanometer is available, we can, of course, measure the resistance of the first one in the ordinary way, in which case its own moving part will be kept clamped. Let us first suppose that such a second instrument is placed in the arm occupied by K_1, and that we watch the indications of *both* galvanometers, whilst going through the operations of obtaining balance. It will be noticed that the first galvanometer is always deflected when the cell circuit is closed, and that, as a rule, closing the key K_1 *alters* that deflection, but that, when balance is obtained, as indicated by the non-deflection of the auxiliary instrument, there is *no alteration* in the deflection when the key K_1 is opened or closed. The reason of this is obvious, and it follows that we may dispense with the auxiliary instrument altogether, using merely the arrangement shown in the diagram. Then, if we adjust the resistance in the other arms until the deflection is *unaltered* by opening or closing K_1, we know that the ordinary law of the bridge is satisfied, i.e. $gr_2 = r_1r_3$, whence g is obtained.

This method is useful when a second galvanometer is not available, but the ordinary method is the more satisfactory.

Mance's Method of Measuring the Internal Resistance of a Cell.

—**Exp. 175.** Place the cell, whose internal resistance (b) is to be measured, in the position occupied by g, and a galvanometer in that occupied by K_1 in the last experiment. Notice that a deflection of the galvanometer is produced, and that the closing of the battery key, K, superposes on this current another due to the testing battery. But when the condition $br_2 = r_1r_3$ is satisfied, the testing battery does not send any current through the galvanometer when K is closed, and the deflection will be unaltered. Remove

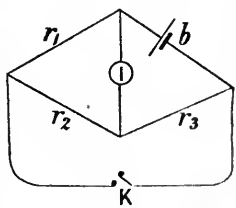


FIG. 214.

the testing battery, and notice that there is still no deflection on opening or closing K. If, however, the above condition is not satisfied, the deflection will be altered. Hence, we have only to arrange the apparatus as shown in Fig. 214, and adjust the resistances until the deflection is unaffected by opening or closing K. Then $br_2 = r_1 r_3$, whence b is obtained. In this experiment, the galvanometer should not be too delicate. It is a very good plan (if a moving magnet galvanometer is used) to place a magnet in such a position as to bring the needle near its ordinary zero, because then we obtain the greatest change of deflection for a given change of current, and the test is correspondingly more sensitive.

Exercise XIV

1. What is the resistance of a column of mercury 2 metres long and ·6 of a square millimetre in cross section at 0° C. ?

2. What length of copper wire, having a diameter of 3 millimetres, has the same resistance as 10 metres of copper wire, having a diameter of 2 millimetres?

3. If the resistance of a yard of iron wire, ·03 inch in diameter, be ·197 ohms, what is the resistance of 15 miles of iron wire, ·3 inch in diameter?

4. What must be the thickness of copper wire which, taking equal lengths, gives the same resistance as iron wire 6·5 millimetres in diameter, the specific resistance of iron being six times that of copper?

5. If the resistance at 0° C. of an iron wire 1 foot long and weighing 1 grain be 1·08 ohms, find the resistance at 0° C. of 1 mile of iron wire weighing 300 lbs.

6. A piece of copper wire 100 yards long weighs 1 lb. ; another piece of copper wire 500 yards long weighs $\frac{1}{4}$ lb. Show what are the relative resistances of the two wires.

7. Two exactly equal pieces of copper are drawn into wire ; one into a wire 10 feet long, and the other into a wire 20 feet long. If the resistance of the shorter wire is ·5 ohm, what is the resistance of the longer wire?

8. Two copper wires, one of which is 4 metres long and weighs 7·5 grams, and the other 5 metres long and weighs 4·2 grams, are joined in "multiple arc" (that is, so that both wires connect the same two points) in a circuit in which a current of total strength 8·09 (amperes) is passing. The current will divide so that part passes through each wire ; what will be the strength of the current conveyed by each?

9. Two wires are joined in parallel circuit, their resistances being 10 and 20 ohms: find the resistance of the conductor thus formed.

10. Three wires are joined in parallel circuit, their resistances being 40, 15, and 55 ohms: find the resultant resistance.

11. The resistance between two points A and B of a circuit is 30 ohms, but on adding a wire between A and B the resistance becomes 20 ohms. What is the resistance of the added wire?

12. A galvanometer of 1000 ohms resistance is shunted with a shunt of 1 ohm resistance. Find the resistance of the shunted galvanometer.

13. A coil concealed in a box, but with its terminals exposed, is found to have a resistance of 126 ohms. A piece of wire, 10 metres in length, of the same material as that of the coil, but of twice its cross-sectional area, is found to have a resistance of 12 ohms. What is the length of the wire in the coil?

(Lond. Univ. Matric., 1906.)

14. A wire, the total resistance of which is 4 ohms, is bent into the form of a square, ABCD, the loose ends being soldered together. Find the resistance of the system when a current enters at B and leaves at D. Will it be modified if the corners A and C are connected by another wire?

15. If a cell has an E.M.F. of 1·08 volts and ·5 ohm internal resistance, and if the terminals are connected by two wires in parallel of 1 ohm and 2 ohms resistance respectively, what is the current in each, and what is the ratio of the heats developed in each. (Lond. Univ. Inter. B.Sc., 1902.)

16. A circuit is made up of (1) a battery with terminals A, B, its resistance

being 3 ohms, and its E.M.F. 2·7 volts; (2) a wire BC, of resistance 1·5 ohms; (3) two wires in parallel circuit, CDF, CEF, with respective resistances 3 and 7 ohms; (4) a wire FA, of resistance 1·5 ohms. The middle point of the last wire is put to earth. Find the potential at the points A, B, C, F.

(B. of E., 1897.)

17. A wire of resistance r connects A and B, two points in a circuit the resistance of the remainder of which is R. If, without any other change being made, A and B are also connected by $n-1$ other wires, the resistance of each of which is r, show that the heat produced in the n wires together will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$. (B. of E., 1900.)

18. Describe carefully the Wheatstone's bridge method for comparing the resistance of two coils. If the two coils are of nearly equal resistance, how would you arrange the experiment so as to obtain great accuracy? (B. of E., 1908.)

19. Explain the terms *specific resistance* and *temperature-coefficient of resistance* of a material. What material would you employ for constructing a standard resistance, and how would you wind the wire? (B. of E., 1906.)

20. Explain the action of a shunt used with a galvanometer. The resistance of a galvanometer is 100 ohms; it is connected in circuit with a battery, whose E.M.F. is 15 volts; the resistance of the battery and wires is 5 ohms; the galvanometer is shunted so that one-tenth of the total current passes through it. Find the resistance of the shunt, and the current through the battery.

(Camb. Local, Senior, 1894.)

21. A closed circuit contains a battery of 1 ohm resistance, a reflecting galvanometer of 4 ohms resistance, and other conductors of 2 ohms resistance. The deflection of the galvanometer is 100 divisions of the scale. What will the deflection be (assuming it to be proportional to the strength of the current) when the terminals of the galvanometer are connected by a wire of 4 ohms resistance?

22. A galvanometer, the resistance of which is $\frac{1}{3}$ ohm, being joined up in circuit with a cell by thick copper wires, the resultant current is noted; and it is found that the current in the galvanometer is halved if, without any other change being made, the terminals of the galvanometer are joined by a wire of resistance ·1 ohm. What is the resistance of the cell?

23. A network is arranged as in Mance's experiment. The resistance of the battery, of the galvanometer, and of each of the other three arms, is 4 ohms, and the E.M.F. of the battery is 2·2 volts; find the current in the galvanometer (1) with the key open, (2) when it is closed. If the resistance of the arm opposite the battery is altered to 5 ohms, find the new values of the current.

(Lond. Univ. B.Sc., 1898.)

24. On passing a current of 1 ampere through a piece of platinum wire, it is found that its temperature rises 10° C. above that of the surrounding objects, which are at 0° C. Assuming that the rate of loss of heat is proportional to the difference of temperature, calculate the temperature of the wire when a current of 2 amperes is passed through it. The temperature coefficient of the resistance of the wire may be taken to be ·004 of the resistance at 0° C.

(Lond. Univ. Inter. B.Sc., 1907.)

25. What difficulties are met with in measuring a very small resistance by the Wheatstone's bridge method? Describe a method of comparing low resistances. (Lond. Univ., B.Sc. Internal, 1909.)

26. A certain electromagnet has to be excited from mains at a definite voltage. Show that if any prescribed number of ampere-turns of excitation are to be given, the coil must be such as to have the resistance per turn of a definite value, independent of the actual number of turns. A coil, of which the mean length of turn is 42 inches, is required to produce an excitation of 6300 ampere-turns, when supplied at 35 volts. Calculate the cross-section of the wire, assuming that a copper rod 1 foot long and 1 square inch in cross-section has a resistance of 9·2 microhms. (City and Guilds, 1908.)

CHAPTER XIX

GALVANOMETERS—MEASUREMENT OF CURRENT

THE word "galvanometer" is a term of very general meaning, covering a wide range of instruments of different types and applications. It may be regarded as signifying an instrument whose primary function is to indicate the *existence* of a current, more especially of weak currents, and which may, under certain circumstances, be capable of *measuring* it.

The simplest and oldest form consists of a coil of wire, within which is pivoted or suspended a small magnet, usually termed the needle (see p. 198). Normally, the needle is at rest in the magnetic meridian, and the position of the instrument is adjusted until it is at right angles to the axis of the coil. On passing a current through the coil, a magnetic field is produced, which at the centre of the coil

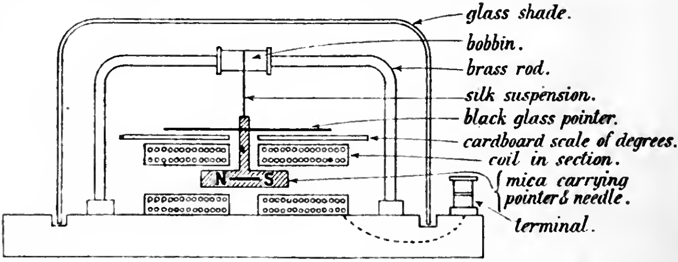


FIG. 215.

is at right angles to the earth's field. The two magnetic fields combine to produce a resultant field, inclined in direction to both, along which the needle points, its deflection being read off on a scale of degrees. It will be noticed that the effect of the earth's field is to produce a restoring or controlling force, which brings the needle back to its original position when the current is interrupted.

A very convenient form of simple galvanometer is shown diagrammatically in Fig. 215. The glass shade is an ordinary flat-bottomed evaporating dish, such as is used in chemical laboratories, about 6 inches in diameter. The rectangular coil (shown in section) is divided into two parts, separated by about $\frac{1}{8}$ inch, so that the needle may be suspended between them. This is a short piece of knitting-needle carried by a strip of mica (shown, in the diagram, as deflected through 90°) which has an extension upwards to which the silk fibre and a black glass pointer are attached. The mica is made nearly as large as the inside of the coils, in order that the oscillations may be checked by air friction. On the coils rests a circular card graduated into degrees, and the needle is raised or lowered by means of a little roller carried by a brass rod. This pattern has been used by the writers for many years, and has proved very serviceable for teaching purposes.

The greater the number of turns in the coil, the closer it is to the needle, and the more delicately the needle is pivoted or suspended, the more sensitive will be the instrument; although as a rule there is no simple law connecting current and deflection, which can be found *by calculation* from the dimensions. For instance, if one current gives a deflection of 40°, and another a deflection of 20°, it is certain that the first current is the greater, but it is impossible to say how much greater, except that it is probably more than twice as great. It is, however, possible to find the relation between current and deflection *by simple experimental methods*, and it will be useful to explain one such method here. It assumes that the resistance of the galvanometer and that of the cell used have been previously measured.

Exp. 176. Connect up the galvanometer in series with a Daniell's cell and a resistance box. Adjust the resistance in the box until a small deflection, say 5°, is obtained. Call this current unity. Add up the total resistance in circuit—resistance box, galvanometer, and cell (although, if the cell has a low internal resistance, this may be negligible). If this total resistance is now halved, the current will be doubled. Adjust the value in the box until this is the case. For instance, if r, g, and b are the resistances of the box, the galvanometer and the cell respectively, the new resistance to be unplugged in the box would be $\frac{r+g+b}{2}-(g+b)$ ohms. Take the new reading, which will be nearly twice the first. Then make the total resistance $\frac{1}{3}$ of its *first* value, *i.e.* unplug $\frac{r+g+b}{3}-(g+b)$ ohms. Proceed in this way as far as possible, thus obtaining a series of deflections due to currents of unknown value, but which are in a known ratio, 1, 2, 3, 4, &c. Plot a graph with these numbers for ordinates and degrees for abscissæ. The graph obtained will be of the form shown in Fig. 216, and from it can be obtained the relative value of the currents producing any given deflections. It will be observed that for a *short* distance from the origin the curve is *nearly straight*, which means that for very small deflections the current may be taken as approximately proportional to the deflection without sensible error. It is more nearly proportional to the *tangent* of the angle of deflection, as will be shown later, but for very small angles the two are

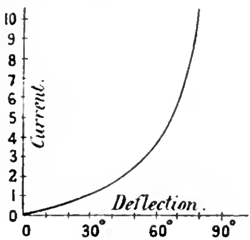


Fig. 216.

practically identical. It will also be noticed that the increase of deflection for a given increase of current diminishes very rapidly above 40° or so (the exact shape depends on the details of construction of the galvanometer), and that an *infinitely* strong current would be required to produce a deflection of 90°.

For many purposes we require only the *relative* values of current, but it is evident that if, in addition, we can determine the *actual* current corresponding to a given deflection in any one case, then by using the curve we can find the actual value of the current corresponding to any deflection throughout the scale. For instance, in an experiment with an instrument of the type shown in Fig. 215, it was found that a current of ·005 ampere produced a deflection of 44°, and that on the calibration curve 44° corresponded to a reading of 1·22 divisions on the scale of current. Suppose that we require to find the value of a current C giving any other deflection $d°$, then if D be the scale reading corresponding to $d°$, we have

$$\frac{C}{\cdot 005} = \frac{D}{1 \cdot 22}$$

$$\text{or} \qquad C = \frac{\cdot 005}{1 \cdot 22} \times D = \cdot 0041 \times D \text{ ampere.}$$

The Tangent Galvanometer.—The difficulty encountered in attempting to deal mathematically with the previous type of instrument is due to the fact that the field produced by the current is not uniform throughout the space traversed by the needle, and hence the direction of the resultant field depends not only on the current strength, but also on the position of the needle. When the field at the needle is uniform, like that of the earth (*i.e.* when it can be represented by parallel equidistant straight lines), the difficulty vanishes, and this condition can be fairly satisfied by a simple modification of the construction. It is only necessary to use a circular coil, whose diameter is at least ten or twelve times as great as the length of the needle. Fig. 217 shows one form of the instrument (made by F. E. Becker & Co.), in which two coils are wound on a brass ring—the number of turns of each being known, and the radii carefully measured. A short needle, carrying a long aluminium pointer at right angles to it, is suspended by unspun silk. The graduated scale is provided with a mirror to avoid parallax.

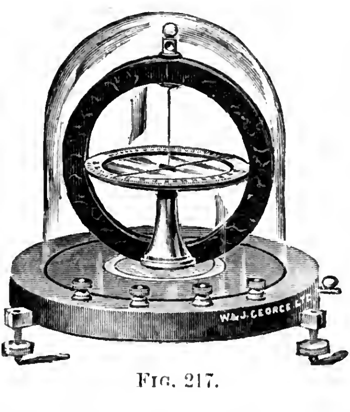


FIG. 217.

In this instrument the field is approximately uniform over a small region at its centre, as may be inferred by examining Fig. 181, p. 244, and therefore we can apply the argument given on p. 142, where it is shown that, if F be the strength of the deflecting field, and H that of the controlling field (both supposed to be uniform), then the direction of the resultant field along which the needle points is determined by the expression $\frac{F}{H} = \tan\theta$. In this case, F is evidently the strength of the field due to the current in the coil, and we have now to find the relation between the two.

Simple experiments show that the field at the centre of the coil is directly proportional to the current strength, and it is customary to write G for its value when the current is unity. Hence, when the current is i absolute units, the field strength is iG, and the resultant field will have the direction determined by

$$\frac{iG}{H} = \tan\theta$$

$$i.e. \qquad i = \frac{H}{G}\tan\theta$$

Of course the same expression holds if we use amperes, but it is then necessary to regard G as being the field due to 1 ampere.

The above formula is important, because (1) for *very small* deflections it holds good for any galvanometer, in which the controlling couple is due to the earth's field (although it may not be possible to evaluate G by calculation), and (2) an expression of similar form may be applied—but only for small deflections—to any type of galvanometer, whatever be the nature of the controlling force. These results follow from the fact that, for any very small range of motion, the field due to the current is practically constant in direction.

We have now to find a value for G, in the special case of the tangent galvanometer. It will, however, be convenient to generalise the problem by finding an expression for the field strength at any point on the axis of a circular coil carrying a current. Let AB (Fig. 218) represent the section of a coil of radius r and of n turns, carrying a current of i absolute units. It is required to find the field strength at any point P

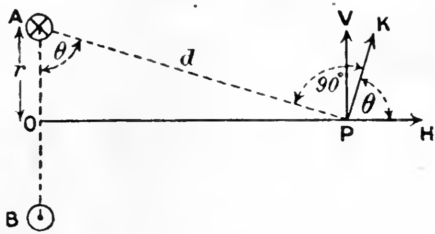


FIG. 218.

on the axis. Let d be the slant distance of P from the coil. (If there is only one turn, then d is measured from the centre of the wire; if there are n turns, we may then, with sufficient accuracy for our purpose, regard d as being the *mean* distance.)

Now, the field at P is due to the joint effect of every part of the wire, and it is convenient to consider first the influence of a very small portion of length s. Some difficulty arises here, as there is no method by which such a portion can be isolated in order to study its action experimentally. All that can be done is to observe the effect of the coil *as a whole*, and experiment then shows that the field at P is directly proportional to the current strength, to the number of turns in the coil, and to the square of its radius (*i.e.* to its area).

Again, when these quantities are kept constant and the distance OP varies, it is found that the field varies inversely as $(\text{OP})^3$, as long as the point P is not too close to the coil.

Laplace pointed out that, if it be *assumed* that the magnetic force due to a very small length s is directly proportional to its length and to the current strength, and inversely proportional to the square of its distance from P, and that the field thereby produced is at *right angles* to the line joining s and P, then the above result follows mathematically. That is, writing H_s for the force, F_s for the field produced by the current in s, we have in C.G.S. units

$$H_s = \frac{i.s}{d^2},\ F_s = \frac{i.s}{d^2} \times \mu$$

(Although μ is unity, we have inserted it in this expression to avoid difficulties that might arise later.)

It must be clearly understood that this equation is *assumed* to begin with, and that it is not capable of direct verification. The justification for using it lies in the fact that in this and in many other cases, it leads to results for complete circuits which are found experimentally to be correct.

In the present instance, the field F_s at P acts along PK, the angle APK being 90°.

$$\text{Let } \angle \text{KPH} = \theta, \text{ then } \angle \text{OAP} = \angle \text{KPH} = \theta$$

Resolve the field along PK into components PH, PV along the axis and perpendicularly to it respectively

The component of F_s along the axis

$$= F_s \cos\theta = \frac{i.s}{d^2}\,\mu \cos\theta = \frac{i.s}{d^2}\mu \,.\, \frac{r}{d} = \frac{i.s.r}{d^3}\,.\,\mu$$

If the whole length of the wire be similarly treated, by dividing it into small portions and considering the effect of each separately, it is evident that all the components along the axis are in the *same* direction, and that their values have simply to be added together; whereas the components at right angles to the axis are radial from P and mutually destructive, their sum vanishing. Hence, the *only* field at a point *exactly* on the axis is along the axis, and its value can be written down by summing the previous expression for every part of the wire.

It then becomes

$$\text{Field at P} = \text{F} = \frac{i \times \text{total length of wire} \times r}{d^3} \times \mu$$

$$= \frac{i \times 2\pi rn \times r}{d^3} \times \mu$$

$$= \frac{2\pi inr^2}{d^3} \times \mu$$

It must be remembered that d is the *slant* distance, whereas, in practical applications it is more convenient to measure the distance *along* the axis. Obviously, if P is fairly distant, there will be no serious error if we write $OP = AP = d$; but, for short distances we have

$$d = (r^2 + OP^2)^{\frac{1}{2}}, \text{ and}$$

$$\therefore \text{F} = \frac{2\pi inr^2}{(r^2 + OP^2)^{\frac{3}{2}}} \times \mu$$

when P is at the centre of the coil $d = r$, and we have

$$\text{F} = \frac{2\pi in}{r} \times \mu$$

And as $F = iG$, and μ is unity, we find that $G = \frac{2\pi n}{r}$

This is the case of the tangent galvanometer, for which, therefore, we may write (see p. 285)

$$i = \frac{H}{G} \tan \theta$$

$$= \frac{H}{\frac{2\pi n}{r}} \tan \theta$$

$$= \frac{rH}{2\pi n} \tan \theta$$

If C is the number expressing the strength of the current in amperes,

$$\text{then } i = \frac{C}{10}$$

$$\text{or } C = \frac{10r.H}{2\pi n} \tan \theta$$

It will be noticed that this expression contains a factor H, which is not, strictly speaking, a constant, for its value will vary in different parts of the earth; hence, the portion $\frac{10r}{2\pi n}$ is sometimes called the *constant of the instrument*, and $\frac{10rH}{2\pi n}$ its *reduction factor*.

For all practical purposes, however, H has a constant value at any given place, and it is more convenient to put $C = K \tan \theta$, where

$$K = \frac{10rH}{2\pi n}$$

An experimental method of finding the value of K is given on p. 335. The theoretical importance of the tangent galvanometer depends upon the fact that H can be evaluated (see p. 160) at any given place by a quite independent method, and hence it is possible to measure a current *absolutely*, if the construction be such that r and n are accurately known.

If, as is often the case, r and n are unknown, the *relative* values of any two currents can be found, and this makes the galvanometer at once available for many purposes (as in Experiments 180 and 186).

It may be pointed out that, from its construction, a tangent galvanometer is *not a sensitive* instrument, and it is, therefore, not suitable for use in "null" methods (*e.g.* Wheatstone bridge and potentiometer). For such purposes the coil should surround the needle as closely as possible, and the nature of the relation between current and deflection is immaterial.

Nor should it be assumed that the tangent law is accurately satisfied in any instrument over the whole range of scale. It is instructive to check it by an independent method, and thus to ascertain the extent of its departure from the ideal law. A number of methods are available, from which we select the following as a useful exercise.[1]

Exp. 177. Place a constant cell (*e.g.* a Daniell's) in circuit with a tangent galvanometer and a resistance box. It is better to provide the galvanometer with a reversing key and to take double readings (Fig. 219). Let b be the internal resistance of the cell, g the resistance of the galvanometer, and r the resistance unplugged in the box. Adjust the resistance in box until a small deflection, say 5° to 8°, is produced. Note deflection, reverse current, and again note deflection, entering both values on paper, and also the mean value. Alter the resistance until the deflection is 10° or 12°, and repeat observations. Proceed in this way until the deflection is as great as possible, say 70° to 80°. A table of values of r and θ is thus obtained. For any one of these we have

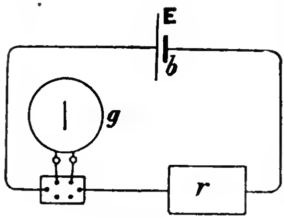


Fig. 219.

[1] This method is due to Carhart and Patterson; see their *Electrical Measurements*.

$$C = \frac{E}{b+g+r}$$

$$\text{also } C = K \tan\theta$$

$$\therefore K \tan\theta = \frac{E}{b+g+r}$$

$$\text{whence } \tan\theta = \frac{E}{K(b+g+r)}$$

$$\text{or } \cot\theta = \frac{K(b+g+r)}{E}$$

Now $b+g$ is constant during the experiment ; let this be r_1,

$$\text{then } \cot\theta = \frac{K}{E}r + \frac{K}{E}r_1$$

If we put $\cot\theta = y$, and $r = x$, this is of the form

$y = mx + c$, where m and c are constants,

i.e. it is the equation of a straight line. Hence (Fig. 220), if we plot a graph with values of $\cot\theta$ for ordinates, and r for abscissæ, the result will be a straight line if the tangent law holds good for the instrument, and if it does *not* hold good, or only so for a portion of the scale, then such deviation will be indicated by the departure of the graph from a straight line. We may note, in passing, that the graph will cut the $\cot\theta$ axis in some point A. Let it be produced to meet the r axis at B, and let α be the included angle. Then, by the well-known property of the straight line,

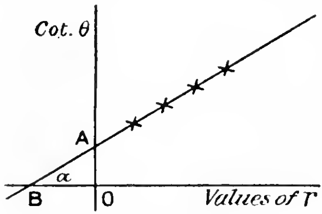


FIG. 220.

$$\frac{K}{E} = \tan\alpha = \frac{AO}{BO} \qquad \text{(i.)}$$

$$\text{and } \frac{Kr_1}{E} = AO \qquad \text{(ii.)}$$

from which it appears that $BO = r_1$; *i.e.* $BO = b + g$.

The Sine Galvanometer.—This is not so much a distinct instrument as a particular way of using a tangent galvanometer, or indeed *any* galvanometer having a magnet directed by a uniform field. Suppose that instead of reading the ordinary deflection θ on a tangent galvanometer, the whole instrument is rotated in the direction of deflection ; at first the deflection will increase, but unless it is too large, the coil will eventually catch up the needle. The actual deflection is now, say, α, which is greater than θ, and is evidently equal to the angle through which the coil has been turned. Hence, in specially made instruments, the coil has an independent motion, and is provided with another scale, on which this angle may be read off; but the reading may be taken on any instrument by simply interrupting the current, when α is the angle through which the needle swings back.

In this method it is evident that, when the reading is taken, the field iG is at right angles to the needle, and the resultant field is along the direction of the needle. The constructiou will be understood from Fig. 221.

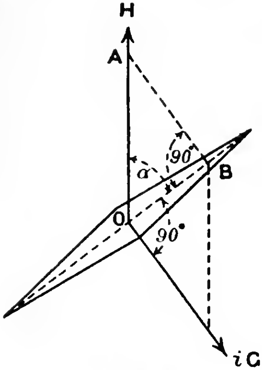


FIG. 221.

$$\text{Now } \frac{AB}{AO} = \sin a$$

$$\text{but } \frac{AB}{AO} = \frac{iG}{H}$$

$$\therefore \frac{iG}{H} = \sin a$$

$$\text{or } i = \frac{H}{G} \sin a$$

A special advantage of this method lies in the fact that it may be applied, as we have previously mentioned, to the simple forms of galvanometer, and does not merely hold good for the tangent type of instrument. The only difference will be that, when thus applied, the value of the constant G cannot be found by calculation, and we must write $i = K \sin a$ where K is unknown, but for all *relative* measurements of current this is immaterial.

Helmholtz or Double-Coil Tangent Galvanometer.—It can be shown mathematically that the best way of producing a uniform field is to use two equal circular coils, placed at a distance apart equal to the radius of either. The coils are connected in series and the needle is placed midway between them. This arrangement is very useful when it is required to produce a uniform magnetic field of moderate strength and known value for any purpose whatever (*e.g.* for deflecting cathode rays in the measurements described on p. 487).

If *each* coil has n turns of radius r, the field due to either, at a point midway between them, is found by putting $OP = \frac{r}{2}$ in the equation given on p. 287, so that

$$\text{F (due to either)} = \frac{2\pi i n r^2}{\left\{r^2 + \left(\frac{r}{2}\right)^2\right\}^{\frac{3}{2}}}$$

As both fields act in the same direction, the actual field is

$$\frac{4\pi i n r^2}{\left(r^2 + \frac{r^2}{4}\right)^{\frac{3}{2}}}, \text{ which becomes } \frac{32\pi i n}{5\sqrt{5}.r}$$

Galvanometers of Greater Sensitiveness.—It is as extremely sensitive instruments that galvanometers are more especially useful in electrical measurements. The tangent galvanometer is of great theoretical interest, but of comparatively little practical use, whereas very sensitive instruments are indispensable for many different purposes.

Speaking generally, there are three methods by which we may increase the deflection due to a current of given strength: (1) by increasing the moment of the deflecting couple due to the current (*e.g.* by increasing the number of turns, by disposing them to get the best effect, &c.); (2) by decreasing the moment of the controlling couple; (3) by increasing the length of the pointer.

The first method may be regarded as obvious; in any case it will be carried out as far as is possible.

The second was first applied practically by using an astatic needle.

Astatic Galvanometer.—As already explained, various systems of magnets may be arranged on which the earth's magnetic field has no restoring couple. The simplest case, as applied to a galvanometer, is shown in Fig. 222. It should be noticed that, if the needles are *perfectly* astatic and there is no friction on a pivot or torsion on a suspension, then *all* currents, weak or strong, would produce a deflection of 90°, and there would be no return of the needle to zero. It is, however, very difficult to make needles perfectly astatic, and there is an advantage in making one slightly stronger than the other, so that a controlling couple may be retained in order to bring the system to zero. As a rule, if special pains are taken to make such a pair of needles astatic, it will be found that the system, when delicately suspended, has a distinct tendency to set itself east and west (see p. 164). In any case, the uncertainty as to the zero position makes comparisons of deflections somewhat difficult, and the instrument is better adapted for indicating the existence of very feeble currents than for measuring them.

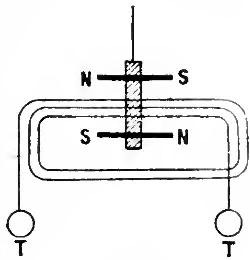


FIG. 222.

The astatic galvanometer in its original form is now seldom used, although the principle is frequently applied in various ways. Speaking generally, we may remark that it is always advantageous for the moving system of a sensitive galvanometer to be "astatic," if that term be extended to mean freedom from disturbance due to the earth's field (or to stray fields due to currents or magnets in the vicinity of the galvanometer). But this property may be obtained in different ways, and it does not necessarily imply a feeble control.

For instance, the suspended coil galvanometers to be subsequently described are ideally "astatic"—the moving part being non-magnetic—but the control due to the suspension may be as strong as we please. When the moving part is magnetic, it is best to make it as astatic as possible, and to work with an independent and adjustable control due to bar magnets attached to the instrument.

As a good example of this construction, we may mention the **Broca galvanometer.** The needle (Fig. 223) consists of two steel wires, placed vertically and very close together, each having consequent poles. A very small mirror is attached below, and the whole is suspended by a fine quartz fibre. This form makes it possible to use comparatively powerful magnets, whilst keeping the moment of inertia small, and at the same time it is very astatic. The coil, wound in two halves for convenience, is arranged as shown with respect to the needle. It will be noticed that all the poles contribute to the turning moment. A movable bar magnet at the back provides the necessary control.

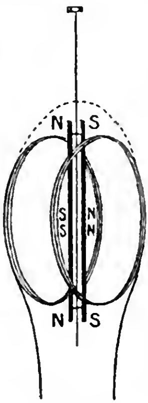


Fig. 223.

Thomson Galvanometer.—The earliest mirror galvanometer was devised by the late Lord Kelvin (when Professor Thomson) for receiving signals through the first Atlantic cable; and a special class of instruments are still known as "Thomson" galvanometers. An early pattern is shown in Fig. 224.

The magnet is very short and light—so small and light, indeed, that when attached to the back of a mirror, A, about a centimetre in diameter, the total weight is not more than one or two grains. The mirror is suspended by a single fibre of unspun silk in a small cylinder, round which the wire is coiled. The length of wire employed for this purpose depends upon the sensitiveness required (as explained on p. 291). Through an aperture in a screen, a beam of light is sent from a lamp upon the mirror, which reflects it on a scale. A permanent magnet is placed on a vertical support above the coil, which controls the magnet in the galvanometer, so that the spot of light is readily brought to the zero on the scale. It may also be used to *weaken* the earth's field if necessary, which still further increases the sensitiveness.

In the most modern forms of this galvanometer the parts are doubled and the moving system thereby made astatic. The principle is indicated in Fig. 225.

Two sets of three or four very small magnets (their size is exaggerated in the figure) are attached to a light aluminium wire, which also carries a mirror, m, and the whole is suspended by a fine quartz or silk fibre, F. Two pairs of coils surround the needles. The mirror is sometimes placed between the coils (as shown) and sometimes under the lower coil on an extension of the aluminium support. In order to obtain a more delicate adjustment, it is now usual to supply two controlling magnets, MM, which may be adjusted either to weaken or to strengthen each other's effect.

The only disadvantage possessed by this type of galvanometer is due to the fact that the deflections are readily disturbed by the presence of iron or of magnets in its vicinity, which makes it very troublesome for ordinary experimental work.

It may be pointed out that, when using a mirror instrument, the angular deflections of the moving system are necessarily very small

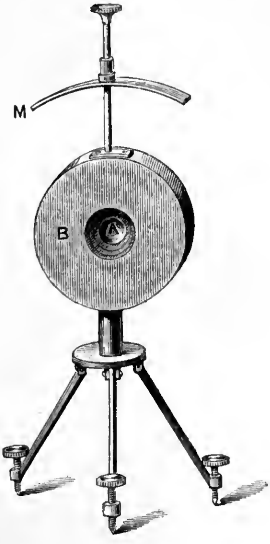


FIG. 224.

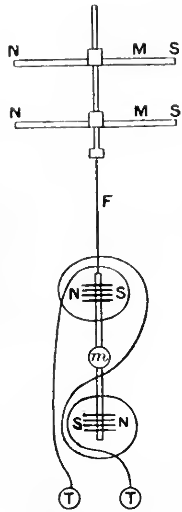


FIG. 225.

(or otherwise the spot of light would move off the scale); and, for such small deflections (as already stated on p. 285), the relation, $C \propto \tan \theta$, is true to a close approximation with almost any type of instrument. Again, it has been shown (p. 142) that the deflection in any arbitrary scale divisions is proportional to $\tan \theta$. Hence, for any mirror galvanometer we may take *the current as proportional to the deflection it produces* as long as we avoid unreasonably large deflections. This is a very great advantage in practical work.

Moving Coil Galvanometers.—These instruments are modifications of the "syphon recorder" devised by Lord Kelvin for use on submarine cables. They are generally known as **D'Arsonval galvanometers**, although improved and developed by many later inventors. Their great value is due to the fact that they are practically unaffected by neighbouring currents or magnets, and hence, they are more generally used both for commercial and for laboratory work than any other type.

The principle involved in the earliest form will be understood from Fig. 226. A narrow rectangular coil of many turns of fine silk-covered copper wire (of which one or two strands are shown in the figure) is suspended by a fine metal wire, so that it can move freely in a narrow annular space between the poles, NS, of a strong steel magnet. Within the coil is a fixed cylinder of soft iron, C, which serves to concentrate the lines of force in the gap. The coil is wound on a light framework (which is either of metal or of non-conducting material, according as a "dead-beat" or a "ballistic" action is required, see p. 296), and which is steadied below by another wire attached to a spring strip to keep it under slight tension. A

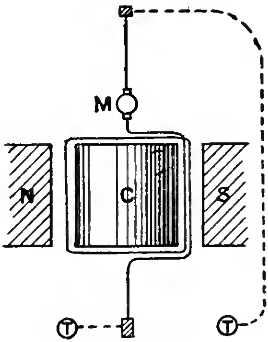


Fig. 226.

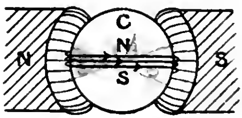


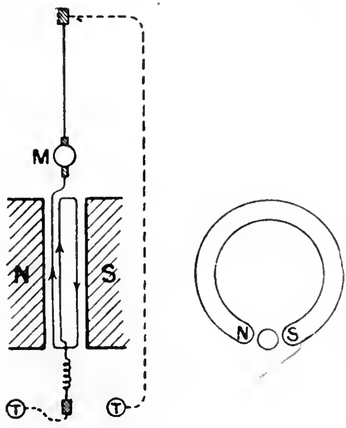
Fig. 227.

mirror, M, is fixed just above the coil. The current is passed through the instrument (as shown) by means of this strip and the metal suspension. There are several ways of regarding the action of this instrument; at present it will be easiest to think of the coil as becoming, whilst carrying a current, the equivalent of a weak magnet. For instance, by applying the swimming rule to Fig. 227 (which represents Fig. 226 looked at from above), it is found that the polarity of the coil is N above and S below, and hence there will be a couple acting upon it, which tends to rotate it through 90° in a clockwise direction. If the current is reversed, the polarity of the coil and the direction of rotation will also be reversed. The controlling couple opposing rotation is provided by the torsion of the suspending wire, and the final position will be acquired when the moments of these two couples are equal.

Evidently, the sensitiveness is increased by making the magnetic field in the gap very strong and by giving the moving coil many turns. The effect of the soft iron core is to make the field in the gap very uniform (except just at the pole tips), and, as a result, the current is proportional to the deflection through a much wider range of motion than is usual in a reflecting instrument. For this reason, the construction just described will be found applied to a very useful class of commercial ammeters and voltmeters (see p. 560).

In 1892, Professors Ayrton and Mather pointed out the great advantage of using a thin strip of metal for the suspension, instead of a wire of circular section—the torsion for a given deflecting couple being much greater, and the stress on the material much less with a strip than with a wire. They also introduced the use of phosphor bronze for this purpose, and modified the shape of the coil and other details in such a way that an entirely new type of instrument was created.

The Ayrton-Mather form is shown diagrammatically in Fig. 228. The magnet, NS, is usually made of cast steel, circular in section, with a very narrow air gap between the poles. These are usually rounded off as shown in Fig. 228 (*b*), in order to concentrate the field; a construction which is advantageous for many purposes (*e.g.* for "null" methods), but which must be modified if the proportionality between current and deflection is required over any large range of scale. There is no soft iron core, and the coil is long and narrow, the current being taken in and out by means of the phosphor bronze strip suspension and a delicate spiral of the same material at the bottom. If the instrument is to be "dead beat," the coil is placed inside a thin silver tube, which moves with it; if it is to be "ballistic," a light non-conducting tube is used, which in this case is merely a mechanical support and protection. It is usual to provide the instrument with a coil of each kind, so arranged that they can be readily exchanged.



(*a*) FIG. 228. (*b*)

Fig. 229 shows the general appearance of a pattern made by R. W. Paul, and Fig. 230 shows one of the coils removed from the instrument. The upper end of the suspension is connected to the brass carrier, which makes contact through the framework to one of the terminals. The spiral at the bottom is attached to a pin passing through an insulating plug of ebonite, and this pin makes contact with a metal strip connected to the other terminal, which is insulated from the framework. A simple form of clip is provided, which serves to clamp the coil when not in use.

Meaning of terms "Dead-beat" and "Ballistic."—As these terms are so frequently used, it is desirable to explain their meaning at some length. It will be understood that the moving part of any instrument, which is controlled by some couple whose moment increases with the angle of deflection, must, when disturbed, oscillate about its final position like a pendulum—the amplitude of the vibrations gradually diminishing until it comes to rest. Hence,

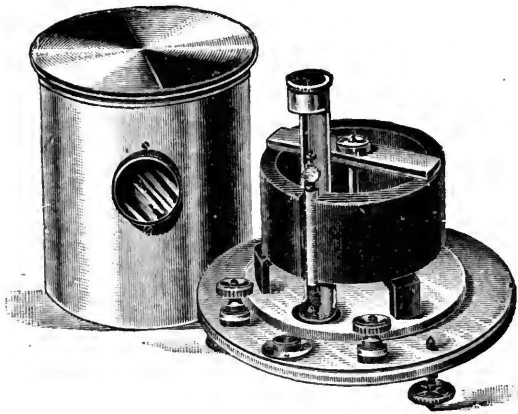


Fig. 229.

Fig. 230.

when a current passes through a galvanometer, the needle does not at once indicate the corresponding deflection, but oscillates about it, so that some time elapses before a reading can be taken; and similarly, when the current is interrupted, time is again lost before the needle comes to rest.

It is evident that the needle cannot come to rest until the energy given to it by the impulse has been converted into heat—in the simplest case, chiefly by air friction. If some additional means of converting the energy into heat are provided, the amplitudes of successive vibrations will diminish more rapidly, and when this effect is marked, the motion is said to be "**damped.**" It must be remembered, however, that such damping does not alter the final position of the needle, but merely enables it to take up that position in a shorter time.

Again, there is a critical amount of damping, for which the vibrations are almost completely stopped, the moving part almost instantly reaching its steady position without overshooting it, and with equal rapidity returning to zero. In this state, the motion is said to be "**dead-beat.**" If, however, the damping is still further increased, the motion becomes inconveniently sluggish.

A dead-beat motion is desirable for all instruments with which readings of steady deflections have to be taken, and it is especially useful in null methods.

There are various methods of obtaining the necessary damping, one of the oldest being the attachment of some light vane to the moving part, thus increasing the air friction, or if this be insufficient the vane may be immersed in some liquid. In certain voltmeters and ammeters, a "pneumatic damping" is used, the principle being similar to that employed in the contrivances fixed on doors to make them close noiselessly, but the most interesting method to the student, and one which is very widely used, depends on the production of induced currents. This method and its application to the moving coil galvanometer are explained on p. 356.

The term "ballistic" may be taken at present to mean "not dead-beat," *i.e.* the moving part is designed so that the damping is very small. It is, however, more convenient to defer the consideration of ballistic galvanometers until the principles of induction currents have been discussed.

"Period," or "Periodic Time" of a Galvanometer.—This is the time required by the moving part to make one complete oscillation, when disturbed from its position of rest. The stronger the control, the shorter will be the period, and *vice versâ*. With null methods, a fairly short period, say 3 to 5 seconds, is desirable, for then the spot of light returns more quickly to its position of rest. For ballistic purposes (as will be shown later), a longer period, say from 5 to 20 seconds, is necessary.

Zero-keeping Property.—This again depends upon the control; when the latter is strong the moving part will return exactly to the same position of rest after being disturbed, whereas when it is weak there is apt to be some uncertainty about the zero, although the instrument is then much more sensitive. Whenever a deflection has to be measured, stability of zero is essential, but for null methods it is relatively unimportant.

Einthoven "String" Galvanometer.—This is an instrument of the moving coil type, but the coil is reduced to a single fine wire or string stretched in a very narrow air gap between the poles of a powerful electromagnet. In the more sensitive forms, this string is frequently a quartz fibre, silvered to make it sufficiently conducting. When a current passes through the string, the latter is deflected sideways, *i.e.* at right angles to the lines of force, an effect similar to that demonstrated in Experiment 204, p. 344. Holes are bored in the pole pieces, through which this movement is observed by means of a microscope, which forms part of the instrument—no mirror being used. The moment of inertia of the moving part is very small (the mirror is dispensed with because it would increase this quantity), and hence the natural time of vibration is also very small. This instrument is extremely sensitive and very dead beat. It was designed more especially for use in physiological research, and, when provided with a photographic recording apparatus, it fulfils, for such purposes, the functions of an oscillograph (see p. 571).

Duddell's Thermo-Galvanometer.—Although this instrument depends upon the properties of thermo-electric currents, dealt with in Chapter XXVIII., it may be alluded to here for the sake of convenience. It is intended to measure extremely small *alternating* or varying currents, *e.g.* it may be used to measure telephone currents or the currents flowing in the receiving circuits of wireless telegraphy. The galvanometers already mentioned will work only with *direct* currents, and the need of *sensitive* instruments for rapidly varying currents has long been felt. A diagrammatic view is given in Fig. 231. A single loop of silver wire, P, is suspended by means of a quartz fibre, Q, between the poles of a permanent magnet. H is a small glass stem attached to the loop, which carries a mirror, M. The lower ends of the loop are connected to a small bismuth-antimony thermo-couple (Bi, Sb). Immediately below this is the "heater," which is a fine filament of high specific resistance (usually a quartz fibre platinised to enable it to conduct), 3 or 4 millimetres in length. The current to be measured passes through the "heater," producing C^2R heat, part of which is radiated and carried by convection currents to the thermo-couple, thus warming it and producing another current in the loop, which then turns like the coil of a D'Arsonval galvanometer. The instrument is an adaptation of a principle first applied by C. V. Boys in his radio-micrometer.

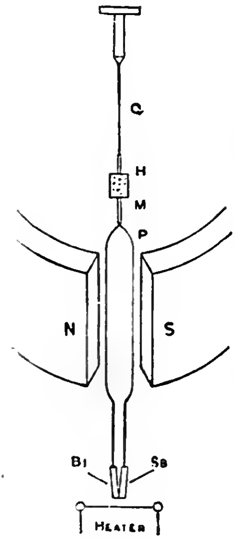


Fig. 231.

Figure of Merit.—This is a term occasionally met with in connection with galvanometers. As originally applied to non-reflecting instruments, it meant the current in amperes required to produce a deflection of 1°. In the case of reflecting instruments, it is often taken to mean the current producing a deflection of one scale division under the particular circumstances of the experiment, but evidently such information is useless unless the distance of the scale from the galvanometer and the length of a scale division are also stated. On the whole it seems best to define it as *the current required to produce* 1 *millimetre deflection on a scale* 1 *metre from the galvanometer mirror.* Using this definition, the values obtained in practice may be summarised as follows :—

Type.	Figure of Merit in Amperes.
Ayrton-Mather moving coil	10^{-6} to 10^{-9}
Thomson	10^{-9} to 10^{-11}
Broca	10^{-10}
Einthoven	10^{-12}
Thermo-galvanometer	2×10^{-6}

Effect of Shunting a Galvanometer.—The laws of parallel circuits and their application to the case of a galvanometer have already been dealt with (see pp. 271–273). It must, however, be remembered that the effect produced by the shunt will depend upon the resistance in circuit, and may vary within wide limits. Let us consider two extreme cases : (1) when the resistance in the circuit is very small, and (2) when it is very large, compared with that of the shunted galvanometer.

(1) Suppose the galvanometer is connected directly to a cell of low internal resistance, and a shunt is inserted having $\frac{1}{9}$ of the resistance of the galvanometer. Then the joint resistance will be $\frac{1}{10}$ of the former value, and as that was practically the only resistance in the circuit, the effect of reducing it to $\frac{1}{10}$ will be to increase the total current to ten times its original amount, and it will be $\frac{1}{10}$ of *this increased* current which passes through the galvanometer, and so the deflection will be but little affected by using the shunt. Thus, when the resistance of the shunted galvanometer, *i.e.* the value of the fraction $\frac{sg}{s+g}$, is large compared with the rest of the circuit, the current passing through it will be practically the same whether shunted or not.

(2) If the resistance in the circuit is very large compared with that of the galvanometer, introducing a shunt will only slightly affect the total resistance, and consequently the total current will be only slightly increased. In such a case the introduction of a $\frac{1}{9}$ or $\frac{1}{99}$ shunt

may reduce the current passing through the galvanometer to nearly $\frac{1}{10}$ or $\frac{1}{100}$ of its original value. For intermediate cases the current passing through the galvanometer will not be reduced so much.

It is instructive to look at the matter from another point of view. If e be the P.D. between the galvanometer terminals, then $C_g = \frac{e}{g}$, and is constant if e is constant whether there is a shunt or not. Hence, if the application of a shunt alters the galvanometer current, it is due to the fact that it has altered the P.D. between the terminals. Now, when the galvanometer is practically the only resistance in circuit, this is also the P.D. between the cell terminals, and it will be found to alter but little when the total current increases due to the addition of a shunt. When there are large resistances in the circuit, e is only a small fraction of $e_\times$, and alters very much with change of current.

The student can easily verify this by taking a particular case. For example, consider a cell of 1 volt E.M.F. and 1 ohm internal resistance, connected up to a galvanometer of 504 ohms resistance. It will be found that the P.D. across the galvanometer terminals is practically 1 volt, and that this value is scarcely altered by shunting it with 56 ohms. But when 5000 ohms is also in circuit, the P.D. across the galvanometer is about $\frac{1}{10}$ volt in the first case, and falls to about $\frac{1}{100}$ volt when shunted with 56 ohms.

Measurement of Current.—It will be noticed that the tangent galvanometer is the only instrument thus far described which is capable of measuring a current "absolutely," *i.e.* without requiring previous calibration by a method involving, directly or indirectly, the use of a *known* current. It may also be pointed out that we can measure a current only by measuring some property of that current, having first arrived at some understanding, based upon experiment, as to the nature of the relation between the two.

Three such properties are available: (1) the magnetic effect, (2) the heating effect, and (3) the chemical effect produced in certain solutions.

The first effect we have seen applied to the tangent galvanometer. It is also applied in various forms of "current-balance" (see p. 567), which depend upon the measurement of the forces of attraction and repulsion between conductors carrying a current, and which (like the tangent galvanometer, only with greater exactness) afford a means of measuring a current absolutely, because the magnitude of the forces in question can be calculated from the dimensions of the apparatus.

The second effect is also available, if Joule's equivalent is accurately known, but it is inconvenient in practice; while the third cannot be applied unless we begin by defining unit current in terms of chemical action.

We are therefore restricted to the use of the first property, but neither current balances nor tangent galvanometers are convenient instruments for the rapid measurement of current under commercial conditions (*e.g.* for standardising ammeters).

Hence, the problem has been solved as follows: The E.M.F.'s of various standard cells (already described) have been determined by very careful and laborious experiments, involving the exact measurement of current by means of a current balance. Having a cell of known E.M.F. and a set of resistances of known value, it becomes possible to measure currents of any magnitude very conveniently and rapidly by the potentiometer method described in the following chapter. Again, by experiments also involving the use of a current balance, the weight of silver deposited by 1 ampere in 1 second from a solution of its salts has been found with great exactness (see p. 590), and consequently the third effect becomes available for the general measurement of current. But, although very exact, the method is so slow and troublesome that its use is confined to special purposes.

Exercise XV

1. A very short magnetic needle is suspended at the centre of a hoop of wire fixed vertically in the magnetic meridian. One current passing through the wire causes a permanent deflection of the needle amounting to 30°; another current causes a deflection of 45°. What are the relative strengths of the two currents?

2. Two tangent galvanometers, alike in all respects except that the hoop of one has twice the radius of that of the other, are employed to measure the strengths of electric currents. If the galvanometers give equal deflections, show what are the relative strengths of the currents passing through them.

3. In a tangent galvanometer a current of strength A causes a deflection of 25°, another of strength B causes a deflection of 20°. What is the relation of A to B? [tan. 25° = ·4663, tan. 20° = ·3640].

4. Discuss the several forces or moments which act on the needle of a tangent galvanometer when deflected by the action of a current passing through the coil of the galvanometer, and deduce the law of action of the instrument.

(B. of E., 1900.)

5. Describe the tangent galvanometer, and the method of using it. The coil of such a galvanometer consists of eight turns of wire, and has a mean radius of 20 centimetres. Find what current will produce a deflection of 45° if the horizontal intensity of the earth's magnetic field is ·18 C.G.S. units.

(Lond. Univ. Matric., 1903.)

6. Calculate the strength of the current in C.G.S. units, and also in amperes, from the following data: Radius of coil, 12 centimetres; number of turns in coil, 10; deflection of needle, 45°; value of earth's horizontal force, ·18.

(B. of E., 1902.)

7. Explain how the sensitiveness of a tangent galvanometer can be (1) decreased, (2) increased by the aid of a bar magnet.

A Daniell cell connected to a tangent galvanometer of ½ ohm resistance produces a deflection of 60°. On interposing a resistance of 2 ohms the deflection falls to 30°. What is the internal resistance of the cell?

(Lond. Univ. Matric., 1906.)

8. A current of ½ ampere passes through a tangent galvanometer, whose resistance is 10·5 ohms. After the terminals of the galvanometer have been joined by a wire, the total current in the circuit remains unaltered, but the

current in the galvanometer is reduced to $\frac{1}{20}$ ampere. If the resistance of the wire is 14 ohms per metre, what length of wire has been used as a shunt?

(Oxford Local, Senior, 1902.)

9. A current flows through two tangent galvanometers in series, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection of the needle be greater? If the greater deflection be 60°, what will the smaller be?

(B. of E., 1897.)

10. An electric current of 1 ampere flows round a circular metal ring, the radius of which is 10 centimetres. Determine the strength and direction of the magnetic field at a point on the line drawn through the centre of the ring perpendicular to its plane and 10 centimetres distant from the plane of the ring.

(B. of E., 1908.)

11. A tangent galvanometer having a coil of one turn of 34 centimetres radius gives a deflection of 45° with a current of 10 amperes. Calculate the strength of the earth's magnetic field at the centre of the coil.

(Lond. Univ. Inter. B.Sc., 1907.)

12. A compass-needle is placed at the centre of two concentric circles which are in the same vertical plane, and are made of wires similar in all respects, except that the outer is copper, the inner German silver. The wires are connected in multiple arc, but so that the currents which flow through them circulate in opposite directions. What must be the ratio of the diameters of the circles so that no effect may be produced on the needle? [*N.B.*—Assume the conductivity of copper to be twelve times that of German silver.]

13. A tangent galvanometer is placed with its coil perpendicular to the magnetic meridian. When no current is passing through it, the needle when set in vibration oscillates 10 times in 15 seconds. Will the rate of vibration be altered when a current is passing through the coil, and if so, will it be increased or diminished?

14. What is the relative strengths of two currents passing through the coil of a sine galvanometer, when the angles through which the coil has been turned, before the needle stands at zero, are 30° and 45° respectively?

15. Why is an astatic galvanometer better adapted for the measurement of weak currents than a galvanometer with a single needle?

16. Describe some type of galvanometer in which the magnet is fixed and the coil is the movable part. How is this type of galvanometer rendered dead-beat? How can such a galvanometer be arranged so as to be used to measure heavy currents?

(Oxford Local, Senior, 1907.)

17. Describe and explain the mode of action of some form of sensitive galvanometer suitable for use in a place where the earth's field is much disturbed by the presence of variable electric currents.

(B. of E., 1906.)

18. Describe the construction and adjustment of a sensitive mirror galvanometer, and explain how you would measure its sensitiveness.

CHAPTER XX

MEASUREMENT OF ELECTROMOTIVE FORCE

It is by no means an easy matter to measure an electromotive force *directly*, although the methods available will be indicated later. It is, however, quite simple to *compare* two electromotive forces with great accuracy, and hence all practical methods depend upon the comparison of the unknown E.M.F. with that of a standard cell.[1]

Potentiometer Method.—This is undoubtedly the most accurate method of *comparing* E.M.F.'s. The principle involved should be thoroughly understood, as the potentiometer, in its more refined forms, not only is of the utmost practical and commercial importance, but is the most generally useful measuring instrument we possess.

In its simplest form, a potentiometer is merely a long piece of uniform wire of fairly high resistance provided with a scale of equal divisions. For the use of a student, a length of 400 to 500 centimetres of 30-gauge platinoid wire is probably the most useful. Such a length of straight wire would naturally be inconvenient, and hence it is usual to zigzag it backwards and forwards on a wooden base; but in doing this, special care must be taken not to introduce errors into the measurement of its length.

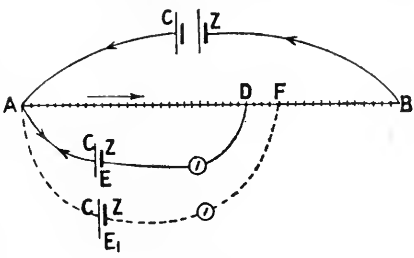


Fig. 232.

Let AB (Fig. 232) represent a potentiometer wire, the numbers on its scale running from A to B. Connect an auxiliary battery, as shown in the figure, to the points A and B, and, for convenience in calculation, always attach the positive terminal to A. This battery has nothing to do with the E.M.F.'s to be compared; its function is to maintain a perfectly steady P.D. between A and B, which must be greater than either of the E.M.F.'s under examination. Hence, it must be capable of giving out a small but steady current for some

[1] Students should use a Daniell's cell as a standard until they have gained some experience, otherwise they may seriously injure an expensive standard cell. Set up with care, its E.M.F. is exceedingly constant within narrow limits.

time. For many purposes two Daniell's cells in series, or one accumulator, is convenient. If r be the resistance of the potentiometer wire, and C the current through it, then the P.D. between A and B is Cr volts.

Let us, for a moment, think of the current as analogous to a stream of water flowing from A to B in consequence of a difference of level between A and B. Then, if any kind of side channel were to be joined on, say to the points A and D, it is evident that the stream would divide at A into two parts both flowing towards D, and the nearer the point D was to B, the greater would be the difference of level tending to urge the water along the new path. We may now suppose that something of the nature of a force pump, driven steadily by some power, is placed in this path. This may evidently be arranged to work in either direction. In one, it merely assists the stream to flow from A to D; in the other, it opposes the flow along the new path, and, according to its power, it may diminish or even reverse the flow. Hence, there will be one case in which it will just stop the flow, in which case, its effect is exactly balanced by the difference of level between A and D. If another more powerful pump be substituted for the first, the side channel might have to reach F in order to satisfy the condition of no flow, and thus we see that the lengths AD and AF might be made a measure of the power of the two pumps.

The electrical case is exactly similar.

Exp. 178. Connect up, *in turn* (as shown in Fig. 232), the two cells whose E.M.F.'s are to be compared, and alter the point of contact until the galvanometer shows no deflection. Let this be at D in the first case, and at F in the second. Then, as no current is flowing through the cells to be compared, the P.D. between their terminals is identical with the E.M.F., and we have

$$E = \text{P.D. between A and D} = C \times \text{resistance of AD}$$
$$E_1 = \text{,,} \quad \text{,,} \quad \text{A and F} = C \times \text{resistance of AF.}$$

Here, C is the steady current in the wire and is the same in each case.

$$\therefore \quad \frac{E}{E_1} = \frac{\text{resistance of length AD}}{\text{resistance of length AF}}$$

Now, the resistance of a uniform wire is proportional to its length, and

$$\therefore \quad \frac{E}{E_1} = \frac{\text{length AD}}{\text{length AF}} = \frac{d}{d_1} \text{ when } d \text{ and } d_1 \text{ are the scale readings.}$$

It will be noticed that the argument fails unless the current in the potentiometer wire is strictly constant during the two readings, and as slight changes may occur, it is desirable to take the second reading as soon as possible after the first.

Fig. 233 shows a convenient arrangement for the apparatus, where a simple switch is formed by removing the cross wires from a reversing commutator (see p. 253), and one cell is exchanged for the other by moving over the rocker.

Accuracy in obtaining balance evidently depends upon the sensitiveness of the galvanometer. If a very delicate instrument is used, it should be shunted until the approximate point of balance is obtained.

If a Daniell's cell be used as a standard, no special precautions need be taken to protect it, but if a Clark or cadmium cell be used, it is necessary to introduce a high resistance, say 20,000 ohms, in the galvanometer circuit, in order to ensure the cell against injury. This resistance does not affect the position of the point of balance, but lessens the sensitiveness of the test, and it may be cut out during the final adjustment.

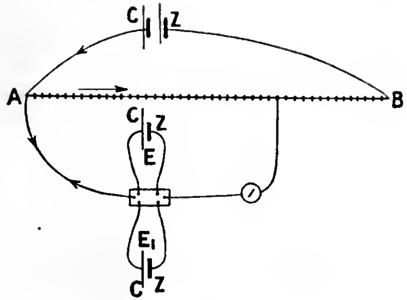


FIG. 233.

The great advantage of the method is due to the fact that the cells, whose E.M.F.'s are to be compared, are not giving out current during the measurement (except very small currents during adjustment), and hence, it gives accurate results even with cells which polarise readily. A second advantage is due to its being a "null" method, *i.e.* it depends on reducing a deflection to zero, a process which can always be carried out with greater exactness than the reading of a deflection.

Measurement of Current by means of a Potentiometer.—This is the most generally convenient method of measuring current for such purposes as checking ammeters, &c. It depends upon the use of standard resistances of known values. For instance, let such a resistance R be placed in series with the ammeter, and a steady current sent through both. Also let the cell E_1 (Fig. 233) be removed, and the wires leading to it connected instead to the extremities of R (taking care that the connections are made so that the current flows through R in the right direction). Then the P.D. across R may be measured by comparing it with the E.M.F. of the standard cell E. If the result is e volts, the current through R and the ammeter will be $\frac{e}{R}$.

The standard resistances used for such purposes are of a special type, as they must be sufficiently massive to carry the current without getting hot enough to be injured or to have their resistance seriously affected by the rise in temperature. Hence, ordinary resistance boxes are as a rule useless. Again, it is desirable that the P.D. to be measured should be of the order of a volt or so, in order to facilitate accurate comparison with the standard cell. Roughly speaking, the standard resistances would be about $\frac{1}{10}$ ohm to measure currents up to 10 amperes, and $\frac{1}{100}$ ohm and $\frac{1}{1000}$ ohm to measure currents up to 100 and 1000 amperes respectively. It is unnecessary to enter into the details of construction here, but it may be remarked that they are usually provided with massive terminals for leading the current in and out, and a pair of "potential leads" for connecting to the potentiometer. These are small flexible connections soldered to the conductor at definite points, between which the resistance is adjusted to have the required value. In this way the varying effects of bad contact at the ends are eliminated; a matter of great importance in the case of such small resistances.

Lord Rayleigh's form of Potentiometer.—A modified form of potentiometer due to Lord Rayleigh is worthy of notice. In this case the potentiometer wire is replaced by two resistance boxes, R_1 and R, arranged as shown in Fig. 234. A steady current is obtained as usual from auxiliary cells, and each of the cells to be compared is *in turn* connected up to one of the boxes (R_1 in figure) in such a direction as to oppose the P.D. across it. The resistances are then adjusted until the galvanometer shows no deflection, *but in such a way that the total resistance of both boxes is kept constant.* If R_1 and R_2 are the values giving balance with cells E_1 and E_2, then, evidently,

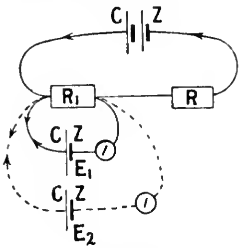


FIG. 234.

$$\frac{E_1}{E_2}=\frac{R_1}{R_2}$$

(A convenient value for the constant total resistance is usually about 10,000 ohms.)

Other Methods of comparing E.M.F.'s.—Many other methods of comparing E.M.F.'s might be given, which are now seldom or never used in practice. As a rule, they require the cells to give out a current during measurement, and hence are subject to serious errors owing to polarisation, and to possible changes in internal resistance due to the passage of a current. A few such methods will be briefly described, mainly as affording simple exercises on Ohm's law.

Lumsden's Method.—Example 6, given on p. 226, illustrates the principle involved in Lumsden's or Bosscha's method. For if the two cells, whose E.M.F.'s are to be compared, are arranged as shown in Fig. 235 (also compare with Fig. 170), and the adjustable known resistances r and r_1 are altered until the galvanometer shows no deflection when the key is pressed down, we have, from the reasoning giving in Example 6,

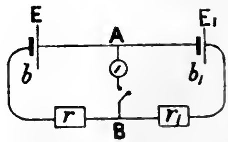


FIG. 235.

$$\frac{E}{E_1}=\frac{b+r}{b_1+r_1}$$

where b and b_1 are the internal resistances of the two cells respectively. These are not known, but if we make r and r_1 very large compared with b and b_1, we have practically

$$\frac{E}{E_1}=\frac{r}{r_1}$$

A much better plan, however, is, after taking the first readings, to alter r and r_1 considerably, and again obtain a balance. If the new values be R and R_1, then

$$\frac{E}{E_1}=\frac{b+R}{b_1+R_1}$$

By a well-known theorem in algebra, we can eliminate b and b_1, thus

$$\frac{E}{E_1}=\frac{(b+R)-(b+r)}{(b_1+R_1)-(b_1+r_1)}=\frac{R-r}{R_1-r_1}$$

Wheatstone's Method.—This is the oldest of all methods, and is noteworthy as not requiring a galvanometer *of known law*, i.e. one for which the relation between current and deflection is known.

Exp. 179. (1) Connect up one of the cells in series with a galvanometer and resistance box, and adjust box until a convenient deflection of $d°$ (about 50°) is obtained. If R be the *total* resistance of circuit (which is unknown), and C the current, we have

$$C=\frac{E}{R}\quad \text{and gives a deflection } d°$$

(2) Increase the resistance by a known amount r so that the deflection is reduced by 10° or 12°, then

$$C_1=\frac{E}{R+r}\quad \text{and gives a deflection } d_1°$$

(3) Remove the cell of E.M.F.$=$E, and replace it with the other cell of E.M.F.$=E_1$, and alter the resistance *until the original deflection $d°$ is obtained.* Then the current must be the same as in (1). The total resistance is unknown; let it be R_1,

$$\text{then}\quad C=\frac{E_1}{R_1}$$

(4) Increase the resistance by a known amount r_1 until the deflection is again $d_1°$. Then the current must be C_1 as in (2), and

$$C_1=\frac{E_1}{R_1+r_1}$$

Now from (1) and (3)

$$C=\frac{E}{R}=\frac{E_1}{R_1}$$

$$\therefore\quad \frac{E}{E_1}=\frac{R}{R_1}$$

And from (2) and (4)

$$\frac{E}{E_1}=\frac{R+r}{R_1+r_1}$$

$$\text{whence}\quad \frac{E}{E_1}=\frac{(R+r)-R}{(R_1+r_1)-R_1}=\frac{r}{r_1}$$

i.e. the E.M.F.'s to be compared are in the ratio of the added resistances.

In a particular experiment, with a chromic acid cell, the resistance required to obtain a deflection of 52° was 30·5 ohms. This resistance was then increased to 50·5 ohms, when the deflection was 40°. The added resistance of 20 ohms thus diminished the deflection by 12°.

A Daniell's cell was then substituted for the chromic acid cell. To obtain a deflection of 52° a resistance of 10 ohms was required, while to bring the deflection down to 40° the resistance was increased to 21·5 ohms; the extra resistance, therefore, was 11·5 ohms. Now,

$$\frac{E}{E_1}=\frac{r}{r_1}$$

$$\therefore\quad \frac{1{\cdot}07}{E_1}=\frac{11{\cdot}5}{20}$$

$$\therefore\quad E_1=\frac{20\times 1{\cdot}07}{11{\cdot}5}=1{\cdot}86\text{ volts.}$$

We can now perceive the practical defects of this and similar methods. In the first place, it would be absurd to apply it to cells which polarise readily, and it is usually such cells whose E.M.F. is required. For the same reason, a standard cell of the Clark or cadmium type cannot be used. Again, it is difficult, under the usual conditions of work, to exactly reproduce a certain deflection with great accuracy.

Sum and Difference Method.—The preceding remarks also apply largely to this method. It differs from the last method in requiring a galvanometer of known law, *e.g.* a tangent galvanometer.

Exp. 180. Connect the two cells in series with a tangent galvanometer (Fig. 236), if necessary including a resistance box to adjust the deflection to a convenient value. Let this be $\theta°$. Without altering the resistance, reverse one of the cells (Fig. 237), and again note the deflection. Let this be $\theta_1°$. Then if E and E_1 are the respective E.M.F.'s, and R the total resistance of the circuit (which is the same in both cases), we have

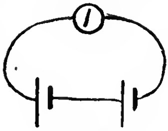


FIG. 236.

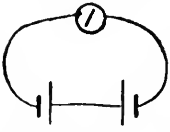


FIG. 237.

$$C=\frac{E+E_1}{R}=K\tan\theta$$

$$C_1=\frac{E-E_1}{R}=K\tan\theta_1$$

$$\text{or} \quad \frac{E+E_1}{E-E_1}=\frac{\tan\theta}{\tan\theta_1}$$

$$\text{i.e.} \quad \frac{E}{E_1}=\frac{\tan\theta+\tan\theta_1}{\tan\theta-\tan\theta_1}$$

In a particular experiment, when the two cells were arranged in series the deflection was 36°; when they were in opposition it was 11°. Now, tan 36° = ·7265 and tan 11° = ·1944.

$$\text{Whence, substituting, we have } \frac{E}{E_1}=\frac{\cdot7265+\cdot1944}{\cdot7265-\cdot1944}$$

$$=\frac{\cdot9209}{\cdot5321}$$

Again, taking the E.M.F. of the Daniell's cell as 1·07 volt, we have

$$E=\frac{\cdot9209\times1\cdot07}{\cdot5321}$$

$$=1\cdot85 \text{ volts.}$$

Electrometer Method.—If an electrometer of the quadrant type is available, electromotive forces may be compared with great ease, once the instrument is set up. It is only necessary to connect each of the cells *in turn* to the terminals of the instrument, and to read the steady deflections thereby produced. If these are d and d_1 divisions respectively, then

$$\frac{E}{E_1}=\frac{d}{d_1}$$

Any kind of standard cell may be used, and, as the cells are always on open circuit, the method is ideally perfect as regards freedom from polarisation. It is inferior to the potentiometer method, because it is more difficult to read a certain deflection accurately than it is to determine zero deflection accurately; and further, a quadrant electrometer is a much more troublesome instrument to use than a potentiometer.

Voltmeter and Allied Methods.—The quickest method of measuring the E.M.F. of a given cell is evidently to connect it up to a voltmeter of suitable range, and to take the reading. Where no great accuracy is required, this is usually done, but the method must be applied with caution. It is not merely that the calibration may be incorrect, for, even if we assume that it is perfect in this respect, the readings may be seriously wrong unless the instrument has a very high resistance, and this, with certain types of voltmeters, is not always possible. As an example (a) let us consider a certain instrument reading up to 2·5 volts, and having a resistance of 12 ohms. Suppose that we connect it up to a cell, otherwise known to have an E.M.F. of 1·5 volts, and an internal resistance of 3 ohms. The current flowing will be $C = \frac{1{\cdot}5}{12+3} = \frac{1}{10}$ ampere. Now, the "drop" due to the internal resistance will be $Cr_b = \frac{1}{10} \times 3 = {\cdot}3$ volt, so that the P.D. at the terminals of the cell will be $1{\cdot}5 - {\cdot}3 = 1{\cdot}2$ volts, and, if the instrument is correctly graduated, this will be the reading—an error of 20 per cent. The student should notice that this source of error is not due to polarisation. It is entirely different in nature.

(*b*) If, however, the resistance of the voltmeter had been 120 ohms, the current would have been $\frac{1{\cdot}5}{123} = \frac{1}{80}$ ampere nearly, and the "drop" would have been $\frac{1}{80} \times 3 = {\cdot}0375$ volt, which would give a reading of $1{\cdot}5 - {\cdot}0375 = 1{\cdot}46$ volt—a fair approximation to the correct value.

It would, however, be unusual for a voltmeter of this range to have so high a resistance, and hence the readings, when applied to a cell, are always slightly too low, although they give quite correctly the P.D. between the terminals under the circumstances.

Method of using a Reflecting Galvanometer as a Voltmeter.—The student has just learnt that the higher the resistance of a voltmeter, the more nearly the P.D. at the terminals becomes equal to the E.M.F., the limit being reached only when that resistance is infinite, as in the quadrant electrometer, which is really only a special form of electrostatic voltmeter.

Now, suppose that a reflecting galvanometer—preferably a dead-beat suspended-coil instrument—of average sensitiveness is (*a*) put in circuit with a resistance of, say, 20,000 ohms at least, and (*b*) shunted with another and variable resistance. Neither of these resistances need be known, but ordinary resistance boxes are convenient for the purpose. The arrangement is shown in Fig. 238. The two resistances are to be regarded as *forming a constituent part of the instrument*, whose terminals are the ends of the connecting wires TT. Then, whatever the actual resistance of the galvanometer may be, the presence of 20,000 ohms in series with it makes the working resistance sufficiently high. If, however, this were used alone, a cell connected to TT would give too large a deflection—perhaps enough to damage the instrument—but the presence of the shunt-box enables the sensitiveness to be adjusted with great nicety, so that a convenient deflection is obtained. (Perhaps it may be pointed out that all such adjustments are to be made with the shunt-box; the other resistance not being altered.) The arrangement is equivalent to an uncalibrated voltmeter of high resistance, and the readings will be proportional to the P.D. between the points to which it is connected.

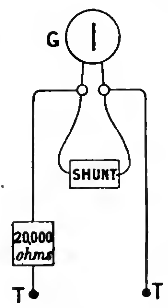


FIG. 238.

Exp. 181. Let each of the cells under examination be connected to TT, *in turn*, giving deflections d and d_1 divisions respectively, then if R = resistance of the instrument between the two terminals, we have

$$C = \frac{E}{b + R} \propto d$$

$$\text{and} \quad C_1 = \frac{E_1}{b_1 + R} \propto d_1$$

where b and b_1 are negligible compared with R

$$\therefore \quad \frac{E}{E_1} = \frac{d}{d_1}$$

This method is especially convenient when it is desired to demonstrate rapidly to a class the fact that different combinations of metals give different electromotive forces when used in simple cells.

Exp. 182. After arranging the apparatus, as shown in Fig. 238, standardise the scale by connecting a Clark's cell to the terminals TT, and adjusting the deflection by varying the shunt until it is 143·4 scale divisions (as nearly as possible). As the E.M.F. of a Clark's cell is 1·434 volts, a deflection of 100 scale divisions means 1 volt, and so the values corresponding to any deflection can be read off at once, *e.g.* a deflection of 95 divisions means ·95 volt. Now, read off the E.M.F.'s due to zinc and copper, zinc and lead, zinc and iron, or any other combination of metals in dilute sulphuric acid or in other liquid.

Results are obtained by this means with very fair accuracy for a class experiment. Moreover, any small pieces or wires of the various materials may be used. It is also excellent for showing that the E.M.F. of a cell does not depend upon the *size* of the plates—the plates of a cell may be raised out of the solution, but the reading does not alter until they break contact.

Methods of measuring the Internal Resistance of Cells.—Several of the previous arrangements may be readily modified to measure internal resistance, and hence it is convenient to consider them here.

The most generally applicable method depends on measuring the ratio of the P.D. at the terminals on open circuit to the P.D. at the terminals when a current is flowing through a known external resistance. For example,

Exp. 183. Using the last arrangement, but without taking the trouble to standardise the scale, join up the cell whose internal resistance is to be measured, and obtain a steady deflection d divisions.

Now connect, across the terminals of the cell, a known external resistance of from 2 to 4 ohms, and again read the deflection. Let this be d_1 divisions. It will be less than before (the external resistance should be chosen so that d_1 is about $\frac{2}{3}$ of d.)

Then, if E = the E.M.F. of the cell, r_b = the internal resistance, $r_\times$ = the known resistance, and $e_\times$ = the P.D. between the terminals when $r_\times$ is in circuit, we have

$$E \propto d \text{ (i.)}, \quad \text{and } e_\times \propto d_1 \qquad \text{(ii.)}$$

whence, by subtraction,
$$E - e_\times = d - d_1 \qquad \text{(iii.)}$$

and dividing (ii.) by (iii.), we obtain

$$\frac{e_\times}{E - e_\times} = \frac{d_1}{d - d_1}$$

$$\text{but} \quad e_\times = Cr_\times, \quad \text{and} \quad E - e_\times = e_b = Cr_b$$

$$\therefore \quad \frac{r_\times}{r_b} = \frac{d_1}{d - d_1}$$

$$\text{whence} \quad r_b = r_\times \left(\frac{d - d_1}{d_1} \right)$$

It must be pointed out that, if polarisation occurs, a serious error may be introduced into the second reading (taken while the current is flowing), hence the method is strictly applicable only to constant cells. But if the galvanometer used is sufficiently dead-beat, the second reading may be taken almost at the instant the circuit is closed, *i.e.* before the cell has had time to polarise seriously.

The same measurements may obviously be carried out whatever be the particular method adopted for obtaining the ratio E to $e_\times$.

A potentiometer is very suitable for the purpose, an experimental arrangement being shown in Fig. 239. A balance is obtained with the key open, and then with the key closed. In practice, a resistance box is used for $r_\times$, and its infinity plug for the key.

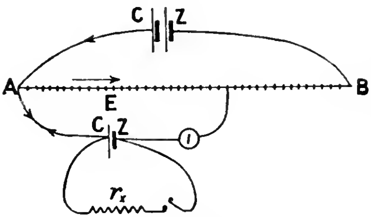


FIG. 239.

If d and d_1 are the two positions of balance, the argument is the same as before.

It is, however, not very easy to obtain the second reading quickly, and, therefore, the method is suitable only for constant cells. To obviate this difficulty completely, the condenser method may be employed.

Use of Condenser.—A condenser suitable for such purposes is made by interleaving many sheets of tinfoil with larger sheets of paraffined paper, or better still, of mica, as dielectric.

As explained on p. 57, the result is essentially the same as a Leyden jar, but, as the coatings are larger and the dielectric thinner, the capacity of even a small condenser is much greater than that of a large Leyden jar; on the other hand, it is intended to stand only comparatively small differences of potential. To charge it from an electrical machine would probably break it down at once, owing to the sparks piercing the insulation.

Let this condenser be connected up to a cell and a ballistic[1]

[1] Practically any galvanometer that is not too "dead-beat" may be used, provided it is sufficiently sensitive.

galvanometer, as shown in Fig. 240. The normal position of the key is as shown; when depressed the condenser is charged by the cell until the P.D. between its coatings is equal to the E.M.F. of the cell. On releasing the key, the cell circuit is opened and the condenser discharged through the galvanometer, producing a swing or "throw" of the needle. For the present purpose, it is sufficient to say that, with a reflecting galvanometer, the first swing of the needle in scale divisions is proportional to the "quantity" discharged through the galvanometer.

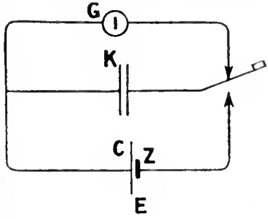


FIG. 240.

Condenser Method of comparing two E.M.F.'s.—**Exp. 184.** Connect up each cell in turn (as explained above) and obtain throws of d and d_1 divisions. If K be the capacity of the condenser,

$$Q = EK \propto d$$

$$\text{and} \quad Q_1 = E_1K \propto d_1$$

$$\therefore \quad \frac{E}{E_1} = \frac{d}{d_1}$$

The equation Q = EK is identical with the equation Q = VC discussed on p. 50, but, when dealing with voltaic measurements, the units of potential, quantity, and capacity are different, and in order to suggest the change, as well as to avoid confusion between the use of C for capacity and for current, we shall write Q = EK, instead of Q = VC.

This method of comparing E.M.F.'s is absolutely perfect with respect to freedom from polarisation. Its only disadvantage lies in the fact that it is necessary to read a "throw" in scale divisions, which means that the possible accuracy is less than that obtainable by using the potentiometer method, which is a *null* method.

Comparison of Two Capacities by the Condenser Method.—If two capacities are to be compared—not differing too widely in magnitude—it is only necessary to use *each in turn* with the *same cell*, obtaining throws of d and d_1, then

$$Q = EK \propto d \quad \text{and} \quad Q = EK_1 \propto d_1$$

$$\text{or} \quad \frac{K}{K_1} = \frac{d}{d_1}$$

Measurement of Internal Resistance by Condenser.—Evidently, we may use a condenser to obtain the ratio of the quantities E and $e_\times$ (as in Experiment 183), and the peculiar advantage of the method lies in the fact that it is possible to obtain the throw corresponding to $e_\times$ at the instant the circuit is completed through the resistance $r_\times$, *i.e.* before the cell has had time to polarise, and hence the method is applicable to all kinds of cells, being especially advantageous with those which polarise quickly.

Exp. 185. The connections may conveniently be made as shown in Fig. 241. In this case the throw observed is that due to the *charging* and not to the *discharging* of the condenser (although for our purposes either may be used indifferently in this and in previous experiments). When obtaining the first throw, remove the infinity plug of the resistance box. We get, as before, $E \propto d$.

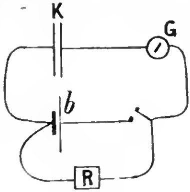


FIG. 241.

Then unplug 2 or 3 ohms in the box, insert the infinity plug, and again take the throw. This will give $e \times \propto d_1$, and the argument is then identical with that already given on pp. 311, 312.

It will be noticed that by closing the key, we *simultaneously* close the cell circuit and charge the condenser.

Condenser Key.—In experiments with condensers, it is convenient to use a special type of key, known as a "condenser key," of which a simple form is shown in Fig 242. A strip of brass, A, is fixed at B, and when depressed it makes contact with a metal block carrying a terminal, C. At the same time it breaks contact with the screw, D, which is carried by a metal bridge-piece, and which is adjustable to any height. When A is allowed to fly up again, it breaks contact at C and makes it again at D. The working parts are supported by ebonite pillars, in order to provide the high insulation required for certain purposes. Such a key can be connected up in a number of different ways according to the purpose for which it is to be used. Fig. 243 shows diagrammatically the connections required in Experiment

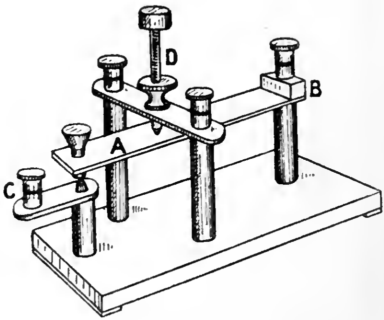


FIG. 242.

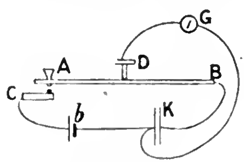


FIG. 243.

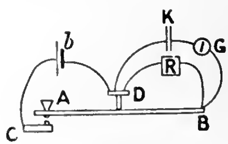


FIG. 244.

184, and Fig. 244 those required for the measurement of internal resistance in Experiment 185. A, B, and C have the same meaning as in Fig. 242, b is the cell, G is the galvanometer, and K is the condenser.

Reduced Deflection Method.—For the sake of comparison, we may mention an older method of measuring internal resistances, sometimes called the "reduced deflection method."

Exp. 186. Connect up the cell, whose internal resistance is to be measured, in series with a tangent galvanometer and a resistance box. Adjust the resistance until a deflection of about 50° is produced. Let this be $\theta°$, and r the resistance unplugged in the box. Increase the resistance until the deflection is reduced by 10° to 12°, and let θ_1 be the new deflection and r_1 the resistance in the box. Then, if g be the resistance of the galvanometer and connecting wires, and b the internal resistance of the cell, we have—

$$C = \frac{E}{b+r+g} = K \tan \theta$$

$$C_1 = \frac{E}{b+r_1+g} = K \tan \theta_1$$

$$\therefore \quad \frac{b+r_1+g}{b+r+g} = \frac{\tan \theta}{\tan \theta_1}$$

$$\text{or} \quad (b+r+g) \tan \theta = (b+r_1+g) \tan \theta_1$$

$$\therefore \quad b(\tan \theta - \tan \theta_1) = (r_1+g) \tan \theta_1 - (r+g) \tan \theta$$

$$\text{or} \quad b = \frac{(r_1+g) \tan \theta_1 - (r+g) \tan \theta}{\tan \theta - \tan \theta_1}$$

In a particular experiment a deflection of 53° was produced with 10 ohms, and a deflection of 40° with a resistance of 20 ohms. The resistance of the galvanometer and connecting wires was ascertained to be 1·25 ohms. Now, tan 40° = ·839 and tan 53° = 1·327, whence from the above formula—

$$b = \frac{\cdot 839(20+1\cdot 25) - 1\cdot 327(10+1\cdot 25)}{1\cdot 327 - \cdot 839}$$

$$= \frac{17\cdot 82875 - 14\cdot 92875}{\cdot 488}$$

$$= 5\cdot 9 \text{ ohms nearly.}$$

Not only is this method tedious and solely applicable to constant cells, but it is also liable to very large errors if the *total resistance* of the circuit is much greater than b. If, for instance (using a galvanometer of low resistance), the resistance of the same Daniell's cell be measured repeatedly, first with small and then with large values of r and r_1, the results will be found to be very variable. If b is about 1 ohm, and if $r = 0$ and $r_1 = 1$ ohm, or if $r = 1$ ohm and $r_1 = 2$ ohms, the result will be fairly correct. If, however, we make $r = 60$ ohms and $r_1 = 100$ ohms, the result will probably be absurd.

This is because the effect of b on the deflections is negligible compared with that of r and r_1, and is less than the unavoidable errors of reading.

When the galvanometer has several coils to choose from, as is usually the case, these considerations will indicate the one to be selected.

Exercise XVI

1. Describe and explain a method of comparing an E.M.F. of two voltaic batteries.

2. Two batteries, known to have different electromotive forces, are found to give equal deflections when joined up, one at a time, with the same tangent galvanometer. Explain how you could find out by experiment which battery has the greater E.M.F., and what proportion the E.M.F. of one bears to that of the other.

3. A Daniell's cell and a Grove's cell were arranged in series with a tangent galvanometer. When their electromotive forces were acting in the same direction, the deflection was 42° ; when in opposition, 14°. Taking the E.M.F. of the Daniell's cell as 1·07 volts, find that of the Grove's cell. Tan 42° = ·9004 ; tan 14° = ·2493.

4. Compare the electromotive forces of two batteries, A and B, if when they are successively connected in series with a tangent galvanometer and a resistance box, the deflection is 20°, and that to reduce the deflection to 10°, 3 ohms must be added when A is in circuit, and 5 ohms when B is in circuit.

5. A cell was connected in series with a tangent galvanometer and a resistance box. When the resistance of the external circuit and galvanometer was 1·4 ohm a deflection of 45° 30′ was obtained, but when an ohm was added to the external resistance the deflection was 32° 20′. Find the internal resistance of the cell. Tan 45° 30′ = 1·0176 ; tan 32° 20′ = ·6331.

6. A cell was arranged in series with a tangent galvanometer and a resistance box. A deflection of 40° was obtained with a resistance of 8 ohms, and a deflection of 35° with 10 ohms. The resistance of galvanometer and connecting wire was ascertained to be 1·05 ohms. Find the internal resistance of the cell. Tan 35° = ·7002 ; tan 40° = ·8391.

7. Two cells, the E.M.F.'s of which are 2 : 1, are joined up in series, with their E.M.F.'s acting in the same direction, and the circuit is completed through a tangent galvanometer, the needle of which is deflected through 60°. If one of the cells is reversed, no other change being made, what will be the deflection of the galvanometer? (B. of E., 1896.)

8. A wire AB, of ·33 ohm resistance, forms part of a circuit through which an electric current flows in the direction from A to B. The points A and B are also connected by another conducting path in which is included a cell of E.M.F. 1·287 volts and a galvanometer, the positive pole of the cell being that joined to A. If the galvanometer is not deflected, what is the strength of the current in wire AB? (B. of E., 1897.)

9. The zinc poles of two batteries, A and B, are connected by a wire, and likewise the platinum poles by another wire. When the poles of the battery A are also connected with each other by a wire whose resistance is to that of the battery A itself as 7 : 3, there is no current in the battery B. Show what relation the E.M.F. of the battery A bears to that of the battery B.

CHAPTER XXI

ELECTROLYSIS

CHEMICAL actions, similar to those already described as taking place in the cells themselves, are also produced outside the cells when a current is passed through certain liquids. Neglecting mercury and molten metals, which behave exactly like solid metals and require no special notice, we may divide liquids into two classes :—

(1) Those incapable of conducting a current.

(2) Those which conduct, but in a manner which differs from metallic conduction.

In the first category must be placed the more complex liquids such as oils, alcohol, &c., some of which are amongst the most perfect insulators. Pure water must also be included in this group.

In the second category we are chiefly concerned with solutions and fused salts. If, for example, one of the large class of bodies, known as chemical salts (themselves non-conductors in a dry state), be dissolved in a non-conducting solvent, such as water or alcohol, the result is a more or less conducting solution or **electrolyte**. Such a liquid solution behaves quite differently from a metallic conductor, for it can only conduct by being decomposed during the process. Ordinary acids, which may be regarded as hydrogen salts, behave similarly. Such decomposition by the electric current is called **electrolysis**. The conductors, by which the current is conveyed in and out of the solution, are known as **electrodes**; the electrode by which the current enters being called the **anode**, that by which it leaves the **cathode**.

Exp. 187. Join up a battery in series with a galvanometer and dip the ends of the connecting wires in some oil or turpentine. Observe that no deflection is obtained.

Exp. 188. Connect the ends to platinum electrodes dipping into the purest distilled water obtainable. There should be no visible effect at the electrodes, and although a sensitive galvanometer may show that *some* current is passing, it will be extremely small.

Exp. 189. Add to the water a drop of sulphuric acid, and stir. Notice that bubbles of gas now appear at *both* electrodes, and that the galvanometer shows that a current of considerable strength is flowing round the circuit.

Exp. 190. Instead of using the acid, show that similar results are obtained when various salts are dissolved in the water. Try common salt, potassium or sodium sulphate, potassium iodide, &c. Notice that no action is visible anywhere except at the electrodes.

When the apparatus used in such experiments is designed to collect the substances liberated, so that they can be weighed or measured, it is known as a **voltameter**. A lecture-table form, especially useful with acid solutions, is shown in Fig. 245. An alternative form of apparatus can be easily made as follows :—

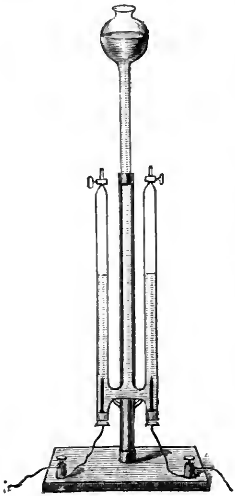


FIG. 245.

Obtain a glass funnel, five or six inches in diameter across the top. File off the stem at a point about half an inch from the bottom of the funnel. Solder strips of platinum foil to two copper wires, and then pass the wires from the inside of the vessel through the stem. Arrange the platinum strips parallel to each other, and then fill the stem and part of the funnel with plaster of Paris, so that the pieces of platinum project above it. If the wires are uninsulated, take care that they are not in contact. In order to make the apparatus water-tight, melt some paraffin-wax and pour it over the plaster of Paris.

Electrolysis of Dilute Sulphuric Acid.—**Exp. 191.** Partially fill the vessel with water acidulated with sulphuric acid. Fill two test-tubes of equal size with acidulated water, and invert them over the electrodes. Connect the free ends of the wires to a battery of three to six cells in series. Observe that bubbles of gas rise from the electrodes. After the action has proceeded for a short time, it will be found that the volume of gas (H) in the tube over the cathode is nearly double that (O) in the tube over the anode.

If the apparatus is filled with a solution of sodium sulphate or of potassium sulphate, *exactly the same effect* is obtained as regards the evolution of gas.

Outline of Theory.—When discussing the actions which occur in voltaic cells, we explained that simple salts may be regarded as made up of a metallic portion and a non-metallic portion held together by electric forces, and that, in solutions, these portions to some extent appear to dissociate and to wander about as free *charged* atoms, or groups of atoms, known as **ions**.

This process of ionisation seems to be essential to conduction in liquids (and gases)—if no free ions are present, they cannot conduct. The extent to which a substance ionises in solution depends partly upon the nature of the solvent; and of all known solvents, water is by far the most effective.

Suppose that two inert electrodes, say of platinum, are placed in such a solution, and that an electric field is created between them by connecting them to the terminals of a cell or battery, then evidently any free charged particle, or ion, will tend to move one way or the other along the field according to the sign of its charge.[1] Such motion is slow, owing to the frictional resistance of the water, but there must evidently be a tendency to set up two opposite processions and to cause an accumulation of ions around the electrodes, as far as the counter tendency of diffusion permits. There can, however, be no current in the circuit, and the process must soon stop, unless the ions in contact with an electrode can give up their charge to it, or receive one from it. When this occurs, the particle or group ceases to be an ion (this term, as already explained, being strictly limited to a *charged* particle or group) and displays at once its ordinary chemical properties. What happens next depends upon circumstances. If it has no chemical action either on the electrode or on the solvent, it appears in a free state; but in many cases such chemical affinity does exist, and then it combines with one or the other, or both, the product actually liberated in a free state depending upon the nature of the reaction. It is therefore convenient to distinguish between primary and secondary effects in electrolysis. The former depend only on the nature of the substance; the latter, also upon the nature of the electrodes and upon the experimental conditions, as will be seen when we apply these ideas to the results of experiments. Let us consider the case of dilute sulphuric acid with platinum electrodes, as given in Experiment 191. Then we have:—

Primary action H_2SO_4 becomes $\underset{+\ +}{HH}$ (at cathode) and $\underset{-\ -}{SO_4}$ (at anode).

Secondary action. There is no secondary action at the cathode, because hydrogen does not combine with either platinum or water, and hence it comes off as a gas. At the anode the (SO_4) group or "sulphion" is unable to attack the platinum, and may be regarded as acting on the water present, according to the equation

$$SO_4 + OH_2 = H_2SO_4 + O$$

and hence, sulphuric acid is re-formed in solution and oxygen gas is evolved. (This is the simplest way of considering the matter, although we might think of the SO_4 group as obtaining the necessary negative charge to keep it in solution as an ion by removing the charge from the oxygen in a molecule of water—the oxygen then becoming free and the two hydrogen atoms becoming charged ions.)

These statements (and others made in this chapter) must be regarded merely as a simple outline of the more important facts and not as an exhaustive treatment. Experiment shows that more complex ions may be present in small quantity, sometimes formed by simple ions uniting with undissociated or neutral molecules—a

[1] The products of electrolysis, which appear at the anode, are called anions; those, at the cathode, are called cations.

tendency especially marked in strong solutions—and sometimes representing a partial stage in the process of ionisation. It is possible, for instance, that H_2SO_4 ionises at first into $\underset{+}{H}$ and $(\underset{+\ -\ -}{HSO_4})$, the latter group then breaking up into $\underset{+}{H}$ and $(\underset{-\ -}{SO_4})$; for a few HSO_4 ions appear to be usually present.

The behaviour of water itself is peculiar, on account of its solvent action on all ordinary containing vessels. The early experimenters tried to electrolyse the purest distilled water they could obtain, and they invariably found that acid and alkaline substances were formed in the liquid around the electrodes (acid at anode, alkali at cathode), and for some time it was thought that a current had some power of creating such substances. Sir Humphry Davy finally cleared up the matter by a series of laborious experiments. He found that, when a marble vessel was used to hold the water, the alkali was sodium hydrate and the acid HCl, both due to a trace of NaCl present in the marble; when agate cups were used, the effect was less, but silica was obtained; with gold vessels, nitric acid and ammonia appeared (traced to the surrounding air); and by working in a vacuum, the effect was finally reduced to a minimum, but then the liquid was almost totally non-conducting, although he used fifty cells in series. Hence, all ordinary water is slightly conducting, although it appears that water itself is not an electrolyte.

Exp. 192. Put a current reverser in circuit with a battery, attach *copper* strips to the free ends, and insert them into a beaker of dilute sulphuric acid. Notice that a gas is evolved as before at the cathode—if collected, it proves to be hydrogen—whilst no gas is evolved at the anode, but observe that the liquid immediately around it gradually becomes blue. Reverse the current and notice that the actions are also reversed.

The primary action is unaltered. As before, there is no secondary action at the cathode, but at the anode the SO_4 group combines with the copper to form copper sulphate, which dissolves in the surrounding water. (If the current strength is great compared with the area of the anode, part of the SO_4 may attack the water as in Experiment 191, and some oxygen gas will be given off in addition to the formation of the sulphate).

Exp. 193. Electrolyse a saturated solution of copper sulphate with *platinum* electrodes (as before, two strips dipping into a beaker will do).

Primary action $CuSO_4$ becomes $\underset{+\,+}{Cu}$ (at cathode) and $\underset{-\,-}{SO_4}$ (at anode). Again there is no *secondary action* at the cathode, copper being deposited. (If the cathode surface is sufficiently great compared with the current strength, the copper will be in the form of a firm coherent coating of bright metal; with greater current strength it may be deposited partly as a loose powder.) At the anode, the secondary action will be the same as in experiment 191—oxygen gas being given off and sulphuric acid re-formed in solution. Reverse the current, and notice that the deposited copper returns into solution.

Exp. 194. Repeat the last experiment, using copper electrodes. No apparent effect will be noticed at the anode, but the cathode will become coated with a deposit of copper.

The primary action is the same as in Experiment 193, and as before there is no secondary action at the cathode, copper being deposited upon it; in fact the nature of the cathode is immaterial ; if it is a conductor, copper will be deposited upon it. At the anode, the secondary effect will be the same as in Experiment 192, copper sulphate being formed and entering into solution, which is unaltered in strength.

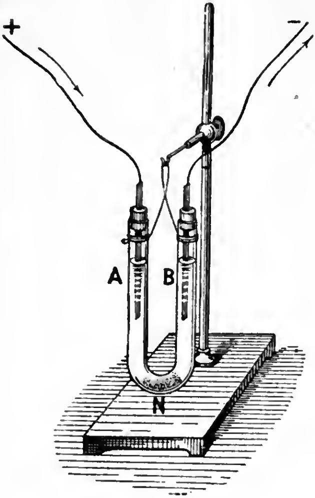


FIG. 246.

Exp. 195. Make a saturated solution of sodium or potassium sulphate, and electrolyse it, using platinum electrodes, in a U-tube, as shown in Fig. 246 (from Carey Foster and Porter's *Electricity and Magnetism*). Gas will be evolved at both electrodes, and we know from Experiment 191 that it consists of oxygen and hydrogen in the proportions to form water. Pour blue litmus solution in each limb. At the cathode, it will remain blue, and at the anode it will turn red. Reverse the current, and notice that these colours are gradually reversed also. These operations may be repeated several times. Hence, an alkaline substance is formed at the cathode, and an acid substance at the anode.

Assuming that potassium sulphate is used, the primary action is evidently K_2SO_4 becomes $\underset{++}{K_2}$ (at cathode) and $\underset{--}{SO_4}$ (at anode).

Secondary actions now occur at both electrodes. At the cathode the liberated metal attacks the water, forming strongly alkaline potassium hydrate, and liberating hydrogen gas, according to the equation

$$K_2 + 2OH_2 = 2KOH + H_2.$$

At the anode, the action is the same as in Experiment 191, and we have (see p. 319)

$$SO_4 + OH_2 = H_2SO_4 + O.$$

Exp. 196. Electrolyse a solution of lead acetate with platinum electrodes. Gas will be evolved at the anode, and feathery crystals of metallic lead will form on the cathode (Fig. 247). Reverse the current, and notice that little or no gas is evolved at the new anode until the previously deposited lead has either redissolved or dropped off.

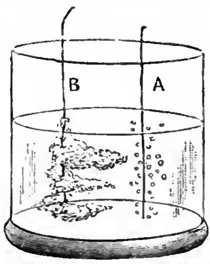


FIG. 247.

This illustrates the tendency of many metals to assume a crystalline form. There is no secondary action at the cathode, but at the anode the radicle of acetic acid behaves like the SO_4 group, taking hydrogen from the water and liberating oxygen, which may again react on the substance to produce a dark red powder,—lead peroxide, PbO_2.

Exp. 197. Repeat the observations, using copper electrodes. Again we find that the nature of the cathode makes no difference, lead crystals being formed as before, but at the anode the solution soon turns greenish blue, owing to the formation of copper acetate.

These experiments sufficiently illustrate the general nature of electrolytic actions. It will have been noticed that the metal (or hydrogen) is invariably liberated at the cathode; this is due to the fact that in simple salts it always carries a positive charge. If an ammonium salt is electrolysed, the group NH_4 behaves like a metal and is set free at the cathode, where it breaks up into ammonia and hydrogen. Sometimes the secondary actions are very complex. If, for example, a not very strong cold solution of ammonium chloride be electrolysed with platinum electrodes, the above action takes place at the cathode, and chlorine gas is evolved at the anode, where it partly dissolves in the liquid and then comes off as gas. But if the solution is warm and saturated, and the current density fairly great, the very dangerous explosive, chloride of nitrogen, is formed. The experiment can be performed safely in an apparatus of the kind previously described and used in Experiment 191, provided a thin layer of turpentine is poured on the top of the liquid (the test-tubes are not required), for, as the chloride is lighter than water, it rises to the surface and instantly explodes on coming into contact with the turpentine. Hence it is never allowed to accumulate, and so results which might be disastrous are avoided.

Practical Applications.—In 1808, Sir Humphry Davy isolated the metals potassium and sodium by electrolysing their hydrates, which were then regarded as elements. The chief difficulty was to exclude secondary actions, and many experiments with solutions in water as well as with the fused hydrate were failures. He succeeded eventually by using a slightly moist piece of hydrate placed on a platinum-plate anode and touched with a platinum-wire cathode. This illustrates the general principle to be adopted in such cases. When secondary actions are unavoidable, on account of the chemical activity of the body required, the details of the experiment should be arranged so that the substance is liberated at a greater rate than secondary actions can remove it. This usually means a small cathode and a relatively large current.

In this way, many years later, Bunsen succeeded in depositing the metals calcium, strontium, and barium from hot aqueous solutions of their chlorides—a result which might well have been deemed impossible. Davy succeeded in obtaining these metals only as amalgams, from which the mercury was afterwards removed by distillation out of contact with oxygen. Such amalgams can be easily obtained by using mercury as a cathode in a solution of any convenient salt of the metal required.

At the present time many substances are produced on a commercial scale by electrolysis, and its applications are being extended daily. Amongst these, we may instance the following:—

Metallic sodium is now almost entirely produced by electrolysing fused sodium hydrate with iron electrodes (Castner process).

Practically all the copper used for electrical purposes is refined by making it the anode in copper sulphate solutions and depositing it on a suitable cathode.

Gold is very largely obtained from poor ores and residues by treating the finely ground material with potassium cyanide. This dissolves the gold, forming a double cyanide of potassium and gold, which is then electrolysed.

Caustic soda and chlorine are made in quantity by the Castner-Kellner method of electrolysing a solution of common salt.

Metallic calcium was obtained many years ago in small quantities and with considerable difficulty by the electrolysis of the fused chloride, but recent improvements in details have enabled it to be put on the market on a commercial scale at a relatively low price. The chloride is contained in a graphite crucible, which also serves as anode, the bottom being kept cool by water. The cathode is an iron rod rather more than an inch in diameter, also water-cooled. At the commencement of the process, the cold chloride is partly fused by forming a temporary arc from the side of the crucible, and, when it begins to conduct, an alternating current is applied until the chloride is completely fused by the heat thereby produced. The alternating current is then cut off, a direct current switched on, and the process begins (the chloride being kept in a state of fusion by the heating effect of the current). It is necessary to keep the end of the cathode only just below the surface, because then the spongy metal first formed melts almost at once, and its surface tension draws it into a compact globule, which is almost instantly solidified by the cooling effect of the cathode. In this way, by raising the cathode very steadily, a solid stick of calcium is gradually built up. If the cathode dips too far below the surface, the spongy metal is apt to be swept away by convection currents, and therefore lost.

Aluminium is obtained by electrolysing the oxide (alumina), dissolved in fused cryolite (a naturally occurring double fluoride of aluminium and sodium). This metal is remarkable for several reasons. It has never been successfully deposited from an aqueous solution, so that aluminium plating is unknown; and when used as an anode in many electrolytes, it offers a very considerable resistance to the passage of a current. If, for example, electrodes of aluminium and lead (or iron) are placed in a solution of ammonium phosphate, the current passes readily when the aluminium is the cathode, but when it is made the anode, the cell is almost non-conducting. It, therefore, acts like a valve, and may be used to obtain a direct current from alternating mains. When used for this purpose, the device is known as an **aluminium rectifier**. The best plan is to use four such cells, arranged as shown in Fig. 248, for then both directions of the alternating current are made available. A and L denote aluminium and lead respectively; MM are the alternating mains; TT the points from which the direct current is taken off to some load (generally accumulators, which cannot be charged with an alternating current). Remembering that A acts only as a cathode, *i.e.* the current through the cells must go from L to A, it will be found, on tracing out the path of the current, that its direction in the load circuit is the same, whichever way it may be flowing from the mains. In practice, an adjustable resistance would be placed in series with the mains, in order to prevent an excessive current at starting, for the action appears to depend on the formation of a thin non-conducting layer of oxide at the aluminium anodes, and this takes a few minutes to form. Then the resistance is cut out. A four-cell rectifier will easily stand alternating pressures of from 100 to 150 volts, and greater voltages may be dealt with by increasing the number of cells; in fact, in this way one of the writers has for several years worked up to 2000 volts. When the rectifier is in good order, the current strength is about the same on both alternating and direct current sides, but the voltage on the latter is only about half that on the former. This means that there is only 50 per cent. efficiency at the best, but in practice, it is not likely to average more than 30 to 40 per cent. One of the most troublesome defects is due to the heating of the cells, for as the temperature rises, the rectifying property diminishes. It is, therefore, necessary to make the cells rather large for a given current output. In any case, they are somewhat troublesome to keep in order, but they are often convenient when no other way of obtaining a direct current is available.

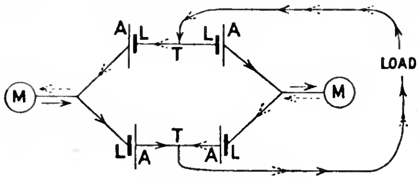


FIG. 248.

Two other practical applications may be mentioned :—

1. *Electrotyping*, by means of which impressions of coins, wood engravings, &c., are obtained.

2. *Electroplating*, by which the surface of a base metal—*e.g.* German silver or copper—is covered with a superior metal—*e.g.* silver or gold —either for the purpose of protecting the article from oxidation, or to give it the appearance of being wholly composed of the superior metal.

Electrotyping.—In order to obtain a copper electrotype of any object, a mould must first be made, on which the layer of metal is to be deposited. For objects, such as medals, which can be submitted to pressure, guttapercha may be advantageously used for this purpose. It is softened in hot water, pressed upon the object, and then allowed to cool. For wood blocks or type, wax moulds are commonly used, which are made by pouring a mixture of wax, tallow, and Venice turpentine into a shallow vessel, and, before it completely sets, pressing the block or type upon it. Moulds of plaster of Paris, or of a fusible alloy, are sometimes used. It is very important that the face and edges of these moulds should be covered very carefully and thoroughly with graphite, so as to make the surfaces conduct. The mould is placed in a saturated solution of copper sulphate, and then made the cathode of a Daniell's cell or battery, while a copper plate forms the anode, which, gradually dissolving in the solution, keeps it at a constant strength.

Electroplating.—This term includes :—

(*a*) *Electro-gilding*, the process by which gold is deposited on a baser metal. The bath is a solution of the double cyanide of gold and potassium. In order that the gilding bath may be of constant strength, the anode consists of a gold plate, which dissolves at a rate equal to that at which the gold is deposited on the cathode.

(*b*) *Electro-silvering.*—In this process, the bath consists of the double cyanide of silver and potassium, and the anode is a silver plate.

(*c*) *Electro-nickeling.*—The bath in this case consists of a solution of the double sulphate of nickel and ammonium (made slightly acid).

The principle of these processes will be understood from the following method of coating a German silver spoon with silver. The spoon must be thoroughly cleansed by (1) boiling it in a weak solution of caustic soda to remove grease, (2) washing with water, (3) immersing it for a moment in dilute nitric acid to remove any film of oxide, (4) brushing it with a hard brush, and (5) plunging it in clean water. Two metal rods are placed across the vessel containing the solution, which should be gently warmed while the deposit is being made. The spoon, hung from one of the rods by means of a wire (waxed all over), is made the cathode, while a silver plate, suspended from the other, is the anode.

Faraday's Laws of Electrolysis.—By a most laborious and extensive research, Faraday established the following important generalisations :—

I. *The mass of a substance liberated during electrolysis is proportional to the "quantity" passing through the solution,* i.e. *to the product of the current strength and the time during which the current flows.*

II. *The mass of a substance liberated by a given "quantity" is proportional to its "chemical equivalent" (where chemical equivalent* $= \frac{\textit{atomic weight}}{\textit{valency}}$*).*[1]

[1] *The chemical equivalent* of an element is obtained by dividing its atomic weight by its valency.

The atomic weight of an element is the weight of an atom of the element compared with the weight of an atom of hydrogen. (At the present time atomic weights are usually referred to that of oxygen, taken as 16, but this introduces a source of confusion, and the older definition is used in the table on next page.)

The valency of an element is the atom-fixing or atom-replacing power of the element; *e.g.* when zinc is acted on by sulphuric acid, the chemical action is represented by the equation $Zn + H_2SO_4 = ZnSO_4 + H_2$, in which it is seen that *one* atom of zinc replaces *two* atoms of hydrogen.

The first law may be written

$$W \propto C.t$$

where W is the weight in grams of the substance liberated by a current of C amperes flowing for t seconds,

$$\text{or } W = C.t.z$$

where z is a number which is constant for a given substance, but which varies for different substances. It is called the "*electro-chemical equivalent*" of the substance.

If we put $C=1$, $t=1$, it is evident that z is the weight in grams deposited by 1 coulomb.

From the second law we learn that the quantity which will deposit W grams of hydrogen, will also deposit $W \times \frac{107{\cdot}03}{1}$ grams of silver, or $W \times \frac{64{\cdot}85}{2}$ grams of zinc, *i.e.* if we know the value of z for hydrogen, its value for any other substance will be the product of this value for hydrogen and the chemical equivalent of the substance.

If a substance has two valencies, like iron in ferrous and in ferric salts, it will be deposited by the same current at different rates from the two classes of salts, in the ratio of its chemical equivalents of each.

The following table gives the atomic weight, the valency, the chemical equivalent, and the electro-chemical equivalent of various elements:—

	Atomic Weight.	Valency.	Chemical Equivalent.	Electro-chemical Equivalent (given in Grams per Coulomb).
Hydrogen	1	1	1	·0000104
Potassium	38·86	1	38·86	·0004065
Sodium	23	1	23	·0002403
Silver	107·03	1	107·03	·0011183
Gold	195·7	3	65·23	·0006816
Copper (cupric) . .	63·06	2	31·53	·0003294
,, (cuprous) . .	63·06	1	63·06	·0006588
Mercury (mercuric) .	198·5	2	99·25	·0010370
,, (mercurous) .	198·5	1	198·5	·0020750
Tin (stannic) . .	118·1	4	29·52	·0003084
,, (stannous) . .	118·1	2	59·05	·0006169
Iron (ferric) . .	55·4	3	18·5	·0001930
,, (ferrous) . .	55·4	2	27·7	·0002894
Nickel	58·3	2	29·15	·0003046
Zinc	64·85	2	32·42	·0003388
Lead	205·45	2	102·72	·0010732
Oxygen	15·88	2	7·94	·0000830
Chlorine	35·18	1	35·18	·0003676
Iodine	126	1	126	·0013166
Bromine	79·36	1	79·36	·0008292

It is instructive to combine the two laws into one general equation, which then becomes

$$W = C.t \times \frac{\text{atomic weight}}{\text{valency}} \times \text{a constant}$$

It is evident that this constant is the same for *all* substances, and hence, it is a number of great importance.

Putting unity for C and t we have 1 coulomb, and as the atomic weight and the valency of hydrogen are both unity, it will be seen that this constant is the mass (in grams) of hydrogen deposited by 1 coulomb, *i.e.* it is the electro-chemical equivalent of hydrogen. Its numerical value is ·0000104 very nearly. For our purpose, it is more convenient to write it in the form $\frac{1}{96,000}$, thus getting

$$W = \frac{C.t \times \text{atomic weight}}{96,000 \times \text{valency}}$$ [1]

It follows from this equation that 96,000 coulombs must pass through a solution in order to liberate 1 gram of hydrogen (*i.e.* 96,000 coulombs of *positive* in one direction, and the same quantity of *negative* in the opposite direction), and from our previous statements as to the nature of the process, *this must also be the amount of positive charge carried by* 1 *gram of hydrogen.* Similarly it will take $\frac{96,000}{23}$ coulombs (= 4174 coulombs) to deposit 1 gram of sodium, and this must be the charge carried by 1 gram of sodium; but, as the atom of sodium weighs 23 times as much as the atom of hydrogen, 1 gram of sodium contains only $\frac{1}{23}$ as many atoms as 1 gram of hydrogen, which means that the monad atom of sodium carries the same charge as the monad atom of hydrogen.

The argument applies also to ions (like chlorine), which carry a negative charge, and thus we arrive at the very important conclusion that *every monovalent ion in an electrolytic solution carries exactly the same amount of charge—negative or positive as the case may be.* It also follows that every divalent ion, like oxygen, or copper in cupric salts, carries exactly twice this charge, and so on. These facts suggest that an electric charge is essentially atomic in nature, being made up of small charges, which are incapable of division by any known process, and this conclusion is strongly supported by other evidence derived from the passage of a current through gases.

The charge carried by one monad ion is hence a natural unit of quantity, being the smallest portion of electricity known to exist.

[1] It must be understood that this number (96,000) is given in round figures, as the value of the last three significant figures is somewhat uncertain. The value of z for hydrogen is probably ·00001045, which corresponds to 95,690.

Now the number of atoms in 1 gram of hydrogen has been determined by many different methods, and is known to be about $6{\cdot}16 \times 10^{23}$.

Hence, the natural "atom" of electricity is $\frac{96{,}000}{6{\cdot}16 \times 10^{23}} =$ $1{\cdot}57 \times 10^{-19}$ coulombs, or $1{\cdot}57 \times 10^{-20}$ absolute units of quantity in the magnetic system of units. This is $4{\cdot}68 \times 10^{-10}$ static units of quantity.

(It will be seen later that free negative charges of this magnitude—known as *electrons*—are present in vacuum tubes, and the above values agree with those determined by direct experiment in various ways.)

Ionic Velocities.—If an electrolytic cell be divided into two parts by a porous partition (to prevent diffusion as much as possible) and a current passed for some time, the solution is no longer of the same strength in each compartment. For instance, if a solution of copper sulphate is used with copper electrodes, after a time it is stronger in the compartment containing the anode, although the solution as a whole contains as much copper sulphate as before. This fact was carefully investigated many years ago by Hittorf, who showed that it could be explained by assuming that different ions moved with different velocities, under the influence of the same P.D., and that the ratio of the velocities could be ascertained by measuring the relative strengths of the solution in the two compartments. This will be understood from Fig. 249, which is a modification of a diagram due to Ostwald.

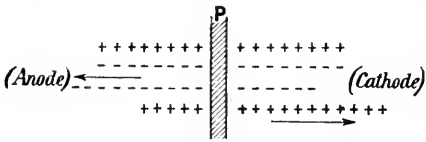


Fig. 249.

P is a porous partition, and the two upper rows represent the state of the solution before the current flows, each compartment containing eight molecules of some simple salt, *e.g.* NaCl.

Let v = velocity of positive ions towards the cathode, and u = velocity of negative ions towards the anode; and assume that $v=3$ and $u=2$. When the current has passed for a certain time, the state of the solution will be as shown in the two lower rows. Now, the electrical conditions determine that charges must be given up to the two electrodes at exactly the same rate, otherwise there would be an accumulation of charge at one or the other. Suppose that five ions are discharged at each electrode; then five molecules out of the original solution have disappeared—three from the anode compartment and two from the cathode compartment.

If the strength of the solution in each compartment is determined experimentally and compared with the original strength, it will be found that

$$\frac{\text{anode loss}}{\text{cathode loss}} = \frac{3}{2} = \frac{v}{u}$$

or anode loss × anion velocity = cathode loss × cation velocity.

Which may be written

$$\frac{\text{anode loss}}{\text{total loss}} = \frac{v}{v+u}, \quad \frac{\text{cathode loss}}{\text{total loss}} = \frac{u}{v+u}$$

If another equation can be found connecting v and u, it will evidently be possible to determine their absolute values. Such an equation was ultimately provided by Kohlrausch; his argument being briefly as follows:—

First let us suppose, for the sake of clearness, that only *one* kind of ion moves—say the positive with velocity v. This would be the equivalent of a current, because an equal number of negatively charged ions would be simultaneously liberated at the other electrode, but as will be seen, by drawing another diagram similar to Fig. 249, all loss of electrolyte must necessarily occur at the electrode from which the ions are moving, which in this case is the anode.

Let there be n charges of *each* sign per cubic centimetre, each of magnitude e coulombs, where e is the charge on one monad ion. (This does not mean n ions unless the valency of each ion is unity; in fact the number of ions will be $\frac{n}{\text{valency}}$.)

Consider a column of liquid 1 square centimetre in section between the electrodes. Then the quantity of charge given up to the cathode per second will be $+nev$ units; an equal charge, $-nev$ units, being simultaneously given up to the anode.

$$\text{But current strength} = \text{quantity passing per second,}$$
$$\therefore \quad \text{current} = nev \text{ amperes.}$$

Secondly, if the other kind of ion moves also in the opposite direction with velocity u, it will, by similar reasoning, also correspond to a definite current *in the same* direction as the previous current, and the diagram will show that if $u = v$, the loss will be the same at each electrode; but if the velocities are unequal, the loss will be greatest near the electrode at which the slower-moving ions are discharged.

The actual current will be the sum of the two components, hence

$$C = nev + neu = ne(v+u) \text{ amperes.}$$

Let l be the distance between the electrodes, and E the PD between them in volts. The resistance of the column in question will be $\frac{l}{1} \times s$, where s = specific resistance of the solution. It is, however, usual to employ specific conductivity in this connection, where specific conductivity $= \frac{1}{s} = K$,

$$\therefore \text{ resistance of column } = \frac{l}{K} \text{ ohms}$$

$$\text{Now } C = \frac{E}{\frac{l}{K}} = \frac{KE}{l} = ne(v+u) \text{ amperes}$$

If E_0 be the P.D. per centimetre of length of solution, then

$$\frac{E}{l} = E_0$$

$$\text{whence } KE_0 = ne(v+u)$$

Now ne is the total charge per cubic centimetre of solution. Let us suppose its strength is N gram-equivalents per cubic centimetre (one gram-equivalent means a number of grams of each ion equal to its chemical equivalent, *i.e.* to its $\frac{\text{atomic weight}}{\text{valency}}$). Then we get the same total charge per cubic centimetre, whatever the nature of the dissolved substance may be;

i.e. each cubic centimetre will contain $N \times 96{,}000$ coulombs of each sign;

$$\therefore \quad ne = N \times 96{,}000$$

$$\text{whence } \quad KE_0 = N \times 96{,}000(v+u)$$

$$\text{or } \quad \frac{K}{N} = \frac{96{,}000(v+u)}{E_0}$$

The ratio $\frac{K}{N}$ is called the *molecular conductivity.* It varies with the strength of the solution, but, for many substances, it approaches a limiting value for very dilute solutions, and it is this value which is to be used in the above equation, for we have, so far, really assumed that *all* the molecules are ionised, which is not the case except in infinitely dilute solutions.

It follows, from the above equation, that v and u are directly proportional to the P.D. per centimetre of length; also, if the limiting value of $\frac{K}{N}$ can be found by measuring the specific resistance of solutions of various strengths (E_0 being easily measured by means of a suitable voltmeter), we can calculate v and u independently by combining it with the ratio determined by Hittorf's method. It is found that the velocity of a given ion in different dissolved salts is always the same for a given P.D. for infinitely dilute solutions.

Of all ions, hydrogen moves the fastest, but even its velocity is very small.

The following table, taken from Carey Foster and Porter's *Electricity and Magnetism,* gives the velocities of a few ions in centimetres per second due to 1 volt per centimetre of length :—

(+)H	320×10^{-5}
(−)OH	182×10^{-5}
(−)Cl	69×10^{-5}
(−)I	69×10^{-5}
(+)K	66×10^{-5}
(+)NH_4	66×10^{-5}
(−)NO_3	64×10^{-5}
(+)Ag	57×10^{-5}
(+)Na	45×10^{-5}
(+)Li	36×10^{-5}

It has since been found possible to measure these velocities *directly* by several methods, the first of which was due to Sir Oliver Lodge.

He made a solution of common salt and added some phenolphthalein,[1] which he made slightly alkaline with sodium hydrate to bring out its red colour. This solution was made into a semi-solid mass with agar-agar jelly and placed in a horizontal glass-tube, which connected two vessels containing dilute sulphuric acid, into which the electrodes were dipped. When a current passed, the hydrogen ions travelled along the tube, forming hydrochloric acid, which decolorised the phenolphthalein. Hence their velocity was found by measuring the rate at which the decolorisation progressed along the tube. The results obtained by this method, which has been greatly improved in detail by later experimenters, afford a strong confirmation of the general correctness of the theory.

Back E.M.F. in Electrolytes.—**Exp. 198.** Remove the cross connections from a current reverser, R, and connect up (as shown in Fig. 250) a battery of a few cells; a galvanometer, G, and any form of voltameter, V, having platinum electrodes in dilute sulphuric acid. It will be seen that in one position of the rocker the battery is connected to the voltameter, and in the other the voltameter is connected to the galvanometer. After passing the current for a few minutes, throw over the rocker, and notice that the galvanometer is deflected, and that the deflection rapidly decreases. Find the direction of the current through the galvanometer, and notice that the voltameter is giving out a current in the opposite direction to that previously passed through it.

---

[1] The indicator phenolphthalein becomes red in the presence of an alkali, but becomes colourless in presence of an acid.

Exp. 199. Repeat the experiment, using copper electrodes of fair size in a saturated solution of copper sulphate. No effect of the kind will be noticed. Replace the copper anode by platinum, and repeat, and notice that there will be a deflection as in Experiment 198.

Exp. 200. Repeat the experiment, using two lead plates in dilute sulphuric acid and replacing the galvanometer by an electric bell. On passing the current through the voltameter, gas will be evolved at each plate, and after a short time the anode will be found to be covered with a dark-brown coating of lead peroxide (PbO_2). When the rocker is reversed, the bell will ring for a few minutes.

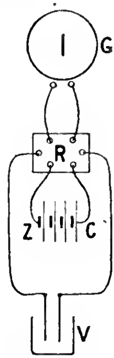


FIG. 250

These experiments show that the passage of a current through a voltameter is, in certain cases, able to make it act temporarily as a voltaic cell, the anode of the voltameter becoming equivalent to the copper or the carbon of a cell. That is, there must be a P.D. produced by the passage of a current, which persists after that current has ceased, and which is in a direction opposite to that of the applied E.M.F. Hence, the equation of a current through an electrolyte must be of the form $C = \frac{E - e}{r}$, where r is the true ohmic resistance and e is a back electromotive force.

This is another instance of the fundamental law that a current can do work only by flowing against a back E.M.F. In the case we are discussing, the work done is the breaking up of a chemical compound, and the back E.M.F. is a definite quantity depending on the nature of that compound. In fact, we can say generally that, when chemical action occurs in a circuit *due to the passage of a current* (*i.e.* excluding "local actions") the *formation* of a compound means the production of a certain E.M.F. *in the direction* of the current flowing, and the *breaking up* of a compound means the production of a *back* E.M.F. of the same value.

Consider Experiment 199, in which a solution of copper sulphate was electrolysed with copper electrodes. Copper is going into solution at the anode with the formation of $CuSO_4$, and producing a forward E.M.F., and going out of solution at the cathode and producing an equal back E.M.F. Hence there is an exact compensation, which explains the absence of any effect in that experiment. When, however, we used a platinum anode, this balance is upset, and a back E.M.F. was found to exist. We may mention that this will be the case to some extent even with a copper anode, if the current density is so great that the SO_4 ion acts on the water as well as on the copper and evolves oxygen.

Exp. 201. Try to electrolyse dilute sulphuric acid, using platinum electrodes, with *one* Daniell's cell. Put a suitable ammeter or galvanometer in circuit. A current will be indicated at first, but it will rapidly decrease and become practically zero.

Add a second Daniell's cell in series with the first. Observe that there is now a current and evolution of gas, but that the action is slow.

Add a third cell, and notice a marked increase in current strength, and consequently a much more rapid evolution of gas.

These results depend upon the fact that the back E.M.F. is, in this case, about 1·48 volts. This is greater than the E.M.F. of a Daniell's cell (about 1·1 volt), and hence the action must stop as soon as the back E.M.F. rises to an equality with that of the cell. With two cells in series, the current is given by

$$C=\frac{2{\cdot}2-1{\cdot}48}{r}=\frac{{\cdot}72}{r},$$ where r is the resistance of the voltameter.

With three cells, $C=\frac{3{\cdot}3-1{\cdot}48}{r}=\frac{1{\cdot}82}{r}$, which is about $2\frac{1}{2}$ times as large as with two cells.

Measurement of Resistance in Electrolytes.—It follows that the apparent resistance of an electrolyte is greater than its true value, and that the ordinary methods of measurements do not apply. Hence, special methods, in which the back E.M.F. is eliminated, must be used in order to determine the specific conductivities required to evaluate $\frac{K}{N}$.

The method most generally used is due to Kohlrausch. It depends upon the fact that, when alternating currents of a sufficiently high frequency are passed through a solution, the chemical actions at the electrodes are reversed so rapidly as to be practically negligible. Increasing the surface area of the electrodes is a further advantage, which is secured by platinising the platinum electrodes, *i.e.* depositing a rough coating of platinum upon them. Kohlrausch employed ordinary Wheatstone bridge methods, using alternating currents obtained from a small induction coil, and substituting a telephone for the galvanometer—the position of balance being determined by adjusting the resistances until the telephone is silent. An induction coil of suitable design is better than alternating currents derived from supply mains, as the abrupt changes of current give more distinct sounds in the telephone. Using alternating currents of frequency 50, it is almost impossible to obtain an exact balance, for the point of contact can be varied considerably without perceptibly affecting the telephone. With a frequency of 100, better results can be obtained, but even they are not really satisfactory.

A telephone is less convenient than a galvanometer (1) in requiring a fairly quiet room, and (2) in not indicating (as the latter does by the direction of its deflection) which way a resistance is to be altered to obtain a balance. These difficulties are avoided by using a dynamometer of suitable form, although such instruments cannot easily be given sufficient sensitiveness (see p. 562). At present a dynamometer may be regarded as a kind of galvanometer, which will work with alternating currents.

Another excellent, but more elaborate, plan, is to use the ordinary bridge method with a galvanometer to detect the position of balance, and to employ a double form of rotating contact-breaker to reverse rapidly the current supplied to the bridge, the galvanometer connections being simultaneously reversed, so that the current is always in the same direction through the galvanometer. This involves the use of a motor of some kind to drive the contact-breaker.

Calculation of E.M.F.—We have been gradually led to the conclusion that any chemical action corresponds to a definite E.M.F.; for instance, the formation of zinc sulphate under circuit conditions, sets up an E.M.F. which is exactly the same in value as the back E.M.F. produced when that compound is electrolysed.

In practice more than one reaction is usually going on simultaneously, and the observed value is a resultant.

Suppose that it is required to find the E.M.F. represented by the formation of a given compound, say, $ZnSO_4$. Let a solution of this salt be electrolysed by a current of strength i flowing for t seconds against a back E.M.F. e, and depositing W grams of the metal.

Then the work done against the back E.M.F. is $e.i.t$ ergs, which must be the potential energy of the liberated metal with respect to that compound. Let H heat units be produced when 1 gram of the metal re-forms the combination, then W grams will give WH heat units, and, as 1 heat unit $= 41{\cdot}8 \times 10^6$ ergs, the potential energy of the metal is $WH \times 41{\cdot}8 \times 10^6$ ergs.

$\therefore \quad e.i.t = WH \times 41{\cdot}8 \times 10^6$

but if $z =$ absolute electro-chemical equivalent of the substance

$W = i.tz$

$\therefore \quad e.i.t = i.tzH \times 41{\cdot}8 \times 10^6$

whence $e = z \times H \times 41{\cdot}8 \times 10^6$ absolute units.

Now, as H is known for many reactions, e can be calculated.

The above expression can be simplified as follows: We know that z for any substance is equal to z for hydrogen multiplied by the chemical equivalent of that substance.

$\therefore$ if $z_h =$ value for hydrogen,

$$e = z_h \times \frac{\text{atomic weight}}{\text{valency}} \times H \times 41{\cdot}8 \times 10^6 \text{ absolute unit.}$$

Now 1 coulomb liberates ·0000104 gram of hydrogen, but, in our equation, we have used absolute units, and the absolute unit of quantity is 10 coulombs.

$$\text{Hence } z_h = {\cdot}000104 \text{ grams.}$$

Again, we can divide by 10^8 to express the result in volts;

$$i.e. \quad e \text{ (volts)} = \frac{{\cdot}000104 \times H \times \text{chemical equivalent} \times 41{\cdot}8 \times 10^6}{10^8}$$

$$\text{whence} \quad e \text{ (volts)} = \frac{H \times \text{chemical equivalent}}{23{,}000}$$

The numerator is the heat given out by the equivalent weight in grams, *i.e.* by 1 gram-equivalent of the substance, when forming the given compound.

$$\therefore \quad \text{we have generally, } e \text{ (volts)} = \frac{\text{Heat produced by 1 gram-equivalent}}{23{,}000}$$

Let us now apply this to a Daniell's cell, in which there are two distinct reactions going on: (1) the formation of $ZnSO_4$, producing a forward E.M.F., and (2) the decomposition of $CuSO_4$, producing a back E.M.F. From chemical data, we find that 1 gram of zinc gives out 1670 heat units in forming $ZnSO_4$, and 1 gram of copper gives out 909·5 heat units in forming $CuSO_4$; we also know that the equivalent weight of zinc (in round figures) is $\frac{65}{2} = 32{\cdot}5$, and that of copper $\frac{63}{2} = 31{\cdot}5$.

$$\therefore \quad \text{for the first reaction, } e = \frac{1670 \times 32{\cdot}5}{23{,}000} = 2{\cdot}36 \text{ volts}$$

$$\text{and for the second reaction, } e = \frac{909{\cdot}5 \times 31{\cdot}5}{23{,}000} = 1{\cdot}25 \text{ volts}$$

$$\because \quad \text{E.M.F. of a Daniell's cell} = 2{\cdot}37 - 1{\cdot}25 = 1{\cdot}1 \text{ volts.}$$

The argument is, however, incomplete, for it does not take into account the fact that the E.M.F. of a cell depends, to a certain extent, upon its temperature. The result just given happens to be correct, because the temperature coefficient of a Daniell's cell is so small as to be negligible. To deduce the necessary correction requires an application of the second law of thermo-dynamics, which must be omitted here. What it means is, that a cell at work may either absorb heat energy from its surroundings, or give out such energy to them, and this effect must be considered in order to obtain the correct E.M.F. It is related to the phenomena discussed in Chapter XXVIII. on thermo-electric currents.

In connection with voltaic cells, it may be pointed out that the number of coulombs obtained by the consumption of a given amount of active material is independent of the nature of the reaction, whereas the E.M.F. obtained, and, therefore, the output of energy, *does* depend upon the nature of the reaction. For instance, the electro-chemical equivalent of zinc is ·000338 gram per coulomb, which means that, if no further loss occurs through local action, the consumption of ·000338 gram of zinc will give out 1 coulomb, *i.e.* 1 gram of zinc will give out 2967 coulombs; and this is true whether the zinc is forming $ZnSO_4$ (as in a Daniell's cell), or $ZnCl_2$ (as in a Leclanché cell. But the *energy* value of the reaction, and therefore the E.M.F., may be widely different, *e.g.* 1 gram of zinc in forming

$ZnCl_2$ gives out 1559 heat units as against 1670 for $ZnSO_4$. In this example the numbers do not differ much, but if (for the sake of illustration) we imagine a cell in which zinc iodide is formed instead of zinc chloride, then, although the number of coulombs would be unaltered, the heat units evolved per gram of zinc would be 821, and the E.M.F. could not exceed $\frac{821 \times 32 \cdot 5}{23,000} = 1 \cdot 16$ volts.

Measurement of Current by Chemical Action.—One of the most exact methods of measuring the strength of a current depends upon the use of the equation $W = C.t.z$, but obviously an exact knowledge of the value of z for at least one substance is required, and it is equally obvious that the selected substance should be one whose liberation is least affected by secondary reactions.

For very accurate work, a solution of pure silver nitrate is electrolysed in a special form of voltameter (which often consists of a platinum bowl as the cathode and a silver plate as the anode), the electro-chemical equivalent of silver having been determined, with the utmost care, by Lord Rayleigh and others.

For ordinary purposes a copper voltameter may be used. A very convenient form for the use of a student is shown in section in Fig. 251. The anode is a U-shaped sheet of fairly stout copper, $2\frac{1}{2}$ to 3 inches wide, the sides being about 5 inches long and $1\frac{1}{4}$ inches apart. A portion of one side is bent as shown to carry a terminal. The anode is screwed to a strip of wood (about $4\frac{1}{2}$ in. $\times$ 1 in. $\times$ 1 in.), which acts as a support in the containing vessel and which carries the cathode through a 3 in. slot cut in it. The cathode is a strip of very thin copper ($4\frac{1}{2}$ in. $\times$ $2\frac{1}{2}$ in.) to which is fastened a screw terminal. This terminal not only serves to take the connecting wire, but also prevents the cathode from slipping through the slot. The arrangement is placed in any convenient vessel to contain the copper sulphate solution; although it is preferable to use a glass vessel, because the position of the cathode can then be seen.

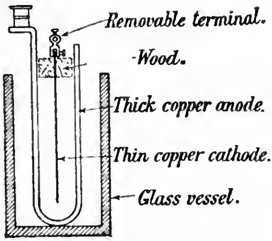


FIG. 251.

Exp. 202, *to find the constant of a tangent galvanometer by means of a copper voltameter*. Arrange the apparatus as shown in Fig. 252. The tangent galvanometer, T, is provided with a reversing key, R, and is in series with a Daniell's cell, b, the voltameter, V, and a plug key, K. The cell should be set up a little time in advance, and it may with advantage be short-circuited for a minute or two before being used.

The first step is to adjust the resistance until a convenient deflection is obtained. For this purpose a dummy cathode of the same size as the real cathode should be employed, and a short length of 30-gauge platinoid wire intercalated in the circuit at P. Alter the length of this wire until a deflection of about 45° is obtained. Then substitute the real cathode, previously cleaned and weighed (taking care that it is connected up the right way, and also that it does not touch the anode beneath the surface of the liquid), and at a noted instant insert the plug key. The current must be allowed to flow for *at least* 20 or 30 minutes, and during the time its strength ought to remain constant, but in practice it is almost certain either to increase, or to decrease slowly. In this case, the value deduced from the amount of copper deposited is evidently the *average* value during that time, and it is necessary to ascertain the corresponding value of the deflection. Again, any inaccuracy in the initial adjustment of the pointer to zero will make the deflections vary with the direction of the current. Both sources of error can be eliminated by reversing the current and noting the deflection at *definite* periods, say every three minutes, thus getting ten readings of the deflection in the course of 30 minutes.

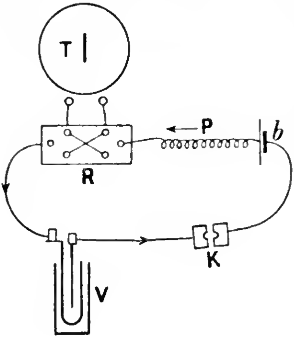


FIG. 252.

Let θ be the *average* value of these deflections. When time is up, break the circuit at a definite instant, carefully remove the cathode, wash it with distilled water, dry—partly with clean blotting-paper, and finally in a current of warm air (remembering that the deposited copper will readily oxidise if unduly heated) —and then weigh it.

Let W be the increase in weight in grams during time t seconds,

$$\text{then} \quad W = C.t.z, \text{ where } z = \cdot 0003294$$

$$\text{or} \quad C = \frac{W}{tz} \text{ amperes}$$

$$\text{also} \quad C = K \tan \theta$$

$$\therefore \quad K = \frac{W}{t.z. \tan \theta}$$

Notes on Method.—A tangent galvanometer is usually provided with several coils, and with some of them it is probable that a current, which will produce a suitable deflection, will be too small to deposit a weighable amount of copper in a reasonable time. In such a case, the resistance of the coil may be measured, and it can then be shunted with a suitable resistance. The total current in the circuit is obtained as above, and the current in the galvanometer is $\frac{s}{s+g}$ of that value.

In order to obtain a satisfactory deposit, at least 30 square centimetres of cathode surface should be allowed per ampere; if the current density is greater than this, some copper may fall off and be lost. The solution should be saturated, and may with advantage contain about 1 per cent. (by volume) of strong sulphuric acid.

Accumulators or Secondary Batteries.—The principle involved in Experiments 198–200 was first employed by Ritter, in

1803, in the construction of secondary batteries. He used large plates of platinum, having pieces of moistened cloth between each pair. Each end of the pile was then connected with the poles of a battery. After receiving a charge it was separated from the battery, when it was found to be capable of producing all the effects of an ordinary voltaic battery.

Later, Sir William Groves constructed his **gas battery** (Fig. 253).

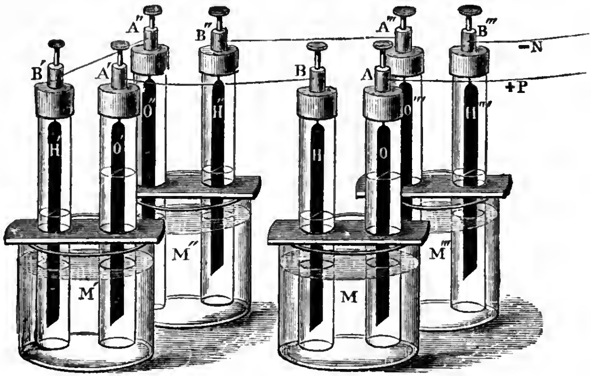


FIG. 253.

A cell, M, consists of two glass tubes, each containing a platinum plate, to which platinum wires are attached, and then connected outside with binding-screws, A, B. The tubes were filled with dilute sulphuric acid and inverted over a similar solution. On passing the current, oxygen and hydrogen are liberated and collected. When the charging battery is removed, and AB joined by a wire, the arrangement is found to give out a current in the opposite direction to the charging current, the liquid meanwhile slowly rising in the tubes as the gas disappears. The E.M.F. of such a cell is about 1·4 volts. It is best to platinise the platinum plates, *i.e.* coat them with finely divided metallic platinum in order to increase the effective surface.

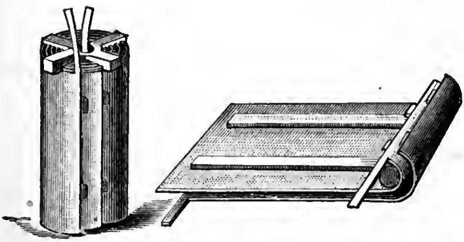


FIG. 254.

Planté, in 1860, constructed a secondary cell by using two sheets of lead, each provided with a tongue (Fig. 254). The sheets were then rolled up—narrow strips of felt being put between them to prevent contact—and then immersed in dilute sulphuric acid. The terminals of a battery were attached to the two tongues, so that a current passed through the cell. By this means the liquid is decomposed, oxygen being liberated at the anode, which combining with the lead forms peroxide of lead (PbO_2), while hydrogen is liberated at the cathode. The current is then reversed until the PbO_2 is reduced to spongy lead by the action of the hydrogen, while the other plate (the first cathode, now the anode) is in its turn oxidised. Thus by sending repeated currents in alternate directions, the plate which served last as the anode is left deeply coated with PbO_2, while that which served last as the cathode is deeply coated with spongy lead. Planté's cells were vastly more efficient and practical than any previously produced, and he must be regarded as the real inventor of the modern accumulator.

In order to obviate the lengthy process of the "formation of the cells," Faure, in 1881, improved the construction by coating the two plates with minium or red lead (Pb_3O_4). When the current is passed through the cell to charge it, the red lead at the anode is oxidised to PbO_2, while at the cathode it is reduced by the hydrogen—first to PbO, and then to the spongy metallic state.

Plates formed by Planté's method are superior to Faure's pasted plates in durability, because the peroxide is more firmly attached; on the other hand, a greater capacity for a given weight is obtained more readily with pasted plates. It is now a common practice to make the positive plates by some modification of Planté's method and to use pasted plates for the negative. Various devices are employed to keep the paste from falling off, one of the most recent being the "box" type of negative, in which the plate is made of two perforated grids, fitting together to form a narrow box, which is filled with the oxide (litharge). As the grids are merely supports for the active material, they are usually made of an alloy of antimony and lead, which is stiffer and stronger than pure lead.

A modern accumulator consists of a number of positive and negative plates sandwiched together (precautions being taken to avoid contact), all the positives being connected together and similarly all the negatives. The outside plates are always negative, and hence there is always one more negative than positive. The electrolyte is dilute sulphuric acid.

In consequence of the large area of surface and the closeness of the plates, the internal resistance is very low, and as the effective E.M.F. is about 2 volts, the possible current on short circuit is very great; in fact great enough to seriously injure the cell by causing the plates to bend and the active material to drop off.

It should be borne in mind that there is no essential difference between a primary cell and a secondary cell. In the former, it is more convenient to renew the active materials themselves when the cell is exhausted, whereas the latter is designed to permit of the materials being brought back to their original state by electrolysis. For this purpose, it is essential that no product formed during normal work (or *discharge*) should be lost. A little consideration will show, for instance, that an exhausted Daniell's cell might be restored to its original state by sending a reverse current through it, but this could not be done with a Leclanché cell on account of the ammonia lost as gas.

A lead accumulator is essentially a single-fluid voltaic cell in which the zinc is replaced by a lead plate of relatively enormous surface area (spongy lead) and the copper by lead peroxide. It is free from "local action" and "polarisation."

The acid produces hydrogen ions and "sulphion" (SO_4) ions, and during discharge (1) the sulphion gives up its negative charge to the lead plate, and forms $PbSO_4$, which, being insoluble, remains on the plate as a white film; (2) the hydrogen has its positive charge neutralised at the peroxide plate, and combines with the oxygen to form water; thus

$$(1)\quad Pb + SO_4 = PbSO_4$$

$$(2)\quad PbO_2 + H_2 = PbO + OH_2$$

The PbO thus formed is acted on by the acid, and also forms $PbSO_4$.

$$PbO + H_2SO_4 = PbSO_4 + OH_2$$

Thus, after discharge both plates are coated with a thin layer of insoluble lead sulphate.

To recharge, a current in the opposite direction is passed through it from an external source. The peroxide plate now becomes the anode, and the lead plate becomes the cathode. When the acid is electrolysed (1) the hydrogen at the cathode reduces the $PbSO_4$ to Pb, thus

$$PbSO_4 + H_2 = Pb + H_2SO_4$$

(2) the oxygen at the anode acts on the $PbSO_4$ and water, re-forming the peroxide

$$PbSO_4 + O + OH_2 = PbO_2 + H_2SO_4$$

The equations given above are neither exact nor complete. They are given merely as useful aids by which the nature of the principal reactions may be indicated.

From what has been said previously, it will be evident that the voltage required to send a given charging current through a battery of accumulators is very much greater than the value calculated from the internal resistance. For instance, suppose that the battery consists of ten cells, each having an internal resistance of $\frac{1}{50}$ ohm, and that the charging current is to be 12 amperes. The total internal resistance is $\frac{1}{5}$ ohm, and to send 12 amperes through $\frac{1}{5}$ ohm requires $12 \times \frac{1}{5} = 2 \cdot 4$ volts. But each cell, *during charging*, will develop a back E.M.F. of at least 2·5 volts, or 25 volts in all, and therefore the charging E.M.F. must be at least 27·4 volts. That is, the equation of current must be written, as already explained, in the form

$$C = \frac{E - e}{r}$$

where e is a back E.M.F. and r is the ohmic resistance.

It will be seen later (see p. 372) that an equation of this form indicates that work of some kind is being done by the current, and it will be found to admit of very simple interpretation—EC being the power in watts supplied to the accumulator, eC the power expended in doing work (in this case chemical action), and the difference being the power wasted as C^2r heat on account of internal resistance.

The Edison Accumulator.—This is the only practical accumulator which does not involve the use of lead. Although little known at present in this country, it seems likely to become of great commercial importance in the near future.

The positive plate is a grid of steel, supporting closely packed vertical rows of perforated steel tubes, made by rolling steel strip into a long spiral. These are filled with thin alternate layers of metallic nickel (in the form of excessively thin flakes) and of nickel hydrate. The negative grids are also of steel and carry rectangular pockets filled with an oxide of iron. The electrolyte is a 21 % solution of potassium hydrate. The containing vessel is also of steel, and all the steel parts, grids, &c., are nickel-plated.

The chemical reactions are said to be as follows: Beginning with iron oxide on the negatives, green nickel hydrate on the positives, and potassium hydrate in solution, the first charging of the cell reduces the iron oxide to metallic iron, and converts the nickel hydrate to a higher oxide, black in colour. During discharge, the iron becomes oxide again, and the nickel is reduced to a lower oxide, but does not again return to its original form of green hydrate. The operation of charging, therefore, amounts to a transference of oxygen from the iron to the nickel, and that of discharge to a transference back again. When fully charged, the plates are no longer acted upon, and hydrogen and oxygen gases are given off as in ordinary cells. The amount of potassium hydrate in solution never changes, and hence the density remains constant. The nickel flakes in the positives and a small amount of mercury in the negatives do not take part in the reactions, and are used merely to ensure good electrical contact between the active material and its supports.

The cell is remarkable for its mechanical strength and perfection of design. The absence of a corrosive acid is a distinct advantage, but even more important is its freedom from the most troublesome defects of lead cells—the plates do not "buckle" or "sulphate," neither do they deteriorate if the cell is left uncharged.

Again, the cell is remarkably light for a given capacity in ampere-hours, and only requires filling up occasionally with distilled water.

Its only disadvantage appears to be its low E.M.F., which is about 1·3 volts per cell.

Exercise XVII

1. How many amperes would deposit 2 grams of copper in 15 minutes, the current being supposed constant?

2. How many grams of copper would be deposited by a constant current of 12 amperes acting for 1 hour?

3. What would be the strength of a constant current which would deposit 36·36 grams of copper in 5 hours?

4. What would be the strength of a constant current which liberates 50 cubic centimetres of hydrogen in 5 minutes?

5. How many amperes would liberate 250 cubic centimetres of hydrogen in 15 min. 32 sec., the current being constant?

6. What is the result of passing a current through a solution of sulphate of sodium (Na_2SO_4), by means of platinum electrodes separated from one another by a porous partition?

7. Explain the term *electro-chemical equivalent.* If 3 amperes deposit 4 grams of silver in 20 minutes, what is the electro-chemical equivalent of silver? (B. of E., 1899.)

8. If the electro-chemical equivalent of silver is ·01118, what is the electro-equivalent of oxygen? (B. of E., 1901.)

9. State Faraday's laws of electrolysis. A current of 1 ampere is passed for two hours through an electrolyte and decomposes 2·4 grams. Find the electro-chemical equivalent of the electrolyte. (B. of E., 1905.)

10. A current is passed through a voltameter and through a coil of wire in series with it. If the current is altered in such a way that the heat produced in the coil is doubled, show what change will be produced in the rate at which chemical action takes place in the voltameter. (B. of E., 1903.)

11. Distinguish between the chemical and electro-chemical equivalents of an element. What weight of hydrogen is separated from water by the passage of 1000 coulombs of electricity, given that the chemical equivalent of copper is 31·5, and its electro-chemical equivalent 0·000328 per coulomb? (B. of E., 1902.)

12. It is stated that, in order to separate 1 gram of hydrogen from acidulated water by electrolysis, 96,500 coulombs of electricity must pass: how would you proceed to verify the statement? (B. of E., 1907.)

13. Six Grove's cells, connected (*a*) in a single series, and (*b*) in two series of three each, are used to decompose water in a voltameter. If there is no local action in the battery, show how much zinc is dissolved in each case (*a*) and (*b*) in the whole battery while one grain of hydrogen is evolved in the voltameter.

Take sulphate of hydrogen (sulphuric acid) as H_2SO_4 (98) and sulphate of zinc as $ZnSO_4$ (161).

14. A battery of eight cells, connected in series, is used to decompose water in a voltameter; the chemical equivalent of zinc being 32·5 times that of hydrogen, show how much zinc is consumed in the whole battery while one grain of hydrogen is liberated in the voltameter.

15. A current is passed through a coil of wire and then through a galvanometer arranged in series with it. If the strength of the current is altered so

that the heat produced per minute in the coil of wire is doubled, show what change will be produced in the rate at which chemical action takes place in a voltameter.

16. After acidulated water is electrolysed, find the electromotive force of hydrogen tending to recombine with oxygen (34,000 units of heat are produced by the combination of 1 gram of hydrogen with oxygen).

17. Find the E.M.F. of zinc dissolving in sulphuric acid, where $z = \cdot 00338$, $H = 1670$.

18. An electromotive force of 3 volts is required to force a current of 1 ampere through a voltameter containing acidulated water. If the work required to separate one gram of hydrogen is 142,000 watt-seconds, and the electro-chemical equivalent of hydrogen is ·00001035, find the resistance of the voltameter. (B. of E., 1904.)

19. Given that the electro-chemical equivalent of copper is ·00033, calculate what must be the output in watts of a dynamo to deposit 10 kilograms of copper per hour, the voltage of the dynamo being 10 volts?

20. What do you understand by the polarisation of the platinum plates employed as electrodes in a voltameter? How would you show experimentally in which direction the polarisation acts?

21. A current flows through two troughs, which are connected in multiple arc, and contains a solution of copper sulphate. If all the circumstances of the two paths which are thus open to the current are the same, except that the metal plates by which it enters and leaves the liquid are, in the one case copper, and in the other platinum, will the currents be equally strong in the two troughs? Give reasons for your answer.

22. A number of cells formed of plates of zinc and platinum, immersed in dilute sulphuric acid, are to be connected in a circuit, so that the platinum of each cell is in contact with the zinc of the next. What effect, if any, would be produced on the current if, by mistake, one cell was made up with two platinums, instead of with one platinum and one zinc plate?

23. A single Grove's cell is joined up in circuit with a voltameter, in which acidulated water is decomposed between platinum electrodes, and the strength of the current is noted. On connecting a second Grove's cell in series with the first, the strength of the current becomes considerably more than twice as great as at first. Explain this.

24. A current of 20 amperes passes through a resistance of 2 ohms and is also sent through a battery of 10 accumulator cells in series to charge them. If each accumulator cell has a counter electromotive force of 2·4 volts and an internal resistance of $\frac{1}{100}$ ohm, how many volts must be applied to furnish this charging current? (Lond. Univ. Matric., 1899.)

25. Describe a method of investigating the relation between the strength of an electric current and the rate of chemical change produced by it. (B. of E., 1897.)

26. Is the strength of the current that passes through a simple circuit the same at all points of the circuit, however its parts differ in resistance? How would you justify your answer by experiment? (B. of E., 1894.)

27. Two liquid resistances, A and B, of 5 and 10 ohms respectively, are connected in parallel, and a battery of electromotive force of 8 volts and 2 ohms internal resistance is used to send a current through them. Find the currents in the two liquids, being given that the electromotive force of polarisation is ·1 volt in A and 1·8 volts in B. (Lond. Univ. B.Sc., 1908.)

28. Describe the construction and method of charging a secondary battery.

CHAPTER XXII

ELECTRO-MAGNETISM

Mutual Actions between Magnetic Fields and Currents.—We have already learnt that a current always produces a magnetic field in the space around it. Hence, if a current is made to flow in another magnetic field of independent origin, the mutual action of the two sets of lines of force will, as a rule (but not in all cases), set up mechanical forces which react upon the conductor carrying the current.

In the case of a straight conductor, the nature of the effect will be easily understood if we draw a simple diagram and then apply the laws already established; *e.g.* in Fig. 255, the conductor, seen in section, is at right angles to the independent magnetic field, which is not necessarily produced between the poles of a magnet, although for the sake of clearness such poles are shown. For convenience also, only a few lines of force are drawn, and these, according to the general rule, are marked in the direction in which a free N pole would tend to move.

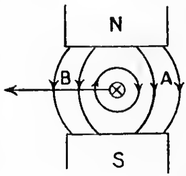


FIG. 255.

We now see that, at A, the two sets of lines are in the same direction, and therefore they repel one another, whereas, at B, they are in opposite directions, and so tend to neutralise each other. The result is a force, which tends to drive either the conductor in the direction shown by the arrow or the field in the opposite direction.

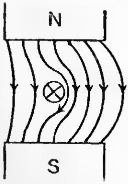


FIG. 256.

It must be clearly kept in mind (1) that such forces always act between the invisible lines of the field, although we can perceive the effect only on the bodies with which the fields are associated; (2) that, in reality, we should not obtain the distribution of lines shown in the diagram, the two fields really producing a resultant field in which no two lines intersect (Fig. 256). But in order to understand the actions taking place, it is better to draw the two components superposed, rather than the resultant field itself.

The diagram also informs us that reversing *either* the direction of the current *or* the polarity of the magnet would reverse the direction of the force, whereas reversing *both* would leave the latter unaltered.

Many experimental illustrations are possible, and as the principle is important—all practical electric motors depending upon it—we shall investigate the matter rather fully.

Exp. 203. Bend a wire, AB, as shown in Fig. 257. Flatten one end, A, and drill a hole through it. Fix AB so that it is in connection with the binding-screw, T. Pass a wire, C, through the hole, keeping it in position by making a small loop at the top, and of sufficient length to just touch some mercury contained in the hollow, D, made in the base. The mercury is connected to the binding-screw T′ by means of a wire passing underneath the base (indicated by the dotted line). Attach a wire from the battery to one end of the coil of an electromagnet, E, the other end being connected to the binding-screw T′. Fasten the other wire from the battery to T, and observe that the wire moves out of the mercury across the lines of force of the magnet, and that, at the moment contact is broken, it falls back again. These motions are repeated while the current lasts. If the direction of the current is altered, the direction of motion of the wire will remain the same as before, because the current is reversed in both wire and magnet.

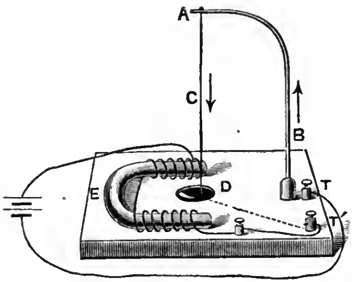


FIG. 257.

(This experiment also works satisfactorily if a fairly strong steel horse-shoe magnet be used instead of an electromagnet.)

The student will find that the direction of motion can be deduced by applying the method indicated in Fig. 255.

A similar experiment can be performed in an interesting manner as follows :—

Exp. 204. Suspend a straight brass or copper wire so that the lower end swings freely between the poles of the electromagnet shown in Fig. 258 (*a*). (To do this it is only necessary to make a loop of copper wire and hang a straight piece loosely on it.) Join it up to a battery of one or two cells through a current reverser, making connection to the lower end by means of a strip of flexible tinsel. Excite the electromagnet from some independent source (not shown in the diagram) and notice that, when a current is sent through the vertical wire, it is driven sideways out of the field. Reverse the current in it, and notice that the direction of motion is also reversed. Determine the polarity of the electromagnet and the direction of the current, draw a diagram to represent

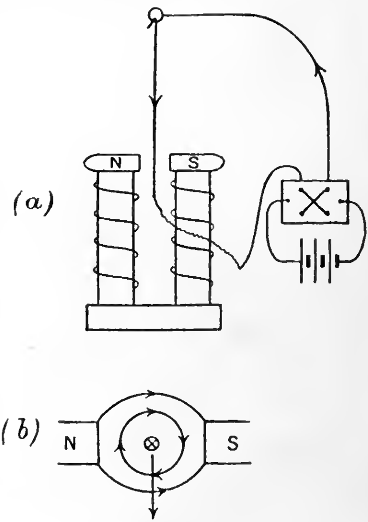


FIG. 258.

the facts (as in *b*), and observe that the direction of motion agrees with that deduced from the diagram. (It will be seen that (*b*) indicates the state of affairs when (*a*) is looked at from above.)

When alternating currents are used for lighting purposes, a very simple demonstration of the existence of the force in question may be given by holding an ordinary bar magnet (or horse-shoe magnet) near an incandescent lamp provided with a carbon filament. The force on the filament is quite small—indeed, if it were carrying a continuous current, the displacement would probably be imperceptible—but as it is reversed many times per second, a vibration occurs, which may be so violent as to break the filament unless the magnet is removed.

These experiments can easily be modified to produce a continuous rotation. One of the oldest devices for this purpose is known as **Barlow's wheel**, which usually consists of a brass or copper wheel

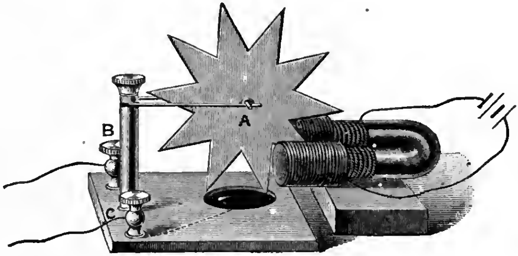


FIG. 259.

cut into the shape of a star (Fig. 259). As it rotates, the points just dip into a mercury cup.

Exp. 205. Connect the binding-screws, B and C, with a battery. The current then passes from one binding-screw, B, up the support to the axis A, thence down a vertical radius of the wheel to the mercury, and so back to C and the battery. When the wheel is placed between poles of a strong magnet, as shown in the diagram, observe that it begins to rotate, and, one point of the wheel after another passing out of the mercury, that the rotation is kept up while the current lasts. As in the last experiment, the direction of rotation can be reversed, by either changing the direction of the current or the polarity of the magnet.

In this experiment a steel magnet may be used, and again the student should deduce the direction of motion by aid of Fig. 255.

Another method of obtaining rotation—due to Faraday—is shown in Fig. 260. If desired, a steel magnet may be used instead of the electromagnet therein indicated.

Exp. 206. Fit two corks, AB (Fig. 260), tightly into a lamp chimney. Remove the cork B, and bore a hole to admit a round piece of soft iron. Pass the iron a short distance through the cork, and then coil insulated copper wire round the outer part, so as to form an electromagnet. Now fix the cork in its place, so that one end of the coil passes between the cork and the glass and the other end is free. Pour mercury in the tube, so that the end of the iron projects slightly above the surface. Through the cork A pass a copper wire, making a loop at the end which is to be inside the tube. In this loop hang a piece of copper wire, D, of such a length that its lower end just rests in the mercury. Connect the wires from a chromic acid battery to the free ends of the wires outside the apparatus, and observe that the wire D revolves round the pole, C, of the magnet.

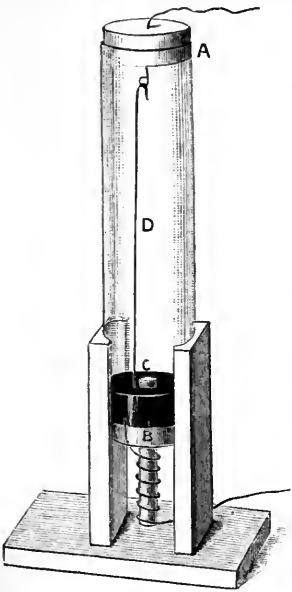


Fig. 260.

The result can be explained, and the direction of rotation predicted, in exactly the same way as before. Looking down upon the apparatus, the conductor and the magnet will be seen in section, and some lines of force will spread laterally from the upper pole towards the lower pole. On a flat surface these must be shown as a series of radial lines. Hence we deduce Fig. 261. The upper end of the magnet is assumed to be a N pole, and the current is supposed to be flowing *down* the wire. Two positions, A and B, of the wire are shown, and only one line of force is drawn as a circle around it to represent the field due to the current. When the direction of the two fields are marked according to the usual rule, it becomes apparent that the two fields are in the *same* direction on one side of the wire, and hence there is a tangential force tending to drive the lower end of the conductor sideways, the final result being a continuous rotation in a clockwise direction.

As usual, reversing *either* the polarity of the magnet, *or* the direction of the current, will reverse the direction of rotation.

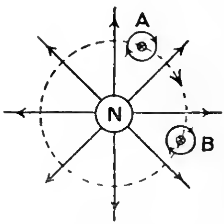


Fig. 261.

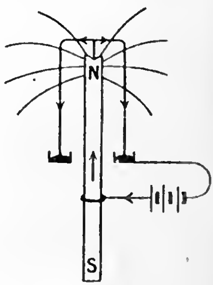


Fig. 262.

Exp. 207. A simple modification of the last experiment can be made by pivoting a wire, bent twice at right angles, on the pole of a magnet (Fig. 262), and letting the ends dip into an annular trough containing mercury, fixed about the middle of the magnet, so that a current can be sent through the magnet and into the wire, returning by way of the mercury trough, or *vice versâ*.

The student should draw an explanatory diagram as in the last experiment, except that the two conductors must be shown 180° apart. He will then see why the current must flow in the same direction in both sides of the wire—for if it were to flow up one and down the other, the conductors would tend to rotate in opposite directions.

Similarly, a *liquid conductor* may be made to rotate.

Exp. 208. Fix a fairly powerful *bar* electromagnet vertically, *e.g.* one of the two uprights of the horse-shoe electromagnet described on p. 249. On the top of the magnet, support a flat-bottomed circular glass dish having a diameter of 5 or 6 inches. Bend a long strip of sheet copper into a circular hoop, which fits just inside the dish. Make a conducting liquid by acidifying water with sulphuric acid, and pour it into the dish.[1] Support another strip of copper, so that it dips into the liquid at the centre. Connect the two pieces of copper to a battery, so that a current may be sent from the centre to the circumference of the liquid, or *vice versâ*. Notice that, when the magnet is excited, the liquid will begin to rotate, as may be made evident by floating small pieces of wood or cork in it.

As before, reversing either the current or the polarity will reverse the direction.

In this experiment, it is the approximately vertical lines of force that are operative, and the diagram (Fig. 263) may be drawn by taking *any* radius of the liquid to represent a conductor. Supposing that the current flows from the centre *outwards*, we obtain a figure which shows that the liquid, as seen from above, is rotating in a clockwise direction.

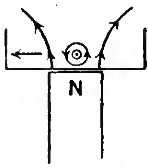


FIG. 263.

Exp 209. Perhaps the most interesting variation of Experiment 207 is to make a magnet rotate under the influence of its own field. For this purpose, prepare a permanent magnet of round steel about $\frac{3}{8}$ to $\frac{1}{2}$ inch diameter, and 6 or 7 inches long, with both ends carefully pointed, so that it may be pivoted to rotate freely about its axis. Twist a copper wire round the middle of the magnet, and bend it so that its end dips into an annular mercury trough. The arrangement is shown in Fig. 264.

Notice that the magnet begins to rotate when a fairly strong current is sent through one half of the magnet.

A better effect will be produced if the two ends are connected in parallel; the current then enters at the middle and flows towards each end, or *vice versâ*.

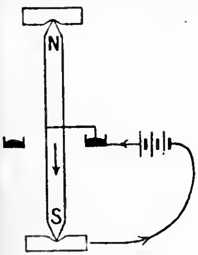


FIG. 264.

The results of such experiments may be expressed in the following terms:—

If a conductor carries a current at right angles to the lines of force in a magnetic field, a mechanical force is exerted upon it which tends to move it laterally, i.e. *in a direction which is at right angles both to the direction of the current and to that of the field.*

If the conductor is *not* at right angles to the field, but is inclined

[1] Mercury or any sufficiently conducting liquid may be used.

to it at some angle θ—as in Fig. 265, where AB represents the portion of the conductor under consideration—then the force in it is the same as if it were of length AC. This means that we can write force $= p \sin \theta$, where p is the value when $\theta = 90°$.

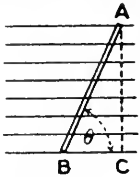


FIG. 265.

When the conductor is parallel to the field, $\theta = o$ and the force vanishes.

The subject is further dealt with on p. 372.

An exactly similar effect is produced when the conductor is gaseous. For instance, the luminous discharge in a vacuum tube may be made to rotate around a magnet pole very much as the wire rotates in Experiment 206, and the electric arc in air at ordinary pressure may be driven sideways until it is blown out, by means of a magnetic field at right angles to its direction. This action is usefully applied in the "controllers" used for driving electric cars, in order to prevent arcs forming when the circuit is broken. It is also applied in a modified form in the "flame" arc lamps now largely used, as explained in Chapter XXXII.

Production of Electromotive Force by the Motion of a Conductor in a Magnetic Field.—Let a straight conductor be placed in a magnetic field at right angles to the lines of force. We have just shown that, if a current be sent through it, a mechanical force is produced which tends to move it sideways. If, however, instead of sending a current through it, the conductor itself, by some external means, is *moved* sideways so that it cuts the lines of force, we find that, during such motion, an E.M.F. is produced in it, *i.e.* a difference of potential is produced between its ends. If a circuit is formed, say by joining the two ends of the conductor by a wire outside the field, then a current flows. Such currents are commonly known as induced currents, but it is important to remember that an E.M.F. is produced whether there is a circuit or not. This action was discovered by Faraday in 1831, and upon it is based the construction of all dynamos for generating current on a large scale, just as all electric motors depend on the mechanical force produced when a current is sent through a conductor.

To demonstrate this fact, it is advisable to employ a reflecting galvanometer of the suspended coil type (preferably fairly dead-beat), as it is not affected by the magnets used during the experiment.

Exp. 210. Connect the terminals of such a galvanometer by means of a long copper wire. Move a portion of the wire rapidly in, say, an upward direction, between the poles of a steel horse-shoe magnet, and notice a very slight movement or "throw" of the spot of light. Notice that, when the wire is moved in the opposite direction, say downwards, the direction of the deflection is reversed. Also notice that the deflection is increased with the speed of motion, and that a slow movement produces no perceptible effect. After trying the experiment with an ordinary horse-shoe magnet, the student should repeat these operations with the electromagnet described on p. 249.

In Fig. 266, AB represents the portion of the wire which is moved through the field.

It is important to mention that the best effect will be obtained when AB is moved at right angles to the field, and that no effect will be produced when it is laid across the poles and moved upwards, *i.e.* when it is moved without cutting the lines of force.

Exp. 211. Wind the wire into a few loose turns, and slip them on and off one of the poles of a steel horse-shoe magnet. Notice that larger deflections are now easily obtained.

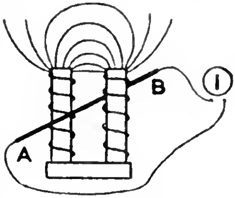


FIG. 266.

The student will readily understand that the loose coil is equivalent to the joining together of a number of conductors *in series*, an arrangement which causes their separate E.M.F.'s to be added together.

Exp. 212. After trying the effect of moving the coil on and off at A (Fig. 267), move the coil from B to C, and notice that little or no effect is produced. Hence, we infer that the action depends upon the turns of the coil *cutting through* the lines of force in such a way that the number of lines threading the coil must either increase or decrease.

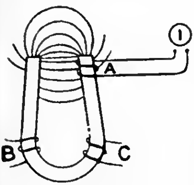


FIG. 267.

Exp. 213. Place the coil anywhere between B and C, and try the effect of putting a soft iron keeper on the magnet, and then suddenly pulling it off. In this position, no induced current will be obtained. Repeat the experiment with the coil at A. The result will be a deflection in one direction when the keeper is applied, and another in the opposite direction when it is removed.

Evidently this is due to the fact that *more* lines of force pass through A when the keeper is in position than when it is absent; in the latter case many of the lines emerge from the magnet pole below A. But the flickering of the lines of force is confined to the neighbourhood of the poles and does not affect a coil near the bend. This experiment illustrates the action of telephones (described in Chapter XXX.).

Similar results may, of course, be obtained by using a bar magnet.

Exp. 214. Connect a hollow coil of wire to the galvanometer and then insert a bar magnet. (Try this gently at first, as, if the coil has many turns, the deflection may be violent enough to injure the galvanometer.) Notice that a "throw" is produced when the magnet pole enters the coil, and that another occurs in the opposite direction when it is removed, whereas no effect whatever is obtained by merely leaving the magnet in the coil. If the other end of the magnet is inserted, similar actions are produced, but the directions of the throws are reversed.

Exp. 215. Remove one of the coils of the large electromagnet (p. 249) and connect it to the galvanometer, using wires long enough to permit of free movement without jerking the instrument. Hold the coil in such a position that as many lines as possible of the earth's field pass through it. This will occur when the axis of the coil is in the line of dip. Turn the coil suddenly through 90°, *i.e.* until it reaches a position where no lines of force pass through it. Notice that a small throw will be produced due to the cutting of the earth's field. Continue the process, turning it through 90° at each step, until the coil has made a complete revolution, and notice the directions of the throws obtained. It will be found that they are in the same direction during one half revolution, and in the opposite direction during the second half. This is due to the fact that during the first *quarter* revolution, the lines threading the coil are decreasing in number, and during the next quarter are increasing again. We might, therefore, expect a reverse throw (and this would be the case if we turned the coil back again), but, when motion is continued for the second quarter, the lines as they increase in number are threading the coil in the *opposite* direction, which again reverses the E.M.F., leaving the direction of throw as at first. Thus the rotation of the coil produces an alternating E.M.F., which will be more fully understood after performing Experiment 217.

The existence of an induced current may be demonstrated without using a galvanometer by a simple method due to Faraday.

Exp. 216. From the electromagnet, remove one coil and its iron core, so that the remaining coil and core form a vertical bar electromagnet. Excite this as strongly as possible by means of a current. Attach two loose bare copper wires to the terminals of the other coil, and bend them until their free ends just touch crossways to complete the circuit. Hold this coil, with its iron core inside, above the bar electromagnet, and bring it suddenly down upon the latter. An induced current will flow during the process, and the shock of contact of the two iron cores will jerk the loose wires apart at the right moment, a distinctly visible spark being produced at the gap.

(It should be noticed that all the experiments given to demonstrate the existence of a force on a conductor carrying a current in a magnetic field are necessarily reversible, *i.e.* if, instead of sending a current through it, the moving part is moved by some external means, then an induced E.M.F. is produced.)

Magnitude of Induced E.M.F.—Faraday established, by experiment, the laws underlying the production of induced currents, and we may conveniently summarise the facts in the following way:—

When a conductor is moved so that it cuts the lines of force of a magnetic field, an E.M.F. is induced in it, which depends only upon the rate at which the lines are cut, i.e. *upon the number of lines cut per second.*

Hence, steady motion in a uniform field will induce a steady E.M.F. during that motion; if, however, the field is not uniform, the E.M.F. will vary from moment to moment according to the rate of cutting at any particular instant.

It is important to notice that the magnitude of an induced E.M.F. is absolutely independent of (1) the material of the conductor, (2) its shape, (3) its size, or even (4) the existence of a circuit. If a circuit does exist, a current will flow therein, the magnitude of which depends, of course, upon the resistance of the circuit, and therefore upon the nature, size, and shape of the conductor.

The conception of an induced E.M.F. is thus simpler than that of an induced current, and should precede it.

From what has been said, the student will see that we may have an induced E.M.F. without an induced current, but that we cannot have the converse.

Absolute Unit of E.M.F.—The fact that an induced E.M.F. is independent of the material and dimensions of the conductor, enables us to define the unit of E.M.F. in the following simple way:—

The absolute unit of E.M.F. is the E.M.F. produced when a conductor moves at such a rate that it cuts one line of magnetic force per second.

(Here, it must be mentioned that the essential point is *relative* motion. The conductor may be stationary and the field may move, but the result is the same if the rate of cutting is unaltered.)

As this unit is too small for practical convenience, a multiple is used for most purposes, which is already known to us as a "volt." A volt is one hundred million (or 10^8) absolute units of E.M.F.

It follows from the definition that, if Z lines of force cut N conductors in t seconds (the rate of cutting being uniform during that time and the conductors being connected in series), then

$$e = \frac{Z \times N}{t} \text{ absolute units} = \frac{Z \times N}{10^8 t} \text{ volts.}$$

If the rate of cutting is *not* uniform, this is the average value during that time.

Sometimes we are concerned with the motion of straight conductors (as in dynamo and motor armatures), and sometimes with the motion of coils—either circular or rectangular in section—and there is some danger of confusion when applying the above formula. Now a single *turn* of a coil is really the equivalent of two conductors in series, but it is convenient to adopt a method which enables us to apply the formula to both cases, and which, therefore, obviates the necessity of using two special equations. Suppose that Z lines of force pass through a coil of N turns, and that the coil be rotated through 90° so that no lines of force *then* pass through it; if we agree to define this as meaning that Z lines have cut N turns, the above equation applies without ambiguity, whether N stands for the number of turns or for the number of conductors, and we then have

$$e = \frac{\text{lines cut} \times \text{turns}}{\text{time of cutting}} \text{ (absolute units)}$$

This definition of "cutting" as applied to a coil means that if the coil makes one complete revolution about a diameter in a magnetic field—its axis of revolution being at right angles to the field—it cuts the lines threading it four times.

Example 1.—A solid fly-wheel, 1 metre in diameter, makes 250 revolutions per minute in a plane at right angles to the magnetic meridian at a place where the horizontal component of the earth's magnetic field is ·18. Find the P.D. between the centre and the circumference of the wheel.

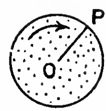


FIG. 268.

The rotating wheel cuts the horizontal component of the earth's field, and is really equivalent, as far as the production of an E.M.F is concerned, to a single conductor OP (Fig. 268) equal in length to its radius r. For if OP rotates round the centre O, a P.D. will be produced between O and P, and this will be true for any number of radial spokes such as OP. Merging them together to form a disc will not affect matters, and, hence, we see that every point on the circumference is at the same potential, and that the P.D. between the centre and the circumference is the same as between O and P. No current will flow unless a connection is made between the centre and some other point on the disc (best at the circumference).

$$\text{Now} \quad e = \frac{\text{lines cut} \times \text{number of conductors in series}}{\text{time of cutting}} \text{ (absolute units)}$$

Consider the time of one revolution, which is $\frac{60}{250}$ seconds. In that time OP has swept through an area πr^2, and has cut $H \times \pi r^2$ lines; hence

$$e = \frac{H \times \pi r^2 \times 1}{\frac{60}{250}} = \frac{\cdot 18 \times 3{\cdot}14 \times (50)^2 \times 1 \times 250}{60}$$

$$= 5890 \text{ absolute units}$$

$$= \frac{5890}{10^8} = \cdot 0000589 \text{ volts.}$$

It will be noticed (1) that this is a uniform and unidirectional E.M.F., and (2) that e varies as r^2, hence the P.D. between O and any point on the disc midway between the centre and the circumference is only $\frac{1}{4}$ of the value just given.

Exp. 217. Make a small rectangular coil by winding a number of turns of 30-gauge copper wire around a match-box; dip it into melted paraffin-wax to hold the turns together, and then trim off the projecting ends of the box. Connect the two free ends of the wire to a dead-beat reflecting galvanometer, and then hold the coil between the poles of an electromagnet, in the position AB (Fig. 269), where the horizontal lines indicate the general direction of the lines of magnetic force. Turn it through a quarter of a revolution and notice the direction of throw; turn it through another quarter of a revolution in the same direction, and observe that the throw produced is in the same direction as before; continue the motion and notice that the throw is reversed in direction during the last two quarters of a revolution.

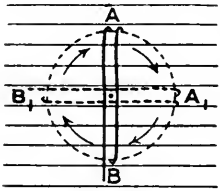


Fig. 269.

Hence we see that in such a rotating coil, the E.M.F. is reversed in direction once in each half revolution. Each turn may be regarded as made up of two active portions, which are really cutting the lines of force, and of two inactive portions which are sliding past the lines without cutting them, and which serve merely as conductors necessary to complete the circuit. The active portions will be the parts at A and B which are perpendicular to the plane of the paper; the inactive portions will be the parts actually shown in the figure from A to B and the corresponding portions on the further side. Consider one active conductor at A. It will be seen that, in passing from A to B, it is moving downwards through the field, and an E.M.F. in the same direction will be produced during the whole of the half revolution, which will have a maximum value at A_1, where the conductor is cutting the lines at the greatest rate, and which will be zero at A and B, where it is sliding along the field, and, for an instant, is cutting no lines. In passing back from B to A, it is moving *upwards* through the field, and hence the E.M.F. will be reversed in direction, again having a maximum value at B_1. This may be represented as in Fig. 270, where the angle of rotation is taken for abscissæ, and the E.M.F.'s at any instant are plotted as ordinates. It will be seen that the alternating E.M.F. passes through one complete cycle in each revolution. The number of complete cycles passed through per second is called the **frequency**. It may be remarked that the symmetrical curve shown in this figure implies that the coil is rotating with uniform velocity, and also that the field is uniform. With the same proviso, a similar curve would be obtained in Experiment 215. Again, it is quite immaterial as to whether the coil is circular or rectangular in shape.

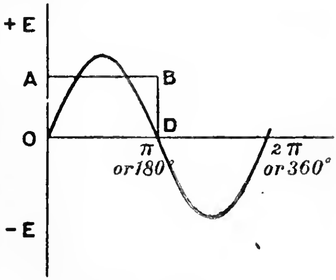


FIG. 270.

Example 2.—A coil of 50 turns, and mean diameter 40 centimetres, rotates about a vertical axis at the rate of 12 revolutions per second, at a place where H = ·18. Find the *average* value of the E.M.F. produced. Generally, we have

$$e \text{ (average)} = \frac{\text{lines cut} \times \text{number of turns}}{\text{time of cutting}} \text{ (absolute units)}$$

The lines threading the coil, when it is at right angles to the field, are given by $H \times \pi r^2$, and these lines are cut four times during one complete revolution. Whence

$$e \text{ (average)} = \frac{4 \times \cdot 18 \times \pi \times (20)^2 \times 50}{\frac{1}{12}} = 542{,}592 \text{ absolute units}$$
$$= \cdot 00543 \text{ volt}$$

The meaning of this result will be understood from Fig. 270. If the curved line represents the actual E.M.F. at any instant, then OA is the *average* value, where OABD is a rectangle having the same area as the space enclosed by the actual curve.

If, for the sake of illustration, we solve the problem from the point of view of active conductors, the argument is as follows :—

1 active conductor in one complete revolution cuts all the lines threading the coil twice—once moving down, once moving up.

∴ 1 active conductor in one revolution cuts $2 \times H \times \pi r^2$ lines,

i.e. 1 active conductor in one second cuts $12 \times 2 \times H \times \pi r^2$ lines.

Whence, by definition, e (average) $= 12 \times 2 \times H \times \pi r^2$ absolute units per active conductor.

Now, as each turn consists of two such active conductors in series, and as there are 50 turns, *i.e.* 100 active conductors in series, we have e (average) for whole coil $= 100 \times 12 \times 2 \times H \times \pi r^2$ absolute units, which is identical with the value previously obtained. We are not in a position to calculate the maximum or instantaneous values at present; for this see p. 427.

To Find the Direction of an Induced Electromotive Force.—An induced current evidently possesses energy, for it may be made to do work in various ways, and in any case heat is produced in the conductor carrying it. This energy must be supplied in some way, and, in the experiments just described, it can have been supplied only at the expense of mechanical work. Whilst an induced current was being produced, the moving part must have experienced a resistance to motion greater than that required to overcome friction. Our experiments were on too small a scale for this resistance to be directly perceptible, but there is no difficulty in demonstrating its existence, which, on a larger scale, is at once obvious. For instance, a dynamo generator requires very little power to drive it when it is not producing a current, but when it is allowed to give out a current, it requires power in proportion to its output of electrical energy, although the moving part is rotating as freely in air as before. It must be understood that no such resistance to motion is experienced when only an E.M.F. is produced. The circuit must be complete, so that a current can flow before any energy is expended in this way.

Exp. 218. Set up a suspended coil galvanometer, using a coil supplied for ballistic work, and produce a deflection by putting the ends of the two wires proceeding from it on the tongue (without touching) for an instant. If necessary, hold a silver coin in contact with one wire, and place this and the end of the other wire on the tongue. Notice that the spot of light oscillates backwards and forwards for a long time before coming to rest. Again set it swinging, and then bring the ends of the wires in contact with each other, thus completing the circuit through the coil. Notice that the result is very marked—the oscillations being powerfully "damped" and the coil speedily coming to rest.

The explanation is very simple. Whilst the coil was swinging in the magnetic field of the permanent magnet, an (alternating) E.M.F. was produced in the coil; but this, as stated above, does not demand any expenditure of energy, and therefore the coil oscillated until its energy of motion became gradually converted into heat (principally by air friction), and this took some time. When the circuit was completed, the induced E.M.F. was able to produce an induced current, the energy of which was necessarily derived from the energy of motion. Hence, this became rapidly used up, the coil moving as if against a resistance, or as if immersed in a viscous medium. Under these circumstances, the energy of motion is not merely converted into heat by air friction, but much of it appears as C^2r heat in the wire of the coil itself, and the process of dissipation is, therefore, more rapid.

This resistance to motion is due to the action of the magnetic field produced by the induced current upon the field produced by the permanent magnet, and we will now apply it to determine the direction of the induced E.M.F. in any given case.

Consider Fig. 271, which is almost identical with Fig. 255, although the method employed in drawing it is different. A conductor, shown in section in the figure, is placed in a magnetic field. It is then moved, say in the direction P, and it is required to find the direction of the induced E.M.F. Assume that an induced current flows, then at once a new field appears around the conductor, which we represent by drawing a circle. If we can find *the direction of this field*, *that of the current* at once follows. Now, we cannot mark it clockwise, because then the two fields would be in the same direction on the right-hand side, and the forces between them would tend to drive the conductor in the way it is being moved. This is evidently wrong, as it would lead to something like perpetual motion, and hence we mark the circle anti-clockwise, which means that a force comes into play tending to oppose the motion, and against which work must be done in moving the conductor, as is found to be the case by experiment. Hence, by applying the results given on p. 241, we find that, in the case shown in the diagram, the current flows outwards from the plane of the paper (and even if there is no current, that must be the direction of the E.M.F.).

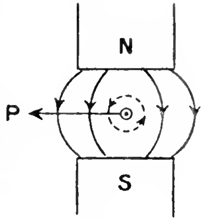


FIG. 271.

The direction can be obtained in all cases by applying the above argument to a suitable figure, although sometimes it is more convenient to adopt the following method. Suppose it is required to find the direction of the induced E.M.F. when one pole of a magnet is inserted into a coil of wire (Fig. 272).

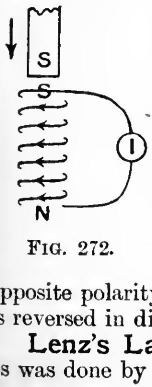


FIG. 272.

If an induced current flows, the circuit being completed, work must be done against some opposing force in inserting the pole, and, therefore, the induced current must produce a polarity in the coil oppositely directed to that of the magnet. For example, if a S pole is approaching the coil, the current in the coil must give it the polarity shown in the figure, from which the direction at once follows. Similarly, work must be done in removing the pole; this requires an opposite polarity, and, therefore, the induced current, during removal, is reversed in direction.

Lenz's Law.—All such results can be conveniently summarised, as was done by Lenz many years ago, in the statement :—

When induced currents are produced by motion, they are always in such a direction that they tend to stop the motion which produces them. Illustrations and applications of this fact, which is commonly known as Lenz's law, are often met with. The damping of a galvanometer, due to closing its circuit, described on p. 354, is an instance of it. It is particularly noticeable in the suspended-coil type, although it occurs to a smaller extent in any form of instrument. Again, it has been stated that, in the original form of suspended-coil galvanometer, the coil is wound on a metal frame, and that in Ayrton and Mather's form it is enclosed in a silver tube, in order to make it "dead-beat." The explanation is now obvious, for induced currents are produced in the frame or in the tube as they swing in the field, and the reactions must damp the motion.

The same principle, for steadying the motion of some moving part, is applied in many commercial instruments. The only essential point is the attachment of a suitably shaped piece of metal (preferably silver or copper, as they are the best conductors) to the moving part, which then is arranged to move in a narrow gap between the poles of a magnet.

Eddy Currents or Foucault Currents.—That induced currents are produced in any piece of metal—and not merely in wires—may be easily demonstrated. Such currents are often known as "eddy currents," or, although they were discovered by Joule, "Foucault" currents. If an exceptionally powerful electromagnet is available, any sheet of metal will show a perceptible resistance to motion when moved rapidly between its poles, and if the motion is continued the metal becomes heated.

If such a magnet is available, the following experiment may be made.

Exp. 219. Attach a thread to a lump of copper (an old twopenny piece answers excellently). Spin it until there is a good twist on the thread, and then let it untwist; whilst rotating rapidly, bring it between the poles of the electromagnet, described on p. 249. Notice that it will at once slow down perceptibly, and that it will speed up again when the magnet circuit is broken.

If an alternating current is available, a simple method of demonstrating the heating effect, due to eddy currents in masses of metal, may be employed.

Exp. 220. Connect one of the electromagnet coils—having first removed its iron core—to alternating supply mains, taking care that the current does not exceed 3 or 4 amperes. Place an iron bar, about $\frac{3}{8}$ in. diameter, inside the coil. Notice that, in a short time, the part inside the coil will become too hot to touch. Repeat the experiment with a bundle of iron wires of about the same section. Notice that this will remain cool, the eddy currents being greatly reduced by thus *laminating* the iron. (Part of the heat produced in the bar, especially if not very soft iron, is due to hysteresis; see Chapter XXV.).

Arago's Rotations.—Arago discovered (some years before Faraday's discovery of induced currents supplied the explanation) that if a compass-needle, or a pivoted or suspended magnet, be oscillated very close to a horizontal metal disc, it came to rest much more rapidly than usual, *i.e.* its motions are damped. He also found that, the better the conductivity of the metal, the greater the effect. Evidently, induced currents are produced in the disc by the moving lines of force of the magnet, and these must tend to stop the motion which produces them. The forces concerned are, however, mutual, and hence we might infer that a moving disc would have some effect on a stationary magnet. Arago rotated a copper disc very rapidly close to a magnetic needle, and found that the needle was also set in rotation in the same direction. The reaction between the field due to the induced currents and that due to the magnet again tends to stop the motion, but as the magnet is now movable, it is dragged after the disc. Conversely, if the magnet is rotated rapidly close to a pivoted disc, the latter is set in motion. This action has become of great practical importance in connection with "induction motors" working with alternating currents.

Actions due to Starting and Stopping of Currents.— Up to the present, we have moved either a magnet or a conductor in order to obtain an induced E.M.F. It is, however, possible to obtain a motion of lines of force by another method.

Exp. 221. Procure a *long* piece of "twin" bell wire (*i.e.* two insulated conductors bound together). Connect the ends of one wire to a sensitive galvanometer, G (Fig. 273), and those of the other to a cell (*b*) and key (*k*). Close the circuit and notice the direction of the "throw." Release the key and notice that the throw is in the opposite direction. In such experiments, it is usual to speak of the wire (AB) connected to the cell as the *primary*, and the one in which the induced current is produced (CD) as the *secondary*. Evidently, the effect obtained in the experiment is due to the lines of force produced in the primary cutting the secondary as they appear and disappear.

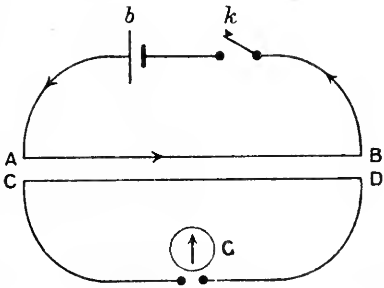


FIG. 273.

To find the direction of the induced E.M.F. at "make" and "break" respectively, we can apply Lenz's law. We may regard the starting of the primary current as equivalent to bringing the current *suddenly* from an infinite distance up to its actual position, and the stopping of it as equivalent to taking it away again. In the first case, the secondary current must produce a field which tends to repel the primary field, and hence, at "make," it must be in the *opposite* direction to that in the primary. At "break," it must oppose the withdrawal of the primary, and must therefore be in the *same* direction as the primary current.

The actual existence of such attractions and repulsions can easily be shown by experiment.

Exp. 222. Drill a small hole through a wooden lath (a metre scale answers well), put a pin through the hole to serve as a pivot, P, (Fig. 274), and support it on a wooden stand slotted at the top. To one end of the lath attach, with wax or any suitable cement, a copper disc, D, and balance the arrangement by means of a weight, W—a small piece of sheet lead is convenient. Bring the disc immediately above one pole of the electromagnet (Fig. 190)—the other pole may be temporarily removed. Excite the electromagnet and notice that the disc is *momentarily repelled.* Demagnetise it by breaking its circuit, and notice that the disc is momentarily *attracted.*

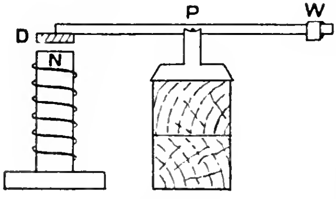


FIG. 274.

Remembering that the magnet may be regarded as a whirl of molecular currents in the same direction as the current in the coil surrounding it, it is evident that the experiment is equivalent to (1) bringing such a current suddenly up to the disc and (2) suddenly removing it. In the first case, the induced current in the disc is in the opposite direction to the molecular currents, and thus we get repulsion; in the second case, the induced current is in the same direction, and attraction takes place.

In Experiment 221, two straight wires were used for the sake of simplicity, but that method is not the most convenient, as a considerable length is needed to obtain a well-marked effect. It would be an improvement to first wind the double bell wire into a coil, but the most satisfactory arrangement is obtained by using two separate coils, one of which will slip inside the other. In the following experiments, it will be found convenient to use one of the coils of the electromagnet shown in Fig. 190, and another of smaller diameter made to fit easily inside it. (This small auxiliary coil is very useful, and will be required for a number of experiments; it may consist of a few layers of 20-gauge wire, wound on a wooden bobbin, with a hole through it in which an iron rod may be placed.)

Exp. 223. Place this small coil inside the larger and connect one of the coils in circuit with a cell and tapper key, and join the other to a galvanometer (if a sensitive instrument is used, protect it by means of a shunt). Notice that a "throw" is produced when the primary circuit is closed, and another in the opposite direction when it is broken. Insert an iron rod, and notice that the effect is very much greater. Interchange the connections, so that the coil previously used as primary is now the secondary, and notice that the general result is unaltered.

Remembering that the primary coil, when carrying a current, behaves as a weak magnet, we see that this experiment is equivalent to inserting such a magnet into, and to withdrawing it from, the secondary coil. It is generally convenient to use the inner coil as primary, although the experiment shows that it is not absolutely necessary; what is really essential is that the field produced by one coil (the primary) should cut the other coil (the secondary) as it appears and disappears.

Now the E.M.F. produced in the secondary depends only upon three factors: (1) the number of lines of force that cut it, (2) the time they take to appear and disappear, and (3) the number of turns in the secondary (for an E.M.F. is produced in *each* turn and these E.M.F.'s are added together). Hence, we learn that the secondary E.M.F. can be made as large or as small as we please, by using many or few turns, and that it is quite independent of the E.M.F. of the cell used in the primary circuit, except in so far that increasing the primary E.M.F. increases the primary current, and, therefore, the primary field. It is to this property that induction coils and commercial transformers owe their usefulness — they enable any desired E.M.F. to be obtained in a simple manner. The ordinary induction coil is more especially a device for obtaining a very high E.M.F., and its actions may be illustrated as follows:—

Exp. 224. Connect the large coil to two pieces of metal of sufficient size to form convenient handles, and join up the smaller inner coil to a tapper key and a battery of from three to six cells in series. Hold the metal handles while contact is made and broken. Nothing will probably be felt when the primary circuit is made, but notice that a distinct, though feeble, shock is felt when the circuit is broken. Place an iron rod inside the coils and repeat the operations. Now notice that a shock may be felt at "make" and also at "break," but that the latter is much the stronger, and that it is very much more powerful than it was without the iron core.

As rather a high E.M.F. is necessary to give a perceptible shock, the experiment shows that the induced E.M.F. in the secondary must have been *greater* than that of the battery; and it also brings out two other facts: (1) the importance of the iron core, and (2) the difference between the induced E.M.F. at "make" and that at "break." The effect of the iron core is readily understood—we know that the core increases enormously the strength of the primary field; but it is not so obvious why the two E.M.F.'s should differ in strength, seeing that they are due to the same lines cutting the same number of turns. The best way to investigate the matter would be to use an oscillograph (see p. 571) to indicate the rate of rise and fall of the current, but such an instrument is not likely to be at the disposal of the student. If it were, he would learn that the primary current, and hence the field due to it, takes *longer* to grow to its full strength when the circuit is made, than it does to die away when the circuit is broken, and, as the induced E.M.F. depends upon its *rate* of cutting, it is necessarily weaker at "make" than at "break."

This difference in time is due to an effect generally known as self-induction.

Self-induction.—**Exp. 225.** Break the circuit of a battery when its terminals are joined by a straight piece of wire. Only a faint weak spark is obtained, although the current strength may be considerable. Put a large coil of wire in circuit, *e.g.* the electromagnet coil used in the previous experiment. The spark is now much brighter, although the current itself is much less on account of the increased resistance in the circuit. Insert the iron core of the electromagnet; the spark is still more intense.

(A more striking method of carrying out these experiments is to connect one wire to a file and to draw the other over its surface. Contact is thus rapidly made and broken.)

Such effects will often be noticed during experimental work. Whenever a circuit contains coils of wires with iron cores, *i.e.* whenever it is linked with strong magnetic fields, a bright spark is produced when the circuit is broken. It is easy to show that this is due to a *new* E.M.F. of considerable magnitude and not to the E.M.F. acting in the circuit before it was interrupted.

Exp. 226. Connect the large electromagnet coil to a few cells in series and a tapper key. Also connect metal handles to the coil (Fig. 275), so that the body of the person holding them may act as a shunt. When the circuit is "made," nothing is felt, whether there is an iron core or not; but, when the circuit is broken, there is a shock, which is greatly increased in intensity by the presence of the core.

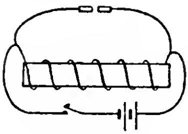


FIG. 275.

We infer from this experiment that the P.D. between the ends of the coil rises suddenly to a high value when the circuit is broken, and it is not difficult to understand that this effect must be due to an E.M.F. generated by the lines of force cutting the turns of *the coil itself* as they disappear.

Such actions are of great importance, and must occur to some extent in any circuit whatever. The student should keep clearly in view the general fact that the existence of a current in a circuit implies the existence of both electric and magnetic lines of force in the space surrounding it. The electric lines must have been present *before* the current began to flow, whereas the magnetic field did not come into existence until the current started. At present we are considering more especially the magnetic field, but in doing so we must not forget the existence of the other, or unnecessary difficulties will be created at a later stage of progress.

It is extremely important to grasp the following fact: When a magnetic field appears and disappears, it produces an E.M.F. in all conductors cut by it, and it is immaterial (*a*) whether these conductors belong to an independent circuit, or (*b*) whether they constitute the circuit in which the magnetising current is flowing.

We shall speak of the E.M.F., which we apply to any circuit to produce a current therein, as the "impressed" or "applied" E.M.F., and we shall denote it by E. We know that when the current is *starting*, an independent induced E.M.F.—which is termed the E.M.F. of self-induction and which we shall denote by e_s—must exist for a certain time (usually a small fraction of a second) due to the magnetic field, as it forms in the surrounding space, cutting the circuit. This E.M.F. is in the *opposite* direction to the impressed E.M.F., and hence the current rises gradually and does not reach its full value until e_s has disappeared. On the other hand, when the circuit is broken and the current is *stopping*, the process is reversed; the field, as it disappears, cutting the circuit in the opposite direction, and another self-induced E.M.F. is produced, which is now in the *same* direction as the impressed E.M.F., and which tends, therefore, to prolong the duration of the current. But whereas the value of e_s at "make" cannot possibly be greater than the applied E.M.F. (otherwise the current could not rise in value), its value at "break" is not limited by any such condition, and when the break is sudden, its instantaneous value may be very great. This explains the results obtained in Experiments 224 and 226. Certain forms of motor ignition—known as the "low-tension" system—depend on this principle. A battery current is sent through a coil of wire wound on an iron core, and the circuit is suddenly broken *inside* the chamber containing the gaseous mixture to be ignited.

Inductive and Non-inductive Circuits.—It is usual and convenient to speak of circuits (or parts of circuits) as being *inductive* or *non-inductive*, according to the power they have of showing these effects in a marked degree or not, *e.g.* a circuit containing electromagnets is typically inductive, and one consisting of short straight wires is practically non-inductive. It must, however, be borne in mind that no circuit is absolutely non-inductive, for, even with a straight wire, some magnetic field must exist around it, and the term non-inductive must therefore be taken to mean that the effects due to self-induction are so small as to be negligible. On the other hand, it is possible, and often necessary, to make some *portion* of a circuit almost completely non-inductive, as for example in the coils of resistance boxes.

Again, the magnetic field around a *steady* current represents a certain amount of energy stored up in the surrounding space. This energy was supplied at the expense of the circuit while the current is starting, and it is paid back to the circuit when the current is stopped (not necessarily in the form of a discharge at a very high voltage, for the applied E.M.F. may be cut off without actually breaking the circuit). Hence, the starting of a current in a circuit resembles, in a marked degree, the starting of some heavy mass, such as a fly-wheel. It cannot be started suddenly; nor does it naturally stop suddenly, and if it be made to do so compulsorily, the result is disastrous. The amount of energy associated with it, when in steady motion, and the magnitude of all the effects depending on that energy, will vary with its mass, so that an inductive circuit is the analogue of a body of great mass, and *vice versâ*.

Such actions are of special importance in connection with alternating currents.

Exp. 227. Measure the resistance of one of the electromagnet coils by Wheatstone's bridge method. Connect it to alternating mains, having put an ammeter in series with it and a voltmeter across it. (Take any necessary precautions to avoid too large a current.) Calculate the current from the measured resistance, and then observe that the ammeter reading is very much less than this value. Hence, when alternating currents are used, an *inductive* resistance cannot be measured by the method indicated in Experiment 165, p. 258, although correct results would be obtained with continuous currents.

Exp. 228. Insert an iron bar in the electromagnet coil, and notice that the current is reduced. Add more iron, and notice that there is a further reduction of current, the smallest value being obtained when the iron core belonging to the coil is inserted. (This should soon be removed, or, for reasons already given, it will get hot.)

This effect is really due to the presence of an E.M.F. of self-induction of variable value, although the experiment seems to indicate that the coil has a variable resistance, and consequently such a coil with a movable iron core may be used, instead of a resistance, to regulate the strength of an alternating current. A coil, specially made for the purpose, is known as a "choking coil." As such coils may be given a very low resistance, this method of regulation wastes much less energy in the form of heat (for the number of heat units varies as C^2r). The subject is further discussed in Chapter XXVI.

Induction Coils.—Induction coils are devices for obtaining very high secondary voltages. During recent years they have acquired considerable commercial importance, owing to their use in wireless telegraphy and in connection with X-ray work. The principle is fully illustrated in Experiment 223, and the practical form merely represents improvements in details. As the secondary E.M.F. is produced only at make and break—chiefly at break—it is necessary to have some device for automatically breaking the circuit many times per second. With large coils, an independent arrangement is now always used for this purpose, but the simplest and best-known **contact-breaker** (shown in Fig. 276, taken from Brooks and James' *Electric Light and Power*) is a modification of the electric bell principle. Its construction will be sufficiently evident from the diagram. A soft iron armature nearly touching the core is fixed at the top of an elastic strip of brass, which on its other side is fitted with a short piece of thick platinum wire, normally in contact with another similar piece of platinum attached to a screw head. It will be seen from the connections that the primary current passes across this contact until the circuit is broken by the pull of the core on the armature. Then the core loses its magnetism and the strip springs back to remake contact. The lower screw, moving through the insulating washer, I, is merely a device for adjusting the tension of the spring, thereby affording a means of regulating the strength of current required to break contact. It is desirable to insert a current reverser, R, in the primary circuit.

The Coil Windings.—In constructing induction coils the point of supreme importance is the effective insulation of the secondary.

The **primary** usually consists of two or three layers of fairly

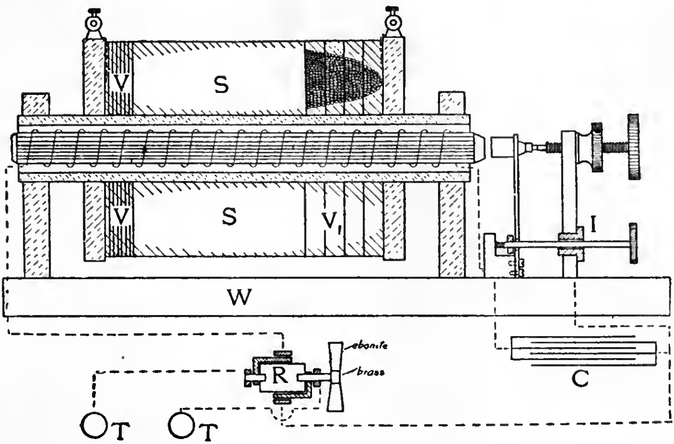


FIG. 276.

S, Space occupied by secondary winding.
V, Vertical insulation.
V_1, Vertical insulation, drawn further apart to show wire more clearly.
W, Wooden base.
C, Condenser.
R, Primary current reverser (actually mounted on base W).
I, Insulating washer.
T, Primary terminals.

stout copper wire (which may be cotton-covered, but is preferably silk covered, especially in large coils), wound upon a core of iron wire. This is placed inside a thick ebonite tube, which may with advantage be somewhat longer than the secondary.

In winding the **secondary**, S (shown in section in the diagram), it is necessary to remember (1) that an E.M.F. is produced in each turn and that the turns are in series, so that the difference of potential between any point of the secondary and the earth will be greatest towards the *ends*, where it will, therefore, be essential to have the most effective insulation; and (2) if the secondary is wound backwards and forwards in the usual way, there will be an enormous difference of potential between the first turn of one layer and the last turn of the next layer which lies immediately above it, and consequently there arises a great danger of sparking through the insulation. To avoid such danger, horizontal layers of insulation are practically useless, and thus it becomes necessary to ensure that there is only a small difference of potential between any turn and those immediately adjacent to it. We can do this best by winding the wire for a short distance (say $\frac{1}{10}$ to $\frac{1}{16}$ of an inch) in a horizontal direction, then returning the wire on itself, and so on. It is convenient, for this purpose, to use a special winding device, consisting of two smoothly turned brass checks, which can be separated by a thin disc of wood to form a narrow deep bobbin.

The wire, which should be fine (say 36-gauge), and must be silk covered, is run through melted paraffin-wax, and then wound on the bobbin until a sufficient depth is obtained. The narrow coil thus formed is held together by the wax, and should be carefully removed, placed between thin discs of paraffined cardboard (or of ebonite), and the whole sweated together with a warm flat-iron. This is paired off with another similar coil—the inside ends being soldered together and pushed out of the way—and the two coils then sweated together as before.

The result forms *one element* of the winding, and *both* the free ends are on the outside. Perhaps 50 to 100 such elements are required for a large coil, and each should be tested separately to stand a voltage much greater than they will have to produce during use. It is usual, as will be seen from the diagram, to wind less depth on those coils which are to be situated near the ends of the instrument (where the need of insulation is greatest), and to gradually increase the depth of the winding towards the middle. They are then placed in the proper order on the ebonite tube containing the primary, the necessary connections being made by soldering. The interstices between the vertical discs of insulation are next filled in with melted paraffin-wax, until the whole forms a solid cylinder (shown in the figure by the shaded portions surrounding the secondary, S), which is then covered with a thin sheet of ebonite.

Coils may be made in this way, which are capable of giving very long sparks without risk of piercing the insulation.

A most ingenious and valuable improvement has recently been patented by Miller, who has devised a method of winding the sections continuously without breaking joint, each section containing only one conductor horizontally—like a watch-spring—so that there are about 1000 separately insulated sections in a coil of moderate size.

The Condenser and Its Functions.—The most important detail yet to be mentioned is the condenser, C, connected as a shunt to the spark-gap of the contact-breaker, and generally placed inside the base, W, although for the sake of clearness it is shown diagrammatically in the figure.

From the result obtained in Experiment 225, we should expect to get considerable sparking at break, due to self-induction, and this would mean serious heating and wear of the platinum contacts, as as well as much waste of energy. This would actually happen if no condenser were used, or one of too small capacity. With a suitable condenser, however, the sparking at the contact-breaker is greatly diminished, and the length of the secondary spark is increased. This action of suppressing or of neutralising the effects of self-induction will not be completely understood until Chapter XXIII. has been read. However, we know from Experiment 226 that, when the primary circuit is broken, the lines of force, in disappearing, set up not only an induced E.M.F. in the secondary but also another in the primary, which is in the same direction as the current previously flowing, and which, therefore, tends to prolong its flow. This, in itself, is very undesirable, for the magnitude of the E.M.F. depends on the *rapidity* with which the field, and, therefore, the current disappear. As the capacity of the terminals forming the spark-gap is small, the P.D. between them rises suddenly to a value sufficiently high to spark across the gap, thus prolonging the current and diminishing the secondary E.M.F.. The presence of the condenser greatly increases the effective capacity of the terminals, and the P.D. across them can only rise at the rate the condenser charges up, and may never reach such a high value as before.

Again, it will be noticed that the two coatings of the condenser are always in conducting communication through the battery and the primary, and hence, immediately after charging, it discharges itself through this path. This discharge means a rush of electricity round the circuit in the *opposite* direction to the dying-away current, and this rush itself tends to produce an induced E.M.F. in the secondary in the right direction, and thus to strengthen the secondary spark at break.

Such a condenser is usually made of sheets of tinfoil interleaved with paraffined paper.

When the primary current is alternating, a contact-breaker and a condenser are not required, but the secondary spark, with an ordinary induction coil, will be shorter and of an entirely different character. The changes in the field strength are now comparatively gradual, and the induced E.M.F. is, therefore, less. A direct current, interrupted by a contact-breaker, is, in fact, desirable when very high voltages are required, in order to obtain the necessary rapid changes in field strength.

Transformers.—For many commercial purposes, it is useful to have a method of obtaining any required E.M.F., high or low, from a given primary E.M.F.; and induction coils made for this purpose and working with alternating currents are known as **transformers.** Their construction and theory are given in Chapter XXVI.; at present, it is sufficient to show that a contact-breaker may be dispensed with when alternating currents are used.

Exp. 229. Connect one of the electromagnet coils to alternating mains, taking the necessary precautions. (Probably it will stand 100 volts pressure for a short time, the current rising only to 3 or 4 amperes on account of the choking effect.) Place inside it the small coil, already used in Experiment 223, to act as secondary, having joined its terminals by a short piece of fine iron wire (*e.g.* that used for bouquets). If the wire is of a suitable thickness, it may become very hot without being luminous. Insert gradually an iron bar. Notice that the wire now becomes dull red, and that the temperature increases as the bar is lowered, until it becomes white hot, and finally breaks.

Exercise XVIII

1. If a current is flowing through a coil, what effect is produced by inserting into the coil and then withdrawing rapidly, (*a*) a bar of wood, (*b*) a bar of iron? (Oxford Local, Senior, 1908.)

2. State generally under what circumstances a current is induced in a coil. Describe some method by which these circumstances can be realised in practice. (Camb. Local, Senior, 1907.)

3. Describe a machine which is set in rotation by passing an electric current through some of its movable parts. What is the effect on the current if, while it is still flowing, the movement of the machine is stopped? (B. of E., 1895.)

4. The binding-screws of two astatic galvanometers, a considerable distance apart, are connected by wires, so that their coils form a continuous circuit. If the needles of one galvanometer are moved, those of the other are disturbed. Explain fully this effect.

5. Two equal magnets, each bent into a semicircle, are fastened together with like poles in contact, so as to form a complete circle, and a copper ring, through which one of the magnets had been thrust before the two were fastened together, is carried round and round the circle at a uniform speed. Show how the currents induced in the copper ring by the magnets vary in strength and direction as the ring passes different parts of the magnetic circle.

6. A metal ring is put round the end of a bar magnet. Upon bringing a mass of soft iron near to this end of the magnet, a current is produced in the ring. Show what motions (*a*) of the magnet and (*b*) of the ring will produce a similar current in the absence of the soft iron.

7. A piece of wire is bent into the form of a rectangle, and the ends are joined. It is laid upon a horizontal table with two sides pointing magnetic north and south. If the rectangle be turned about the east side as a hinge so as to lie on the table to the east instead of to the west of it, what will be the direction of the current which circulates in the wire during its motion, in consequence of the earth's magnetism?

8. When a circular metallic hoop is rotated in the earth's magnetic field, electric currents are generally produced in it. In what positions of the axis of rotation will the induced currents be the greatest and least respectively? Give reasons for your answer.

9. A circular hoop of wire is suddenly twisted half round about a vertical axis. What is its electrical condition during this movement? Determine in what position of the hoop as it moves the E.M.F. is the largest. (B. of E., 1897.)

10. A vertical hoop of wire, at right angles to the magnetic meridian, is

quickly but with uniform speed turned through 180° about a vertical axis, its originally eastern half moving northward at first. State the direction in which the induced current passes round the wire, and determine the position of the hoop in which the induced E.M.F. is the greatest. (B. of E., 1898.)

11. A coil of wire, whose ends are joined to the terminals of a galvanometer, is continuously and rapidly rotated about a given axis. Explain the effect upon the needle. (B. of E., 1900.)

12. A light metal ring is suspended over the end of a solenoid: if a large current is suddenly sent through the solenoid, show that the ring will be repelled. (B. of E., 1903.)

13. An iron hoop is held in the magnetic meridian, and is allowed to fall over towards the east. Explain why an electric current traverses the hoop, and state whether the current would flow north or south in the part of the hoop which touches the ground if the experiment were performed in England. (B. of E., 1894.)

14. State the law of the induction of currents; illustrate your statement by describing the behaviour of two parallel coils, A and B, placed side by side, when currents are started and stopped through A; and when A, while conveying a current, is moved towards and from B. (B. of E., 1901.)

15. A bar magnet is suspended on a stirrup by a string, and oscillates in a horizontal plane. How are the oscillations affected (if at all) when a thick non-magnetic metal plate is placed horizontally beneath the needle so as to be close to without touching it? (B. of E., 1896.)

16. Explain how it is that a disc of copper, revolving in a horizontal plane below a magnetic needle, causes the needle to turn in the same direction as the disc.

17. Describe an induction coil, and explain why the iron core is made of wire. (B. of E., 1902.)

18. In an induction coil, show how the condenser is connected, and explain its function.

19. A copper ring is hung by a torsionless thread between the vertical parallel and flat opposed poles of an electromagnet. The plane of the ring is vertical, and is inclined at 45° to that of either pole face. If a current is started round the magnet, the ring turns through a moderate angle, but quickly comes to rest. If it is replaced in its former position and the current is stopped, it starts twisting in the opposite direction and keeps on spinning. Account for these actions. (Lond. Univ. B.Sc., 1902.)

CHAPTER XXIII

ELEMENTARY THEORY OF INDUCTION

SUPPOSE that Z lines of force cut N turns in t seconds, then, from the definition of unit E.M.F. (p. 351), we have

$$\underset{\text{(average)}}{e} = \frac{Z \times N}{t} \text{ absolute units.}$$

If the current has a total resistance of R absolute units, then by Ohm's law the *average* current is given by

$$\underset{\text{(average)}}{i}[1] = \frac{Z \times N}{t \times R}$$

Now in most cases the actual E.M.F. and current vary rapidly in strength during their brief duration, and their average values are of little use to us. If, however, we rewrite the last expression in the form :—

$$\underset{\text{(average)}}{i} \times t = \frac{ZN}{R}$$

the product (average current × time) is a definite "quantity" (Q), and we have :—

$$Q = \frac{ZN}{R} \text{ absolute units.} \qquad \text{(i.)}$$

Hence we learn (*a*) that the *quantity*, which flows round a circuit due to induction occurring therein, is *independent of the time* taken by the process ; and (*b*) that it varies directly as "the amount of cutting"[2] which takes place, and inversely as the resistance of the circuit.

In the above expression, the absolute unit of quantity is the quantity conveyed by 1 absolute unit of current in 1 second. This is 10 coulombs, where 1 coulomb = 1 ampere-second. Hence, if we

[1] As already stated, where i occurs in an equation, it indicates that absolute units are to be used ; and when C occurs, that practical units are to be employed.

[2] When Z lines cut N turns, we shall call the product ZN "the amount of cutting." It is often said to be ZN "Maxwells," but this term has scarcely passed into general use.

write $Q = \frac{10ZN}{R}$, the result is in coulombs, where R is expressed in absolute units.

If we wish to use practical units throughout, we must write

$$\underset{\text{(average)}}{e} = \frac{Z \times N}{10^8 t} \text{ volts}$$

$$\underset{\text{(average)}}{C} = \frac{Z \times N}{10^8 t.R} \text{ amperes, where R is in ohms}$$

$$\therefore \quad C.t = Q = \frac{ZN}{10^8 R} \text{ coulombs.} \qquad \text{(ii.)}$$

In equation ii., if we put R in ohms, Q is in coulombs; and if we put R in absolute units, Q is in absolute units also. Equations i. and ii. are important, because Q can be measured by means of a ballistic galvanometer (p. 296). Various applications will be met with later.

Example 1.—A coil of inductive area, 15,000 square centimetres, is connected to a galvanometer. The coil lies flat on the table, and when it is turned over, the galvanometer is momentarily deflected. Discharging a condenser of 1 microfarad capacity, charged to 1·5 volts through the same galvanometer, produces the same throw. If the vertical component of the earth's field is ·4 C.G.S. units, what is the resistance of the coil and galvanometer together with the connecting leads?

By the term "inductive area" is meant the product of the area and the number of turns, *i.e.*

$$\text{Area} \times \text{number of turns} = 15{,}000$$

(such a coil, which is usually of rectangular or of circular shape—although it may be of any shape whatever—is often called an "earth inductor").

If A = area of surface, N = the number of turns, and V = the vertical component, then, as it lies on the table, V × A lines of force pass through it, and when it is turned over these lines are cut *twice* (see p. 351).

From equation ii. we have

$$Q = \frac{ZN}{10^8 R} \text{ coulombs, where R is in ohms}$$

$$\therefore \quad Q = \frac{2 \times V \times A \times N}{10^8 R} \text{ coulombs, where } A \times N = 15{,}000$$

$$\textit{i.e.} \quad Q = \frac{2 \times \cdot 4 \times 15{,}000}{10^8 R} \text{ coulombs.}$$

For the condenser discharge, we have

$$Q = EK, \text{ where, if E is in volts and K in farads, Q will be in coulombs}$$

$$\therefore \quad Q = 1{\cdot}5 \times \frac{1}{10^6} \text{ coulombs.}$$

Now these two quantities are equal by the terms of the question,

$$\therefore \quad \frac{2 \times {\cdot}4 \times 15{,}000}{10^8 R} = \frac{1{\cdot}5}{10^6}$$

$$\text{whence} \quad R = 80 \text{ ohms}$$

It should be noticed that the question as set ignores the influence of "damping," considered later in Chapter XXIV., the solution being only exact, when we assume that no damping exists. Neither is it correct to say that the damping factor will be the same in both experiments, and, therefore, will cancel out. The conditions are different, and so will be the damping. In practice, serious errors may arise when these facts are ignored.

It should also be noticed that, when R is known, the experiment suggests an instructive method of measuring V.

Example 2.—Find the strength of the field between the flat-ended poles of a large electromagnet from the results of the following experiment. A small coil of fine wire, having 30 turns and a mean area of 2 square centimetres, is connected in series with a ballistic galvanometer and an earth inductor coil, having 500 turns and a mean area of 480 square centimetres. The small coil is placed between the poles, and suddenly pulled out of the field, the resulting throw being 153 divisions. The inductor coil is then quickly turned over (as in Example 1), and the throw is 102 divisions. Neglect any corrections due to damping, and assume that $V = {\cdot}4$.

If F be the strength of the field due to the electromagnet, then, when the small coil of area A square centimetres is placed in the field, it is threaded by F × A lines, and when it is removed, these are cut *once*.

Let Q be the quantity producing the throw of d divisions.

$$\text{Now, as we have seen, } Q = \frac{Z \times N}{R} \text{ absolute units} \propto d$$

$$i.e. \quad Q = \frac{F \times A \times N}{R} \propto d$$

$$\text{whence} \quad Q = \frac{F \times 2 \times 30}{R} \propto 153. \qquad \text{(i.)}$$

In the second case, let Q_1 be the quantity producing the throw of d_1 divisions,

$$\text{then} \quad Q_1 = \frac{2V \times A_1 \times N_1}{R} \text{ absolute units} \propto d_1$$

$$\text{whence} \quad Q_1 = \frac{2 \times \cdot 4 \times 480 \times 500}{R} \propto 102$$

$$\text{We, therefore, have} \quad \frac{F \times 2 \times 30}{2 \times \cdot 4 \times 480 \times 500} = \frac{153}{102}$$

$$\text{whence} \quad F = 4800 \text{ lines per square centimetre.}$$

(In this example the resistance of the circuit is the same for both throws, and the damping factors will be equal and cancel out.)

Back Electromotive Force.—Consider any one of the experiments mentioned in Chapter XXII. in which motion is obtained by sending a current through a conductor in a magnetic field. The current is produced by creating a P.D. between the ends of the conductor, and we will assume for convenience that this impressed P.D., which we will denote by E, is kept constant. If r be the resistance of the conductor, then, as long as it is *at rest*, the current through it will be $\frac{E}{r}$; but, as soon as it *moves*, an induced E.M.F., e, is produced in it, due to its motion in a magnetic field, just as it would be if it were moved by any other means. This is in the opposite direction to the impressed E.M.F., and is usually called a "back E.M.F." The current through the conductor is now $C = \frac{E - e}{r}$, and is, therefore, *less* than before. Again, e is proportional to the speed, and, as we have assumed E to be constant, the *faster* the conductor moves, the *smaller* will be the current through it.

The student has been accustomed to consider only one E.M.F. in a circuit, the current flowing naturally with it. It must now be pointed out that whenever this occurs, *all the energy expended in the circuit is converted into heat*, and is necessarily wasted, unless it is required in that form for lighting or heating purposes.

A current flowing in the *direction* of an E.M.F., i.e. *down* a slope of potential, resembles a stream of water flowing downhill, and dissipating its energy of motion as heat by friction. In order to make the stream do some useful work, *e.g.* to drive a mill, it must turn a wheel or turbine, which *resists* being turned, *i.e.* sets up a back electromotive force, and the greater this resistance to motion the slower will be the flow of water, which, it should be noticed, is still flowing *down* a slope of potential. We see that a maximum current of water does not mean that a maximum amount of work is being done; it merely indicates that there is the least resistance to its flow, and that occurs when *no* work is being done. Exactly the same reasoning holds good with regard to a falling body. Falling freely, it does no work, and its energy is ultimately converted into heat. If it is to do work, it must act against some resistance to motion (other than friction), which is analogous to the back E.M.F. we are now considering. (Further remarks on this point are made on p. 465.)

An electric current can do no work (other than heat production) except by flowing against an opposing E.M.F.—being, of course, impelled by a greater E.M.F. This is expressed by the equation $C = \frac{E - e}{r}$, already given in one important case in the chapter on electrolysis. Various other applications will be met with later.

We may interpret it by noticing that the watts *given* to the circuit (or the part of the circuit we are considering) must be EC, and the portion expended as heat must be C^2r, so that $EC - C^2r$ watts are left to be accounted for.

$$\text{But} \quad EC - C^2r = (E - Cr)C = eC,$$

i.e. *the product of the back E.M.F. and the current represents the watts converted into some form of work.* We shall now use this principle to obtain certain important formulæ in a simple way.

To Find the Force on a Conductor carrying a Current in a Magnetic Field.—Let a straight conductor of length l centimetres carry a current, i, at right angles to the lines of force of a uniform magnetic field of strength B. We know that a force, tending to drive it sideways across the lines, acts upon it. Let this force be p dynes. If we allow it to move under the influence of this force, a back E.M.F. will be produced in it, and the current does work in moving it. (If, by mechanical means, we move it in the *opposite* direction, we do work *against* the force. and the induced E.M.F. is now assisting the impressed E.M.F., so that the energy we expend is thereby converted into electrical energy.) As the current, when the conductor is at rest, is not the same in magnitude as when the conductor is in motion, it must be postulated that i is the current flowing while the conductor is moving with uniform velocity.

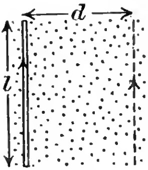


FIG. 277.

Let it so move in the direction of the force through d centimetres in t seconds. The work done in moving it will be force × distance, *i.e.* $p \times d$ ergs.

Again, if e be the *back* E.M.F. during motion, the work done by the current $= e.i.t$ ergs.

$$\therefore \quad p.d. = e.i.t. \qquad \text{(i.)}$$

Now the lines cut by the conductor in its motion are those lying within the area $l \times d$ (Fig. 277), *i.e.* $B \times l \times d$ lines, and as

$$e = \frac{\text{amount of cutting}}{\text{time of cutting}}, \text{ we have}$$

$$e = \frac{B \times l \times d \times 1}{t}$$

(unity is inserted in this expression because we are dealing with one conductor),

$$\therefore \quad e.i.t = \frac{B \times l \times d}{t} \times i.t = B.l.d \times i \qquad \text{(ii.)}$$

Whence we have from equations i. and ii.

$$p.d. = B.l.d \times i$$
$$\text{or} \quad p = B.i.l \text{ dynes.}$$

That is, *the force per centimetre is B.i dynes.*

If the conductor makes an angle θ with the field, then the force is given by $p = B.i.l \sin \theta$ dynes, as already explained on p. 348.

Example.—A conductor carries a current of 50 amperes at right angles to the lines of force in a field of 10,000 lines per square centimetre. Find the force in lbs.-weight per foot of length.

By calculation, we first obtain 1 ft. = 30·48 centimetres, and 50 amperes = 5 absolute units of current.

$$\text{Now } p = B.i.l \text{ dynes}$$
$$= 10{,}000 \times 5 \times 30{\cdot}48 \text{ dynes}$$
$$= 1{,}524{,}000 \text{ dynes}$$

But 1 lb. = 453·6 grams, and 1 lb.-weight = 981 × 453·6 dynes
= 445,000 dynes (approximately)

$$\text{Whence } p = \frac{1{,}524{,}000}{445{,}000} \text{ lbs.-weight}$$
$$= 3{\cdot}4 \text{ lbs.-weight per foot-length.}$$

To Find the Field Strength at any Point P, at Distance *d* from an Infinitely Long Straight Wire carrying Current *i*.—Let B be the field strength at P, Fig. 278 (as the lines of force are circles, it will be the same all round a circle of radius d). Imagine that an ideal unit magnetic pole is placed at P, then, by definition, the force acting on this pole is H dynes, where $B = \mu H$ or $H = \frac{B}{\mu}$. As μ is unity in air, the two quantities are numerically equal, but as in many previous instances, we must be careful not to confuse them.

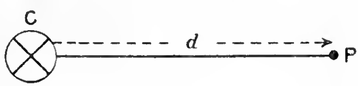


FIG. 278.

This force will tend to urge the pole in a circle around the conductor. Let it do so, making one revolution in t seconds (i being the current strength *during* the motion).

Then work done = force × distance = $H \times 2\pi d$ ergs.

Also work done = $e.i.t$ ergs, where e is the back E.M.F., produced by the lines due to the pole cutting the conductor.

Now the pole has 4π lines (see p. 136), and in one revolution these have cut the conductor once,

$$\therefore \quad e = \frac{4\pi \times 1}{t}$$

$$\text{whence} \quad e.i.t = \frac{4\pi}{t} \times i.t = 4\pi i \text{ ergs}$$

$$\therefore \quad H \times 2\pi d = 4\pi . i$$

$$i.e. \quad H = \frac{2i}{d}$$

$$\text{But} \quad B = \mu H$$

$$\text{So that} \quad B = \frac{2.i.\mu}{d}.$$

This result shows that the field strength varies inversely as the *distance* from the wire, and not as the square of the distance. This was originally discovered by direct experiment. (The field due to a very small part of the conductor *does* vary as the *square*, as already mentioned (see p. 286), but when the effects of *all* such parts are added together, the result is as stated above.)

To Find the Force of Attraction or Repulsion between Two Infinitely Long Straight Conductors carrying Currents i and i_1 respectively at Distance d Centimetres apart.—Consider the conductor carrying the current i_1. By the preceding proposition, it is in a field of strength $\frac{2i\mu}{d}$, and the force upon a portion of it of length l centimetres is given by

$$p = B.i_1.l$$

$$\therefore \quad p = \frac{2i\mu}{d} . \times i_1.l$$

$$i.e. \quad p = \frac{2ii_1}{d} \times l \times \mu.$$

This force is *mutual*, and hence each conductor exerts a force on the other, whose value per centimetre of length is $\frac{2ii_1\mu}{d}$ dynes.

To Find the Field Strength inside a Solenoid having N turns, Length l Centimetres, and carrying a Current i Absolute Units.—Let B be the field strength within the solenoid (assumed to be uniform throughout). Suppose that a *unit* pole is placed at one end and that it is driven by the force H through the solenoid in t seconds.

Now, work done = force × distance = $H \times l$ ergs

Also, work done = $e.i.t$ ergs, where $e = \frac{4\pi.N}{t}$

$$\therefore \quad e.i.t = \frac{4\pi N}{t} \cdot i.t = 4\pi.i.N$$[1]

$$\text{or} \quad Hl = 4\pi.i.N$$

$$\text{Whence} \quad H = \frac{4\pi.i.N}{l} \text{ dynes, and as } B = \mu.H$$

$$B = \frac{4\pi.i.N.\mu}{l} \text{ lines per square centimetre,}$$

where μ is unity for air.

It is important to notice that the assumption, made respecting the uniformity of the field throughout the solenoid, is not strictly true. Evidently it must change, at any rate near the ends, and a more exact investigation would lead to the result that the field at any point P on the axis, subtending angles α and α_1 at the end of the coil, as shown in Fig. 279, is given by

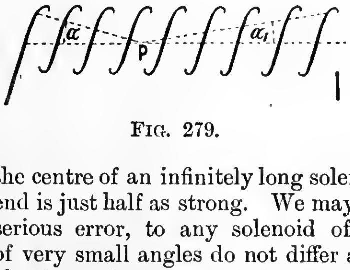


Fig. 279.

$$B = \frac{2\pi i N}{l} \mu(\cos\alpha + \cos\alpha_1) \text{ lines}$$

per square centimetre.

Hence, we see that our value is correct for the field strength at the centre of an infinitely long solenoid, and that the field exactly at an end is just half as strong. We may, however, apply our result, without serious error, to any solenoid of reasonable length, as the cosines of very small angles do not differ appreciably from unity. It should also be noticed that B is independent of the diameter of the solenoid.

Coefficient of Self-induction.—For any circuit (or portion of a circuit), the coefficient of self-induction may be defined as "the amount of cutting" which takes place in that circuit when unit current is stopped or started therein. It is usually denoted by the letter L, and is often called the *inductance* (or the *self-inductance*) of the circuit. In the case of a long solenoid, its approximate value can be easily calculated as follows:—

When the solenoid carries a current of i absolute units, the field within it is B, where

$$B = \frac{4\pi.iN}{l} \times \mu \text{ lines per square centimetre.}$$

[1] It follows from the definition on p. 168 that $4\pi.i.N$ is the *difference of magnetic potential* between the ends of the solenoid.

When the current is unity, $B = \frac{4\pi N}{l} \times \mu$, and the total number of lines passing through the solenoid = B × area of section.

∴ Total number of lines inside solenoid $= \frac{4\pi N A}{l} \times \mu$, where A = area of section.

When the current is stopped or started, these lines cut N turns, and ∴ "amount of cutting" $= L = \frac{4\pi N^2 A}{l} \times \mu$.[1]

It will be seen from this expression that when the length of the solenoid is constant, L varies as the square of the number of turns.

When no iron core is present, L is a constant for any given coil or circuit, although it is not always easy to calculate its value; but when iron is present, μ is usually much greater than unity, and, as will be shown later, its value depends upon the state of the iron, hence, L is no longer a constant (unless, indeed, the circumstances are such that μ may be regarded as constant).

It will therefore be perceived that the presence of iron enormously increases the value of L.

Subject to these restrictions, some of the equations, previously obtained, may be conveniently expressed in terms of L. For instance, if the current flowing is not unity, but i absolute units, then the amount of cutting which takes place, when that current is stopped or started, is Li. Now, we have shown that when Z lines cut N turns in t seconds, the average induced E.M.F. is given by

$$e \text{ (average)} = \frac{Z \times N}{t}$$

If this is a self-induced E.M.F., then $Z \times N = Li$, and

$$e_s \text{ (average)} = \frac{L.i}{t}$$

This does not apply merely to the stopping or starting of a current: it is simply necessary for the current to change in value by i units in t seconds. Hence, we have an alternative method of defining L, for suppose that the current is increasing (or diminishing) at a uniform rate of 1 absolute unit per second, then the induced E.M.F. is constant in value, and is numerically equal to L, so that a circuit may be said to have unit inductance when a current changing at the above rate produces a steady induced E.M.F. of 1 absolute unit.

If we employ volts and amperes, the expression given above becomes

$$e_s \text{ (average)} = \frac{L\frac{C}{10}}{10^8 t} = \frac{LC}{10^9 t} \text{ volts,}$$

[1] For a straight solenoid this expression is approximately correct. For an *endless* solenoid (*i.e.* for a circular coil like that shown in Fig. 289, wound on a non-magnetic ring), it is strictly correct, and hence such a coil may be used as a "standard inductance, although only small inductances can be readily obtained in this way.

which may be interpreted by saying that, when the current is changing uniformly at the rate of 1 ampere per second in a circuit for which $L = 10^9$ absolute units, the induced E.M.F. will be 1 volt.

In this case, the circuit is said to have an inductance of 1 *henry*, or of *one practical unit of self-induction*, where 1 henry $= 10^9$ absolute units of self-induction.

In a similar manner, equation (i.), p. 368, may be written in the form

$$Q = \frac{L.i}{R}$$

If L and R are expressed in absolute units, Q is in absolute units. If the current is expressed in amperes, L in henries, and R in ohms, then Q is in coulombs, or

$$Q = \frac{LC}{R}$$

Energy Stored up in a Circuit.—When a current is flowing steadily in a circuit, a certain amount of energy becomes latent. This energy is set free when the current ceases, and it produces, among other effects, the self-induction spark when the circuit is suddenly broken. As already mentioned, it is analogous to the kinetic energy of a mass in motion, but it must be carefully borne in mind that *the energy is not stored up in the circuit itself, but in the magnetic field linked with the circuit.*

Now, the equation $Q = \frac{Li}{R}$ shows that when a current i is stopped in a circuit of inductance, L, and resistance, R, a quantity, Q, flows round the circuit, *i.e.* the circuit behaves as though a quantity, Q, was stored up in it. We know that this is not really the case, but the effect is very similar to that obtained when, by the application of a steady E.M.F. to a circuit containing a condenser, we charge that condenser to an electromotive force, E, and store therein a quantity, Q. As a matter of fact, in the one case the energy is stored up in the form of a magnetic field, and in the other in the form of an electric field; in both cases, however, the amount of energy so stored will be $\frac{1}{2}Q \times E$ ergs.

$$\text{Now,} \quad E = iR \text{ by Ohm's law,}$$

$$\text{and} \quad Q = \frac{Li}{R}$$

$$\therefore \quad \text{Energy stored up in the circuit } = \tfrac{1}{2} \cdot \frac{Li}{R} \times iR$$

$$= \tfrac{1}{2}Li^2 \text{ ergs.}$$

Hence, when L is great, as in circuits with many turns and much iron, a relatively enormous amount of energy may be stored up, and, in practice, special precautions may have to be taken to avoid breaking such circuits too suddenly; otherwise, serious injury to the insulation, due to the sudden rise in voltage, may occur.

Coefficient of Mutual Induction between Two Circuits or Parts of a Circuit.—Let Fig. 273, p. 357, represent two independent circuits of any shape and size, at any distance apart. When a current is started in one of them, the field thereby produced spreads outwards through space, and some of its lines must cut the other, although the effect, except at short distances, may be negligibly small. Then, M, *the coefficient of mutual induction*—often called the "*mutual inductance*"—between them (in any fixed relative position) may be defined as being *the "amount of cutting," which takes place in one of them, when unit current is stopped or started in the other.*

As before, M may only be regarded as constant when iron is absent, and, in that case, if the current in one circuit is i absolute units, the amount of cutting in the other is $M.i$.

Hence, when current i is stopped or started in the primary circuit, the quantity Q passing round the secondary is given by $Q = \frac{Mi}{R}$, where R is the resistance of the secondary. The relation is *mutual*, i.e. either circuit may be used as the primary without altering the value of M.

The simplest case occurs with two solenoids, one inside the other, as in Experiment 223. Suppose that the primary has N_1 turns, and the secondary N_2. Then if l be the length of the primary, we have, by a repetition of the argument given on p. 376, a total of $\frac{4\pi NA\mu}{l}$ lines within it when the current is unity.

Assuming that all these lines cut all the turns of the secondary, when the primary circuit is made or broken, we have, by definition,

$$M = \frac{4\pi N_1 N_2 A\mu}{l}$$

As a matter of fact, some of the lines would miss the secondary unless very special precautions were taken, and hence, this must be regarded as the limiting value.

Methods of measuring L and M are given in Chap. XXVII.

Magnetic Moment of a Coil carrying a Current.—It has been shown on p. 287 that a circular coil carrying a current produces a magnetic field, which for a point on its axis has the value

$$F = \frac{2\pi n i r^2 \mu}{d^3},$$

where d is the distance of the point, and r is the radius of the coil (its being understood that d is great compared with r).

Now, a short magnet of moment M, placed along the axis with its centre at the centre of the coil, would produce at the same point a field of strength $\frac{2M}{d^3}$.

If, therefore, we agree to define the moment of the coil as being numerically equal to the moment of the equivalent magnet, we have

$$\frac{2M}{d^3}=\frac{2\pi n i r^2 \mu}{d^3}$$

$$\text{or} \quad M = i.n \times \pi r^2 \times \mu = i.n.A.\mu$$

where A is the area of the coil.

This result has been obtained by simple reasoning for a circular coil, but if we investigate the matter more fully, it will be found that it is quite independent of the *shape* of the coil.

Couple Acting on a Coil carrying a Current in a Uniform Magnetic Field.—Let a coil of n turns and area A square centimetres carry a current i in a uniform field of strength B lines per square centimetre. We have already had occasion to consider such a coil, under somewhat similar circumstances, in connection with suspended coil galvanometers (see p. 294), and we know that it may be regarded as a magnet, which tends to place itself along the lines of force, *i.e.* the coil will tend to turn until its axis is parallel to the field.

If its axis be inclined at a direction θ to the field, we know that, in the case of an actual magnet, the moment of the restoring couple is MH sin θ, and we can at once apply this result to the coil, if we put $M = i.n.A.\mu$, and $H=\frac{B}{\mu}$.

Then moment of couple becomes $i.n.A.B \sin\theta$, and is independent of the shape of the coil.

It is a useful exercise in fundamental principles to verify this result in the particular case of a flat rectangular coil by applying the equation $p = B.i.l$ obtained on p. 373 for the force on a straight conductor in a magnetic field.

Let the sides of the coil be of lengths l and b centimetres respectively, and, for simplicity, let it be placed with its axis at right angles to the field, as in Fig. 280, with the sides of length l perpendicular to the plane of the paper. The sides of length l are acted upon by a force p, whereas the sides of length b, being parallel to the field, are inactive. Evidently the moment of the couple is $p \times b$, and $p = B.i.l \times n$ (for n conductors).

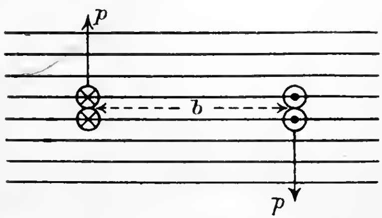


FIG. 280.

$\therefore$ moment of couple $= B.i.l.n \times b$,

but $l.b$ is the area of the coil,

$\therefore$ moment of couple $= B.i.n.A$,

which agrees with the above result, as $\sin\theta = 1$.

Magnetic Shell.—The magnetic effect of a flat coil is often more conveniently treated by regarding the coil as producing a "magnetic shell." For instance, we may regard the whole of one side of the coil as a north pole, and the whole of the other side as a south pole, and thus it resembles a thin plate transversely magnetised, *i.e.* the area of the polar surfaces is relatively large, and the distance between the poles very small. Such a transverse slice of magnetisation is known as a magnetic shell.

Let the area of each end surface be A, the thickness t, and let m be its pole strength. Then the intensity of magnetisation (I) will be given as usual by

$$I=\frac{m}{A}=\text{pole strength per unit area,}$$

and the "strength" of the shell is defined as being the product $I\times t$, or from above

$$\text{Strength of shell}=\frac{m}{A}\times t.$$

Now, mt is the magnetic moment, and, therefore,

$$\text{Strength of shell}=S=\frac{M}{A}=\text{magnetic moment per unit area,}$$

which is independent of the *shape* of the shell.

Now, it has been shown (see p. 169) that the magnetic potential at any point P near a *short* magnet is $\frac{M.\cos\theta}{\mu d^2}$, where d is the distance of the point from the centre of the magnet, and θ is the angle which this direction makes with the axis.

Applying this to the shell, for which $M=SA$,
and writing V_m for the magnetic potential at the point P, we have

$$V_m=\frac{S.A.\cos\theta}{\mu d^2}.$$

But $\frac{A\cos\theta}{d^2}$ is the solid angle subtended by the shell at the point P, and writing ω for this solid angle, we have

$$V_m=\frac{S\omega}{\mu}.$$

Or, the potential at any point due to a magnetic shell is equal to the strength of the shell multiplied by the solid angle it subtends at that point, and is quite independent of the size and shape of the shell.

Now, we have shown that in the case of a circular coil of n turns, area A, and carrying a current i,

$$M=i.n.A.\mu$$

$$\text{but } S=\frac{M}{A}$$

$$\therefore\ S=i\times n\times\mu.$$

Hence, for *one turn*, the absolute unit of current is numerically equal to the strength of the magnetic shell it produces.

Again, the magnetic potential at any point due to the coil will be

$$V_m=\frac{S\omega}{\mu}=n\times i\times\omega.$$

Quite close to the surface of the coil, and on its axis, $\omega=2\pi$. Hence, for a single turn the potential is $2\pi i$. At a similar point on the other side of the turn, the potential has the same value, but with the sign reversed. Hence, the magnetic P.D. between the two sides is $4\pi i$. The argument may be extended to any number of turns, and thus we deduce the fact that the P.D. between the two ends of a solenoid is $4\pi in$ (as stated on p. 375).

Exercise XIX

1. A copper disc having a diameter of 40 centimetres is rotated about a horizontal axis perpendicular to the disc and parallel to the magnetic meridian.

Two brushes make contact with the disc, one at the centre and the other at the edge. If the value of the horizontal component of the earth's field is 0·2 C.G.S., find the potential difference in volts between the two brushes when the disc makes 3000 revolutions per minute. (B. of E., 1906.)

2. What is the magnitude and direction of the force acting on a straight conductor, 10 centimetres long, placed at right angles to a magnetic field of 50 lines per square centimetres, the current through the conductor being 5 amperes? In what unit is your result expressed? (B. of E., 1905.)

3. A square conducting frame cut through at one place rotates in the earth's magnetic field about a vertical axis passing through the middle point of opposite sides. Describe the variation in E.M.F. between the two sides of the break which consequently occurs, and calculate its maximum amount when there are 120 revolutions per minute, if the edge of the square is 25 centimetres, and the intensity of the earth's horizontal force is ·18. (Lond. Univ. Inter. B.Sc., 1904.)

4. A circular coil traversed by a current is placed horizontally in the earth's field. What is the nature of the force acting on the coil, and how does it vary with (1) the strength of the current, (2) the strength of the field, (3) the number of turns of wire in the coil, (4) the size of the wire, (5) the radius of the coil? (B. of E., 1904.)

5. Two points in the circuit of a voltaic battery are connected by two long covered wires arranged in multiple arc (that is, each wire would complete the circuit by itself if the other were removed). The resistances of the wires are in the proportion of 3 to 4. The one is now bent into a zigzag, the other is wrapped in a continuous coil round a soft iron core. Show in what proportion the battery current is divided between the wires (1) when the battery contact is made continuously, (2) when it is made momentarily.

6. How may the intensity of the magnetic force inside a solenoid be approximately calculated? What is it in one of 300 turns, 15 centimetres long, which carries a current of ·2 ampere? What effect has the diameter of the solenoid? (Lond. Univ. Inter. B.Sc., 1904.)

7. When a conductor l centimetres in length, carrying a current c (in C.G.S. electro-magnetic units), is placed at right angles to a magnetic field of strength H, the force acting on the conductor is equal to Hlc dynes. Use this result to determine the value of the electromotive force generated when a conductor is moved with velocity v c.ms/sec. in a direction perpendicular both to its length and to a magnetic field of strength H. Deduce the result that the E.M.F. generated is equal to the rate at which the magnetic lines of force are cut by the conductor. (Lond. Univ. B.Sc. Internal., 1909.)

8. A solenoidal coil 70 centimetres in length, wound with 30 turns of wire per centimetre, has a radius of 4·5 centimetre. A second coil of 750 turns is wound upon the middle part of the solenoid. Calculate the coefficient of self-induction of the solenoid, and the coefficient of mutual induction of the two coils. Will the inductance of the solenoid be affected by short-circuiting the ends of the secondary coil? (Lond. Univ. B.Sc. Internal., 1909.)

9. In what way may a circuit carrying a current be considered equivalent to a magnetic shell? Find the work done in taking a magnetic pole round a closed curve which threads an electric circuit once. (B. of E., Stage III., 1909.)

CHAPTER XXIV

BALLISTIC GALVANOMETERS

GALVANOMETERS are not required merely as indicators in null methods, or as instruments for reading steady deflections. In dealing with condenser discharges, as in Experiment 184, p. 313, and with measurements involving induced currents, we are concerned with a sudden rush of current through the instrument, which is completed in a small fraction of a second, and thus the term *current* is no longer applicable. We have then to measure a "quantity," not as a continuous flow but as a sudden discharge, the difference being somewhat analogous to that involved in measuring (*a*) a steady flow of water, and (*b*) a sudden rush due to emptying, say, a single bucket of water through some apparatus. Such a discharge of electricity will produce a swing or "throw" of the needle of the galvanometer, and then it will settle down to rest again. We can no longer obtain a relation between current and deflection by equating the moments of opposing couples when the needle is in equilibrium, but we may estimate the kinetic energy given to the needle by the quantity passing through the instrument, and then equate this to the potential energy of position at the instant it reaches the end of its first swing. The method implies

(1) that the discharge through the instrument is completed *before* the needle has moved sensibly from its zero position;

(2) that no energy has been lost during the swing, in other words, that the deflection is entirely undamped.

Condition (1) is necessary, because it enables the energy due to the impulse to be easily estimated, and it is satisfied by making the natural time of vibration of the moving part rather great, so that it starts slowly from rest. About five seconds for one complete vibration is a convenient value; it may be greater than this with advantage, but should not be much less.

Condition (2) cannot be completely satisfied, neither is it always desirable to approximate too closely to it; it is sufficient if the damping is comparatively small in amount. The effect of departure from this condition is evidently to make the throw, as measured on the scale, less than it would be if there were absolutely no damping, but as the undamped value is required by the theory, it is necessary to multiply the observed throw by a "damping factor," which is calculated from data obtained during the experiment. When only comparative values are required and the throws are not very different in magnitude, the factor cancels out, and hence it is not introduced in the experiment mentioned in the worked Example 2, Chapter XXIII. For further details, the student is referred to the Appendix to this chapter.

It must be distinctly understood that the term "ballistic," when applied to a galvanometer, does not mean any special type or construction. Any pattern may be used, if only conditions (1) and (2) are satisfied, and these do not involve any serious structural changes. In practice, instruments of both the moving magnet and the moving coil types are employed, but, on the whole, the former are the more generally useful, for although the latter possess the great advantage of being unaffected by external fields, in some cases the damping effect due to induced currents in the moving coil is apt to be inconveniently great, as will be seen in some of the following experiments.

Hence, we shall first consider the simple type in which the moving system is a permanent magnet controlled by the horizontal component (H) of the earth's magnetic field.

Let M = the magnetic moment of the needle system,
I = its moment of inertia,[1]
and ω = its angular velocity.

Now, the "impulse" due to the discharge is equivalent to the action of some couple, C, acting for a very short time t, and we have, by a well-known theorem in mechanics,

$$Ct = I\omega \tag{1}$$

Let G be the strength of the magnetic field at the needle, due to unit current flowing through the instrument. Then, if i be the *average* current during the time t, the *average* field at the needle is Gi. This field is at right angles to the undisturbed position of the needle, and from p. 152, the moment of the couple acting on the latter is $Gi \times M$.

$$\therefore \text{ from (1)} \quad GiMt = I\omega$$
$$\text{but} \quad i.t = Q$$
$$\therefore \quad GMQ = I\omega \tag{2}$$

Again, the kinetic energy given to the moving system is $\frac{1}{2}I\omega^2$, and if θ be the angle of the first swing, the potential energy at the end of that swing is (from p. 168) $MH(1 - \cos\theta)$.

$$\therefore \quad \tfrac{1}{2}I\omega^2 = MH(1 - \cos\theta)$$
$$= MH \times 2\sin^2\frac{\theta}{2} \tag{3}$$

[1] The letter "I" had a different meaning in the last chapter, and will shortly be used for yet another purpose. Experience shows that there is no real danger of confusion.

Finally, if T be the time of one complete vibration (see p. 155)

$$T = 2\pi\sqrt{\frac{I}{MH}} \qquad (4)$$

We have, therefore, equations (2), (3), and (4) from which to obtain an expression for Q.

$$\text{From (2)} \quad \omega^2 = \frac{G^2M^2Q^2}{I^2}$$

$$\text{from (3)} \quad \omega^2 = \frac{4MH\sin^2\frac{\theta}{2}}{I}$$

$$\therefore \quad Q^2 = \frac{4IH\sin^2\frac{\theta}{2}}{MG^2}$$

$$\text{Also from (4)} \quad \frac{I}{M} = \frac{HT^2}{4\pi^2}$$

$$\therefore \quad Q^2 = \frac{T^2H^2\sin^2\frac{\theta}{2}}{\pi^2G^2}$$

$$\text{or} \quad Q = \frac{T.H}{\pi.G}.\sin\frac{\theta}{2} \qquad (5)$$

Therefore, the "quantity" passing through the galvanometer is proportional to the sine of half the angle of the first throw.

In practice, H is usually some unknown field due to the joint effect of the earth and a controlling magnet. This does not affect the argument, although it means that in order to obtain an expression suitable for general use, the factors H and G must be evaluated in some way by experiment. This can be done by noticing (from p. 285) that if a *steady* current i passes through the instrument producing a *steady* deflection $a°$, then if a *is very small*,

$$i = \frac{H}{G}\tan a$$

$$\text{or} \quad \frac{i}{\tan a} = \frac{H}{G}$$

$$\text{Hence,} \quad Q = \frac{T.i}{\pi.\tan a}.\sin\frac{\theta}{2} \qquad (6)$$

Here, Q will be in absolute units or in coulombs according as i is expressed in absolute units or in amperes.

In actual practice, ballistic galvanometers are always reflecting instruments, which means that θ is always fairly small, for under ordinary circumstances, a deflection of 6° or so would send the spot of light off the scale. Hence, for many practical purposes, we may regard the sines and tangents as numerically equal to the angles themselves,[1] and these can be expressed in terms of the observed deflection and the distance of the mirror from the scale. Thus, if d and d_1 are the scale readings corresponding to deflections θ and a, and l the distance of the scale from the mirror, we have (remembering that the angular motion of the spot of light is twice that of the mirror)

$$\sin 2\theta = 2\theta = \frac{d}{l}, \quad \text{and} \quad \tan 2a = 2a = \frac{d_1}{l}$$

$$\therefore \quad \frac{\sin \frac{\theta}{2}}{\tan a} = \frac{\theta}{2a} = \frac{d}{2d_1}$$

and equation (6) becomes

$$Q = \frac{T.i}{2\pi.d_1} \times d \qquad (7)$$

There is some risk of confusing the i and d_1 in this expression with similar letters necessarily used in subsequent work, and hence we shall write I and D respectively for them. Again, to indicate when practical units are being used, *i.e.* when i is in amperes, we shall write A instead of I; so that equation (7) becomes

$$Q = \frac{T.I}{2\pi.D} \times d \text{ (absolute units)}$$

$$\text{or} \quad Q = \frac{T.A}{2\pi.D} \times d \text{ (coulombs)} \qquad (8)$$

If we examine the case of a suspended coil galvanometer with metallic strip suspension, we shall find that Q is proportional to θ and not to $\sin \frac{\theta}{2}$, although the resulting expression, in its final form, will be identical with (8). In fact, we may say that this equation, which is the most convenient form for ordinary use, holds generally for all types of galvanometer, *provided that the deflections are sufficiently small*, so that we have

$$Q = m \times d,$$

where m is a constant depending on the instrument.

This result has been obtained by assuming that no energy is lost during the swing, *i.e.* that the oscillations would continue for ever,

[1] These approximations are sufficiently exact for most purposes, but when necessary a correction can easily be applied.

but we know that this is not true, and that the observed throw d is less than it would be if there was no damping. Hence, we have to multiply the observed throw by a "damping factor" before we can use it in the above expression (unless we are dealing with comparative values under similar conditions, as in Experiment 184, p. 313, in which case this factor may cancel out), and it is shown (see p. 393) that this factor is $\left(1+\frac{l}{2}\right)$, where l is the "logarithmic decrement." Hence, the complete expression is

$$Q = md\left(1+\frac{l}{2}\right). \qquad (9)$$

One word of warning may be given—the use of shunts should be avoided as much as possible. When it is necessary to reduce the sensitiveness of a ballistic galvanometer some other method must be adopted (unless, of course, it is being used with steady currents, as in the experiment given below). This is because a momentary rush does not divide up between two possible paths inversely as their *resistances*, but inversely as their *impedances*, and the galvanometer behaves as if its resistance were temporarily increased. Hence, the results given on p. 273 cannot always be applied.

Practical Applications.—**Exp. 230**, *to find the constant of a ballistic galvanometer. First method.*—We have $Q = md\left(1+\frac{l}{2}\right)$, where m is the constant to be determined.

Now $m = \frac{T \times A}{2\pi \times D}$, and T can be readily found by setting the needle swinging and observing the time in seconds taken to execute a certain number of complete oscillations.

To find a value for $\frac{A}{D}$ we may proceed as follows: Insert the galvanometer (Fig. 281) in a circuit containing a cell of *known E.M.F.* and a resistance box, R. Connect up another resistance box, S, as a shunt to the galvanometer (in order to keep the spot of light on the scale, and also to avoid injuring the instrument). A Daniell's cell is suitable, and if necessary its E.M.F. may be determined by comparison with a standard cell, using the method described in Experiment 178, p. 304. For ordinary purposes, it may be taken as 1·07 volts with sufficient exactness. Make R large—say 10,000 to 20,000 ohms—and adjust the shunt until a convenient deflection is obtained. Let this be D divisions, then we have

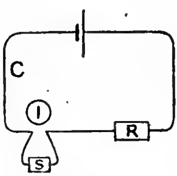


FIG. 281.

$$C = \frac{E}{R + \frac{sg}{s+g}} = \frac{E}{R}\left(\text{because } \frac{sg}{s+g} \text{ is usually negligible compared with R}\right)$$

$$\text{also } C_g = A = \frac{s}{s+g} \times C = \frac{s}{s+g} \times \frac{E}{R}$$

$$\therefore \frac{A}{D} = \frac{s}{s+g} \times \frac{E}{R} \times \frac{1}{D}$$

$$\text{and} \therefore \quad m = \frac{T \times A}{2\pi \times D} = \frac{T \times s \times E}{2\pi \times (s+g) \times R \times D}$$

The following values (taken from a student's note-book) were obtained in an actual experiment:—

Galvanometer resistance	$=960$ ohms
Shunt	$=15$ ohms
R	$=20,000$ ohms
D	$=158$ divisions
T	$=4\cdot1$ seconds

Daniell's cell was used taken as 1·07 volts.

$$\therefore \quad m = \frac{4\cdot1 \times 15 \times 1\cdot07}{2 \times \pi \times (15+960) \times 20,000 \times 158} = 3\cdot4 \times 10^{-9} \text{ coulombs,}$$

which is the quantity that would give a "throw" of *one scale division*, if no damping existed.

Second Method.—This method requires a condenser of known capacity, and a cell of known E.M.F., but as the latter is not needed to give a current, some form of standard cell may be used.

Exp. 231. Arrange the apparatus as in Experiment 184, p. 313, using a condenser key. Observe several throws, due to the discharge of the condenser through the galvanometer, and take the mean. Let this be d divisions. Also determine l, the logarithmic decrement (see Appendix to Chapter).

$$\text{Then} \quad Q = EK = md\left(1 + \frac{l}{2}\right)$$

$$\text{or} \quad m = \frac{EK}{d\left(1 + \frac{l}{2}\right)}$$

The following result was obtained in an actual experiment with the same galvanometer as before:—

$E = 1\cdot0183$ (a standard cadmium cell)

$K = \cdot5$ microfarads $= \dfrac{\cdot5}{10^6}$ farads

$d = 150$ scale divisions

$l = \cdot06$

$$\therefore \quad m = \frac{1\cdot0183 \times \frac{\cdot5}{10^6}}{150\left(1 + \frac{\cdot06}{2}\right)} = \frac{1\cdot0183 \times \cdot5}{150 \times 1\cdot03 \times 10^6} = 3\cdot3 \times 10^{-9} \text{ coulombs.}$$

Third Method (by using a standard solenoid).[1]—In this case we require an accurate ammeter of suitable range, a current reverser, an adjustable resistance, a battery of a few cells capable of maintaining a small *steady* current for a short time, and (possibly) a resistance box.

Exp. 232. The apparatus should be set up as shown in Fig. 282. It will be seen that the battery is arranged to send a steady current through the solenoid, whilst the secondary coil within it is connected to the galvanometer. The resistance box, r, is inserted to diminish the damping, and with some instruments may not be required.

Adjust the current to some convenient value. Then observe the throw when

---

[1] See Appendix to this Chapter, Section II.

that current is suddenly reversed. If the throw is inconveniently large, either reduce the current or suddenly interrupt instead of reversing it; this is done by lifting up the rocker, instead of moving it across, and it means that the lines of force cut the secondary coil *once* instead of twice, so that the throw is half as great. (Increasing the resistance r will also diminish the throw.) Then determine the logarithmic decrement, l, *taking care not to alter the conditions*, i.e. circuit remains closed through the same resistance.

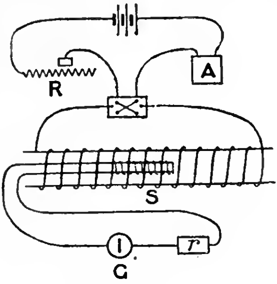


FIG. 282.

Let C_s = current in amperes, as read on ammeter,
d = throw produced when this current is reversed.
R = *total* resistance in the galvanometer circuit.
Z_s = number of lines threading secondary coil when the steady current of C_s amperes flows through the solenoid.
N_s = number of turns in the solenoid.
l_s = length of solenoid in centimetres.
t_s = number of turns in the secondary coil.
A = Area of section of secondary coil $= \pi r^2$.

Then, if Q = quantity passing through the galvanometer,

$$Q = md\left(1 + \frac{l}{2}\right)$$

also $Q = \frac{2Z_s t_s}{R}$ (for the lines Z_s cut t_s twice. If current is merely broken, the 2 disappears).

$$\therefore \; m = \frac{2Z_s t_s}{Rd\left(1 + \frac{l}{2}\right)} \text{ (in which all the quantities are known except } Z_s\text{).}$$

It will reduce the risk of error in working, if we express R in absolute units; then m will also be in absolute units.

To find a value for Z_s, we proceed as follows:—

Number of lines per square centimetre inside solenoid $= \frac{1{\cdot}257 C_s N_s}{l_s}$ (see p. 401)

$$\therefore \text{ Number of lines threading secondary coil} = Z_s = \frac{1{\cdot}257 C_s N_s}{l_s} \times A$$

$$\therefore \; Z_s = \frac{1{\cdot}257 C_s N_s}{l_s} \times \pi r^2,$$ where C_s is in amperes, and r is the radius of the section of secondary coil.

Substituting this value in the expression for m, we obtain

$$m = \frac{2 \times 1{\cdot}257 C_s \times N_s \times \pi r^2 \times t_s}{l_s \times (R \times 10^9) \times \left(1 + \frac{l}{2}\right)}$$

(This is in absolute units; for coulombs, the value will be 10 times as great.)

In an experiment, using the same galvanometer as before, the values obtained were—

$$C_s = \cdot 504 \text{ amperes}$$
$$N_s = 869 \text{ turns}$$
$$r = 2\cdot 53 \text{ centimetres}$$
$$t_s = 1000 \quad ,,$$
$$l_s = 136 \quad ,,$$
$$R = 1957 \text{ ohms}$$
$$d = 194 \text{ scale divisions}$$
$$l = \cdot 6$$

$$\therefore \quad m = \frac{2 \times 1\cdot 257 \times \cdot 504 \times 869 \times \pi \times (2\cdot 53)^2 \times 1000}{136 \times 1957 \times 10^9 \times 194 \times 1\cdot 3}$$

$$= 3\cdot 2 \times 10^{-10}$$

or $m = 3\cdot 2 \times 10^{-10}$ absolute units $= 3\cdot 2 \times 10^{-9}$ coulombs.

In this experiment, the damping was excessive, and should have been reduced by putting more resistance in the circuit. For it will be seen in the Appendix that the expression $1 + \frac{l}{2}$ does not hold good when l is large.

The preceding experiments should be performed on account of the useful training in manipulation and theory thereby obtained, but, in other respects, they are of no particular importance. For instance, any alteration in the distance of the scale or in the nature of the working conditions will alter the value of m, and, for this reason, it is customary, as will be seen in most of the experiments subsequently described, to either eliminate or evaluate m in the course of the work. The following experiment is given as an application of the previous results :—

Exp. 233, *to determine the vertical component of the earth's magnetic field.* Connect up to a galvanometer an "earth-inductor" coil. This may be simply a rectangular or circular coil of known area and number of turns. Insert a resistance box in the circuit if necessary in order to diminish the damping. Lay the coil flat upon the bench, and then turn it *quickly* over. Note the throw, and repeat several times, and then obtain the mean. In this operation the turns of the coil cut the vertical component twice (see p. 351). Also determine the value of l.

Let V = vertical component
R = total resistance of the galvanometer circuit
t_e = number of turns in earth inductor
A = area of inductor
d = throw obtained.

Then $Q = md\left(1 + \frac{l}{2}\right)$ (absolute units)

Also $Q = \dfrac{2V \times A \times t_e}{R \times 10^9}$

$$\therefore \quad V = \frac{R \times 10^9 \times m \times d \times \left(1 + \frac{l}{2}\right)}{2A \times t_e}$$

The results in an actual experiment were—

$g=960$ ohms

$R=3041$ ohms (being the resistance in box, 2000 ohms, resistance of inductor coil, 81 ohms, and of galvanometer, 960 ohms)

$d=171{\cdot}5$ divisions

$1+\frac{l}{2}=1{\cdot}025$

$A=415$ square centimetres

$t_e=500$

m taken as $3{\cdot}3\times10^{-10}$ absolute units

$$\text{As}\quad V=\frac{R\times10^9\times m\times d\left(1+\frac{l}{2}\right)}{2A\times t_e}$$

$$\therefore\quad V=\frac{3041\times10^9\times3{\cdot}3\times10^{-10}\times171{\cdot}5\times1{\cdot}025}{2\times415\times500}$$

$$={\cdot}43\text{ nearly.}$$

Exp. 234, *to measure the capacity of a condenser.* Connect up a constant cell, *e.g.* an accumulator or a Daniell's cell, to a potentiometer; charge the condenser by means of the P.D. between the ends of the potentiometer wire, and then discharge it through the galvanometer. For this purpose a condenser key may be used, the general arrangement being as shown in Fig. 283. It will be seen that depressing the key charges the condenser, which is discharged through the galvanometer when the key is released. Also determine the damping factor.

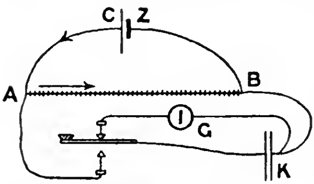


FIG. 283.

Let e=P.D. between the ends of the potentiometer wire, AB, in volts,

and K=the capacity of the condenser in farads.

$$\text{Then}\quad Q=eK \tag{1}$$

$$\text{and}\quad Q=\frac{T\times A}{2\pi\times D}\times d\left(1+\frac{l}{2}\right)$$

$$\therefore\quad K=\frac{T\times A\times d\left(1+\frac{l}{2}\right)}{2\pi\times e\times D} \tag{2}$$

We can determine T and $\frac{A}{D}$ substantially as in Experiment 230, and the potentiometer wire is merely used because it affords a ready means of eliminating e. Let the apparatus be now arranged as in Fig. 284. Place a large resistance, r, in series with the galvanometer, and connect it across a portion of the potentiometer wire, AP, of such a length that a steady deflection of a convenient amount is obtained. Let this be D divisions. Then if A is the current through the galvanometer in amperes, and e_1 the P.D. across AP, we have

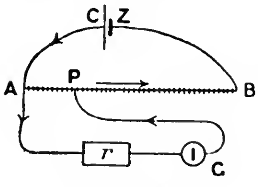


FIG. 284.

$$A=\frac{e_1}{r+g},$$

$$\text{or} \quad \frac{A}{D}=\frac{e_1}{(r+g)D}$$

$$\text{also} \quad e_1=\frac{AP}{AB}\times e$$

$$\therefore \quad \frac{A}{D}=\frac{AP}{AB}\times\frac{e}{(r+g)D}$$

Substituting this value in (2), we obtain

$$K=\frac{T\times d\left(1+\frac{l}{2}\right)\times AP}{2\pi\times(r+g)D\times AB}\text{ farads.}^1$$

Sometimes it is desirable also to shunt the galvanometer in order to avoid making AP inconveniently small.

The results of an actual experiment (using another galvanometer) were—

$T=3$ seconds
$d=61$ scale divisions
$l=\cdot 145$
$g=250$ ohms
$r=20{,}000$ ohms
$s=40$ ohms
$AP=550$ centimetres
$AB=1000$ centimetres
$D=113$ divisions.

In this case, the student also *shunted the galvanometer* in the second part of the experiment, which makes a slight difference in the calculation, for

$$\text{A will be } \frac{e_1}{r+\frac{sg}{s+g}}\times\frac{s}{s+g}$$

and as $\frac{sg}{s+g}$ is negligible compared with r,

$$\frac{A}{D}=\frac{e_1}{rD}\times\frac{s}{s+g}$$

$$\text{Again,} \quad e_1=\frac{AP}{AB}\times e$$

$\therefore$ substituting in (2), we have

$$K=\frac{T\times d\left(1+\frac{l}{2}\right)\times AP\times s}{2\pi\times r\times D\times AB\times(s+g}$$

$$=\frac{3\times 61\times 1\cdot 07\times 550\times 40}{2\times\pi\times 20{,}000\times 113\times 1000\times(40+250)}$$

$$=1\cdot 04\times 10^{-6}\text{ farads}$$

$$=1\cdot 04\text{ microfarads.}$$

The condenser used in the experiment had not been finally adjusted ; it was known to be nearly 1 microfarad.

---

[1] It must be remembered that such measurements of capacity may be seriously affected by "absorption" in the dielectric (see p. 68).

APPENDIX TO CHAPTER XXIV

(1) DAMPING CORRECTION

LET us suppose that the needle is set in motion by the passage of a discharge through the instrument, and that the scale readings are observed as the oscillations die away. Let these be plotted as ordinates against time, as shown in Fig. 285, where $+d$ merely means a deflection to the right, and $-d$ a deflection to the left. We are really only able to observe the turning points of the spot of light, but the complete curve is of the type OCFG, the amplitudes of successive swings gradually decreasing. If, however, the assumption underlying our formula held good, the curve would be that shown in OAH, the amplitudes remaining constant. Hence, in using the instrument we actually read the first

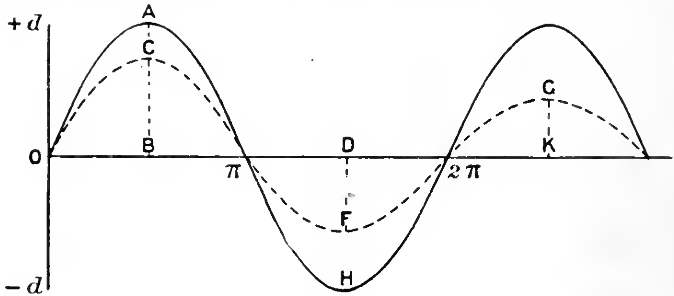


FIG. 285.

throw BC, whereas we require the value of the undamped throw AB, and we have therefore to show how this value can be found.

For this purpose, it is necessary to assume an elementary knowledge of simple harmonic motion. Students possessing such knowledge will know that the ideal *undamped* curve can be represented by an equation of the form $d=a \sin \theta$ where a is the maximum amplitude AB, and positions on the horizontal axis are defined in terms of angle by regarding one complete oscillation as equivalent to 2π radians or 360°.

Similarly, if we take into account the influence of a frictional retarding force proportional to the velocity, we find that the actual curve is represented by

$$d=ae^{-bt} \sin \theta.$$

Where a has the same meaning as before, e is the base of the natural logarithms, b is a constant depending on the instrument, and t is the time that has elapsed since the needle started from rest.

Now, the successive maximum deflections (*i.e.* the scale readings we can actually observe) will correspond to $\sin \theta=1$, and will occur when $\theta=\frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi$, &c.

Let T be the time of one complete oscillation, then when $\theta=\frac{\pi}{2}$, $t=\frac{T}{4}$; and when $\theta=\frac{3}{2}\pi$, $t=\frac{3T}{4}$, and so on.

Hence, we can tabulate the successive readings as under :—

To right.		To left.	
$ae^{-\frac{bT}{4}}$	(BC in figure)	$ae^{-\frac{b3T}{4}}$	(DF in figure)
$ae^{-\frac{b5T}{4}}$	(GK ,,)	$ae^{-\frac{b7T}{4}}$	
&c.		&c.	

Let A_1, A_2, A_3 be the successive *amplitudes* of swing (*i.e.* if the *first* throw is d divisions, and then the spot of light swings back to d_1 on the other side of the zero, the amplitude A_1 is $d+d_1$, and similarly A_2 is d_1+d_2 divisions, and so on), then

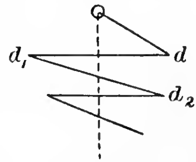


FIG. 286.

$$A_1 = ae^{-\frac{bT}{4}} + ae^{-\frac{b3T}{4}} = ae^{-\frac{bT}{4}}\left(1+e^{-\frac{bT}{2}}\right)$$
$$A_2 = ae^{-\frac{b3T}{4}} + ae^{-\frac{b5T}{4}} = ae^{-\frac{b3T}{4}}\left(1+e^{-\frac{bT}{2}}\right)$$
$$A_3 = ae^{-\frac{b5T}{4}} + ae^{-\frac{b7T}{4}} = ae^{-\frac{b5T}{4}}\left(1+e^{-\frac{bT}{2}}\right)$$

From the symmetry of these results, it will be seen that if m, n represent any two amplitudes (not necessarily consecutive),

$$A_m = ae^{-b(2m-1)\frac{T}{4}}\left(1+e^{-\frac{bT}{2}}\right)$$
$$A_n = ae^{-b(2n-1)\frac{T}{4}}\left(1+e^{-\frac{bT}{2}}\right)$$
$$\therefore \frac{A_m}{A_n} = e^{-b(2m-1)\frac{T}{4}+b(2n-1)\frac{T}{4}}$$
$$\text{or } \frac{A_m}{A_n} = e^{(n-m)\frac{bT}{2}}$$

As b and T are constant, this ratio also remains constant as the oscillations die away.

$$\text{Let } b\,.\,\frac{T}{2} = l, \text{ then } \frac{A_m}{A_n} = e^{(n-m)l}$$
$$\text{or } l = \frac{1}{n-m} \log_e \frac{A_m}{A_n} \qquad (1)$$

l is known as the "logarithmic decrement" of the swing.

In order to apply this result, we notice that the first throw of d divisions, which is the throw actually observed in practice, may be written

$$d = ae^{-\frac{bT}{4}}.$$

Let d_o be the corresponding undamped throw, then $d_o=a$, and this is the value we really need.

$$\therefore \frac{d_o}{d} = \frac{a}{ae^{-\frac{bT}{4}}} = e^{\frac{bT}{4}} = e^{\frac{1}{2}l}$$
$$\therefore d_o = d \times e^{\frac{1}{2}l} = d\left(1+\frac{l}{2}+\frac{(\frac{1}{2}l)^2}{\underline{|2}}+\&c.\right)$$

And when l is small (as it always should be in practice) we may neglect l^2 and higher powers, and, therefore, we have

$$d_o = d\left(1+\frac{l}{2}\right)$$

The value of l must always be obtained under the working conditions, for it will depend upon the nature of the circuit, *e.g.* upon whether it is open or closed and upon the amount of resistance in it. The needle is set in vibration in any convenient way, the extent of its excursions being approximately the same as the observed deflection, and, as the oscillations gradually die away, three readings corresponding to three successive swings are taken. For example, if the readings are 128, 122, 117 divisions, then

$$\frac{A_m}{A_n}=\frac{128+122}{122+117}=\frac{150}{139}$$

and as $n-m=1$, we have from (1)

$$l=\log_e\frac{150}{135}=\log_{10}\frac{150}{139}\times\frac{1}{\cdot 4343}$$

When the damping is small, it may not be easy to determine with accuracy the difference between three successive swings, and then it is convenient to make $n-m$ greater than unity.

In practice, students should remember that the numerical value of $1+\frac{l}{2}$ is to be only slightly greater than unity. If it is large, the working conditions should be altered. Little trouble will be met with when using galvanometers, whose moving part is a steel magnet, for, even on short circuit, the damping will be small, but it is very different with instruments of the moving-coil type. In this case (as shown in Experiment 218, p. 354), on a closed circuit the motion is powerfully damped by induced current produced in the coil, and unless this effect is kept in check by introducing a sufficiently high resistance into the galvanometer circuit, our results will only hold good approximately.

It will help the student to realise the fact that the amount of damping depends upon the working conditions, if he performs the following experiment:—

Exp. 235, *to find the resistance of a galvanometer by observing the "damping" under different conditions*: 1. When the galvanometer is on open circuit (l_1); 2. When the galvanometer is on short circuit (l_2); 3. When a resistance or r ohms is in circuit with it (l_3).

Now, the damping is partly due to air friction, and partly to induced currents whose magnitude is inversely proportional to the resistance in the galvanometer circuit, and it may therefore be represented by an equation of the form,

$$l=A+\frac{B}{R}, \text{ where A and B are constants.}$$

Hence, we have

$$l_1=A+\frac{B}{\infty} \qquad \text{(i.)}$$

$$l_2=A+\frac{B}{g} \qquad \text{(ii.)}$$

$$l_3=A+\frac{B}{r+g} \qquad \text{(iii.)}$$

If we eliminate A and B between these equations, we obtain

$$g=\frac{l_3-l_1}{l_2-l_3}\times r$$

and hence the resistance of the galvanometer may be found, although, of course, it is not suggested that this is a method of determining g with exactness.

The results of an actual experiment with a suspended coil galvanometer (taken from a student's note-book) were: $l_1=\cdot 0162$; $l_2=\cdot 366$; $l_3=\cdot 174$ when $r=1000$ ohms. This gives $g=820$ ohms. The real value was about 900 ohms.

It will be noticed that the damping on short circuit is excessive, and hence the result is unreliable. Under the circumstances it would have been better to have inserted a resistance in (2) as well as in (3), say 1000 ohms in (2) and 2000 ohms in (3).

Simplified Expressions for Damping.—We have treated the question generally, on account of its importance in other branches of the subject, but simplified forms of the expressions obtained are frequently given in text-books which obviate the necessity of determining the logarithmic decrement and at the same time are sufficiently exact for most purposes *when the damping is small.*

We have, from the results given on p. 393, and using the notation indicated in Fig. 286,

$$d = ae^{-\frac{bT}{4}}$$

$$d_2 = ae^{-\frac{b5T}{4}}$$

$$\therefore \quad \frac{d}{d_2} = e^{bT} = e^{2l}$$

$$\text{or} \quad 2l = \log_e \frac{d}{d_2} \qquad (1)$$

Again, we have shown that

$$\frac{d_o}{d} = e^{\frac{1}{2}l}$$

$$\therefore \quad \tfrac{1}{2}l = \log_e \frac{d_o}{d} \qquad (2)$$

From (1) and (2) we obtain $\log \dfrac{d_o}{d} = \tfrac{1}{4} \log_e \dfrac{d}{d_2}$ (3)

This may be written in the form

$$\log d_o - \log d = \tfrac{1}{4}(\log d - \log d_2)$$

and when the damping is sufficiently small, this becomes approximately

$$d_o - d = \tfrac{1}{4}(d - d_2)$$

$$\text{or} \quad d_o = d + \frac{d - d_2}{4} \qquad (4)$$

For instance (taking numbers at random), let d, d_1, d_2, be 90, 85, and 81 respectively. Then, if we use the expression

$$d_o = d\left(1 + \frac{l}{2}\right)$$

it will be found that $d_o = 92{\cdot}37$.

If we use equation (4), we obtain $d_o = 92{\cdot}25$.

(2) STANDARD SOLENOID

It is shown on p. 375 that the field strength at the centre of a long solenoid—without an iron core—is given by the expression :—

$$B = \frac{4\pi.i.N}{l} \times \mu, \text{ where } \mu = 1 \text{ for air,}$$

and hence the field strength can be readily calculated when i, N, and l are known. (It will be noticed that this result is independent of the area of cross section.)

Such a solenoid is very convenient when a field of known value, but not of great strength, is required for standardising a ballistic galvanometer, as in Experiments 232 and 236.

A secondary coil is necessary, which is sometimes merely wound outside the middle part of the solenoid. It is, however, distinctly preferable to wind it on an accurately turned wooden bobbin, which can be placed *inside* (as indicated in Fig. 282).

The solenoid used in Experiment 232 (and in several subsequent experiments) was made by winding one layer of 18-gauge cotton-covered wire on a pasteboard tube about 8 centimetres in diameter, fitted with wooden ends to which the terminals are attached. It has 869 turns, and is 136 centimetres long. The secondary coil sliding within it has 1000 turns of 30-gauge silk-covered wire (although 500 turns would have been amply sufficient), and is wound in one layer on a truly turned boxwood bobbin. This is done because its area of section must be accurately known (when its position is *inside* the solenoid), whereas such information is not required for the solenoid itself.

The above dimensions are given as an illustration, but they are largely immaterial, provided that the solenoid is long compared with its diameter, and that the construction is such that N and l can be readily evaluated.

CHAPTER XXV

THEORY OF MAGNETISATION

We have already indicated in Chapter XII. the importance of the ideas embodied in the fundamental equation $B = \mu H$, and it now becomes necessary to discuss them more fully. In the first place, we must remember that both B and H may be represented by a system of "lines of force"; and by many writers, diagrams such as those shown in Chap. XI. (and the term "line of force") are understood to represent H. *In air*, these diagrams may be regarded as representing either B or H, but *inside magnetisable bodies*, B and H have different numerical values, and then it becomes necessary to distinguish between "lines of force" (H), "lines of magnetic induction" (B), and (sometimes) "lines of magnetisation," which may be regarded as the difference of the two quantities.

There is, however, some risk of confusion of thought in dealing with two or three distinct kinds of "line," and we prefer (unless otherwise stated) to use the term "line" in one sense only, viz. in a manner analogous to that already adopted in connection with "lines of electric force."

Let a magnetising force (H) be produced in any region of space. (We are not at present concerned with its origin; it may be produced in various ways, *e.g.* by a current in a wire or coil, or by a permanent magnet.) At any point therein, H is measured by the force in dynes exerted upon a unit pole placed at that point. This magnetising force produces a magnetic field, which we shall represent by "lines of force," its strength being measured by the number of lines per square centimetre. This number is denoted by B, and is often called the "magnetic induction." By a perfectly arbitrary convention, we then agree to regard B as being numerically equal to H in air (more precisely, in a vacuum, but the difference is very slight), which means that the permeability (μ) of air is taken as unity.

Now, suppose that a substance other than air—*e.g.* iron—is placed in this field. If we can regard the substance as filling up the whole space so completely that we need not consider its boundaries, then *the magnetising force* acting upon it will be the same as it was before the substance was introduced. But it is quite otherwise when we have to deal with substances which are not unlimited in extent. In such a case, if the body is more strongly magnetic than air, from one point of view we may think of it as concentrating the lines of force, which will tend to pass through it in preference to the surrounding air.

The distribution of the *field* inside the body depends upon the shape of the latter; it is not as a rule uniform, but it can be shown to be uniform in two cases, (*a*) when the body is a sphere, (*b*) when it is an ellipsoid with its major axis parallel to the field.

The state of affairs when the body is a sphere of soft iron is shown in Fig. 287, which is taken from Carey Foster and Porter's *Electricity and Magnetism*. There is no particular difficulty in measuring the value of B inside a sphere (by methods subsequently to be described), but it is important to notice that B is much less than it would be under similar conditions inside a mass of iron of unlimited extent. This is due to the self-demagnetising effect of short chains of iron particles already explained on p. 117. It is an effect which is greatest at the poles, *i.e.* where the lines pass

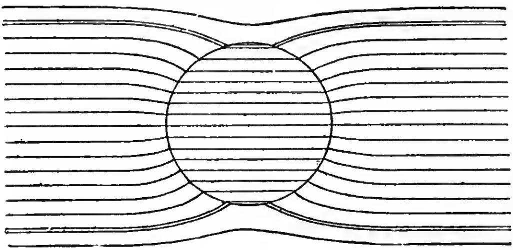


FIG. 287.

from iron to air, and is least at the middle of a long chain of particles, and hence it is particularly marked in the case of a sphere. These facts are usually expressed by saying that the poles developed in the iron exert a "demagnetising force," and hence the effective value of H inside the iron, on which B depends, is much less than it is outside in air. Now, the value of H inside the body cannot be found experimentally, and we therefore see that it would lead to grossly incorrect results if we were to measure the values of B produced in an iron sphere by various magnetising forces, and to regard them as being solely due to those magnetising forces. If we use a bar, instead of a sphere, the difficulty still exists, but we can easily see that it will become of less importance as the length of the bar increases relatively to its cross section, and it can be shown that if the length is from 500 to 1000 times as great as the diameter, there will be no serious error if we regard the magnetising force at the centre of the bar as being identical with its value before the bar was introduced into the field.

It follows that *long* bars or wires should be used in such measurements, when we are dealing with strongly magnetic substances. It also follows that, when the substance is only feebly magnetic, this condition is not so important, for the disturbing influence due to the induced magnetisation will be small, and then good results may be obtained with relatively short specimens.

Induced Polarity.—In order to attain clearness of ideas, it is necessary to regard the phenomena of magnetisation from several points of view. For instance, we may think of a substance, like iron, as becoming magnetised and developing poles, from which emanate a field of its own, superposed upon the field due to the original magnetising force. These induced poles may be treated mathematically as points, and in the case of a sphere they will be very near together on each side of its centre. Let m be the strength of each induced pole, then, as stated on p. 136, each may be regarded as a source producing a radial magnetising force, whose value at distance d is $\frac{m}{\mu d^2}$.

In order to show what these statements mean, it will be convenient to represent the three magnetising forces (not the field) by a line diagram. Let NS, Fig. 288, be the induced poles, from which are drawn a series of radiating lines to represent their "magnetising force." Let these lines be superposed upon a set of parallel lines to indicate the original magnetising force H. Taking the resultant, we obtain the effective value of H at each point, and it can be shown that this would lead to the state of affairs shown in Fig. 287.

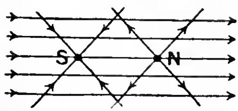


FIG. 288.

Now it will be seen from Fig. 288 that, in the space between the induced poles, the direction of the magnetising force due to the poles themselves is *opposite* to the direction of the original magnetising force; from which it follows that *in the body itself* the poles give rise to a "demagnetising force," in consequence of which the effective value of H will be less than its value outside. Again, as the effect of these poles varies inversely as the square of the distance, the longer we make the body (thus separating its poles) the smaller will become this demagnetising force at its centre, until at last it becomes small enough to be negligible, although by this means it can never be completely eliminated.

Method of completely Eliminating Effect of Induced Poles.—In practice, however, such complete elimination can be obtained in a very simple way. For most purposes, the original magnetising force is produced by a coil of wire carrying a current, and if, instead of experimenting on a long bar inside a long solenoid, we suppose the ends of the bar (and also the winding) to be bent round to form a closed ring, then there are no free poles, and the lines of the field are wholly inside the iron, so that the theory becomes as simple as in the case of a medium of unlimited extent.

We have now to apply these ideas to the more important experimental methods adopted in practice.

Exp. 236. Procure a ring[1] as nearly circular as possible, having a diameter of about 6 inches, and made out of ¼ inch soft iron bar. Anneal it as carefully as possible. Measure the diameter of the iron in several places, and then calculate its cross section. Also measure the mean diameter of the ring. Cover the iron with a thin layer of silk or of paraffined paper, and then wind one layer of 18 or 20 gauge double cotton-covered copper wire uniformly all round it. Where the ends meet, wrap a strip of silk around each of them to obtain good insulation, twist them together, and bring them out to serve as terminals. Finally varnish or dip in melted paraffin-wax. *Count the number of turns* N.

Now wind upon the ring a small "test coil," or secondary, of say 10 or 20 turns. This is conveniently made of fine wire, say 30 gauge, and is one layer deep, so that the number of turns can be easily counted. (The best number to use will depend upon the sensitiveness of the galvanometer, and is unknown at present, but this coil is a very simple affair, readily removed and altered.)

Connect up the arrangement as shown in Fig. 289, in which R is an adjustable resistance; A, an ammeter; *ab*, a current reverser; and B, a battery. By these means, any required current up to about 5 amperes may be sent through the magnetising coil, N, in either direction. The test coil, *t*, is connected to a reflecting ballistic galvanometer, G, preferably of the suspended coil type, and in the circuit is included some device for calibrating the instrument.

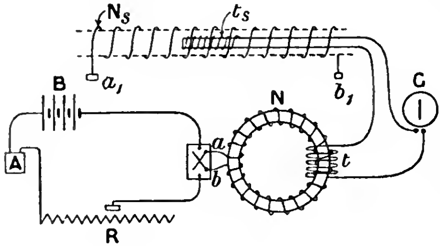


FIG. 289.

Many such devices are possible; for instance, if the vertical component of the earth's field is known with sufficient accuracy, an earth inductor coil may be used, as explained in Experiment 233; but perhaps the most convenient and accurate method is to use a standard solenoid, described on p. 395. Its secondary, t_s, is included in the galvanometer circuit during the experiment, as shown in the figure.

(1) Adjust the resistance and the number of cells until the maximum current intended to be used is obtained—say 5 amperes. Suddenly reverse the current, and notice the throw obtained. If the spot of light moves off the scale, the number of turns in the test coil, *t*, must be reduced until it remains on the scale; if the throw is not fairly large, increase the number of turns in the test coil.

(2) Then adjust the current to some small value until the throw on reversal is just large enough to be measurable—say 5 to 10 scale divisions.

[1] Such a ring is readily procured, and is useful for teaching purposes. When the ring is made with a view to exact measurements, it is better to give it the form of a hollow cylinder, which can be trued up in a lathe. A convenient size will be about 1 inch long, 4 inches mean diameter, and thickness of iron ¼ inch. Before winding, these dimensions must be very exactly measured in centimetres.

Note current and throw, repeating the readings once or twice to avoid mistake. Slightly increase the current, note its value, then suddenly reverse and note the throw. Proceed in this way (taking all the readings on the same side of the scale), until the maximum current is reached.

(3) The next step is to calibrate the galvanometer. Without, in any other way, disturbing the apparatus, disconnect the magnetising coil from the reverser at the points a and b, and, in its place, join up the ends a_1, b_1 of the solenoid. Adjust the current very exactly to some definite value, and note the throw on reversal. This is due to the lines of the solenoid cutting the turns of the secondary coil, t_s. If the throw is too small to be read accurately, either increase the current or increase the number of turns in the secondary. If possible, take two or three readings with different currents, and notice if the readings are—as they should be—exactly proportional to the current. By doing this, accidental errors are avoided.

We are now in a position, from the various data obtained in this experiment, to work out the values for both H and B from equations already obtained.

To obtain Values of H.—In the case of H, this is a simple matter, for $H=\frac{4\pi i N}{l}$, where N is the number of turns in the magnetising coil of the ring, and l is its mean circumference, *i.e.* the mean length of the path of the lines of force inside the iron expressed in centimetres.

As, in the experiment, the current was measured in amperes, it is convenient to write the equation in the form

$$H=\frac{4\pi CN}{10l}$$

where C is in amperes; this therefore becomes

$$H=\frac{1\cdot257N}{l}\times C$$

Work out the value of the constant $\frac{1\cdot257N}{l}$: let this be m. Then for each reading obtained in the experiment, $H=m\times C$.

To obtain Values of B.—Let t = number of turns in the test coil on iron ring; t_s = number of turns in the secondary coil inside the solenoid (as *time* does not enter into the calculation, we can use t for turns in this case without risk of error); N_s = number of turns in the standard solenoid; and l_s = length of standard solenoid in centimetres.

Consider any one of the "throws" obtained in (2) of Experiment 236. *Before* reversing the current, the iron was magnetised up to a certain value of B; on reversal, it was suddenly demagnetised and then remagnetised in the *opposite* direction up to the *same* value of B. The throws due to each operation are in the *same* direction, and the observed throw is, therefore, due to all the lines present in the iron cutting the turns of the test coil *twice*.

Let Z = number of lines present in the iron, and Q the quantity passing through the galvanometer, which produces the throw of d divisions,

$$\text{then} \quad Q = \frac{2 \times Z \times t}{R} \propto d \qquad (1)$$

where R is the total resistance in the galvanometer circuit.

Again, in the calibration experiment (part (3) above), let Z_s = the number of lines present inside the secondary, and Q_s the quantity passing through the galvanometer, which produces a throw d_s, when the current in the solenoid is C_s amperes,

$$\text{then} \quad Q_s = \frac{2 \times Z_s \times t_s}{R} \propto d_s \qquad (2)$$

Dividing equation (1) by equation (2), we obtain

$$\frac{Zt}{Z_s t_s} = \frac{d}{d_s}$$

$$\therefore \quad Z = \frac{Z_s \times t_s}{t \times d_s} \times d \qquad (3)$$

Now all the values on the right-hand side of this equation are known except Z_s, which we can find as follows:—

Applying the general formula for H to the standard solenoid (it is convenient to use the affix s to denote this application), we have

$$H_s = \frac{1{\cdot}257 C_s N_s}{l_s}$$

Now let B_s be the strength of the field *inside* the solenoid, then $B_s = \mu H_s$, but $\mu = 1$ for air.

$$\therefore \quad B_s = \frac{1{\cdot}257 C_s N_s}{l_s}$$

Let A_s = area of cross section of the secondary coil; then the total number or "flux" of lines inside it is $B_s \times A_s$, and we have

$$Z_s = B_s \times A_s = \frac{1{\cdot}257 C_s N_s A_s}{l_s} \text{ (all of which are known)}$$

whence, substituting this value in (3), we obtain

$$Z = \frac{1{\cdot}257 C_s N_s A_s \times t_s}{l_s \times t \times d_s} \times d$$

Now as Z is the number of lines per square centimetre *inside* the iron, and as the area A_i of cross section of the iron in square centimetres is known, we have

$$B = \frac{Z}{A_i} = \frac{1{\cdot}257 C_s N_s A_s t_s}{l_s \times t \times d_s \times A_i} \times d$$

$$\text{or} \quad B = m_1 \times d, \text{ where } m_1 \text{ is a constant.}$$

Work out its value, and calculate the various values of B by multiplying each throw by it.

Finally work out the various values for μ, which equals $\frac{B}{H}$. The results may be tabulated as under:—

Current in Amperes (C)	Throw $(=d)$	H $=m \times C$	B $=m_1 \times d$	$\mu = \frac{B}{H}$

Magnetisation and Permeability Curves.—Plot the "magnetisation curve," taking values of H for abscissæ, and of B for ordinates.

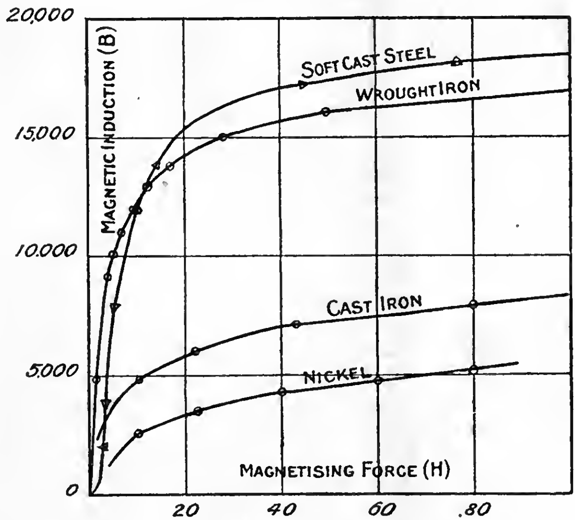


FIG. 290.

Also plot the permeability curve, taking values of B for abscissæ, and of μ for ordinates.

Figs. 290 and 291 show such curves for various materials. If the graphs were straight lines, the ratio $\frac{B}{H}$, *i.e.* the permeability, would be constant. It will be observed that the permeability increases at first and then decreases, and it is for this reason that those expressions which contain μ as a factor (*e.g.* the coefficient of self-induction), become indefinite when iron is present, unless information is available as to the state of the iron.

Three well-marked stages are indicated by a complete magnetisation curve :—

(1) A very short initial stage (just perceptible near the origin in the curve for soft steel, Fig. 290, and perhaps better seen in Fig. 294). During this stage, the iron is under the influence of a very weak, but increasing, magnetising force, and the magnetic induction B is increasing relatively slowly—a given change in H

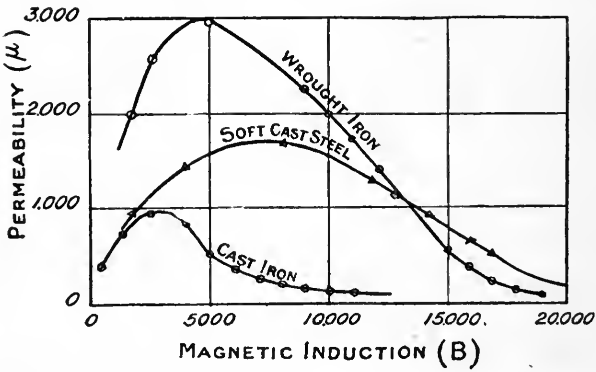


Fig. 291.

producing only a small change in B. In other words, the permeability is small.

(2) A stage indicated by the nearly straight and vertical portions of the curve. During this stage, a very slight change in H produces a relatively enormous change in B, and the permeability increases rapidly, passing through a maximum value.

(3) A stage indicated by the portion of the curve beyond the bend. Here, the iron has reached "saturation," and further increases in H produce relatively little effect. The curve slowly rises, however, because B consists of two parts, (*a*) the field due to the iron, and (*b*) the field due to the coil itself (*i.e.* without the iron), and the latter part increases without limit. Meanwhile the permeability falls steadily to quite low values.

In practice, such curves are absolutely necessary to furnish data for calculating dimensions and windings in designing motors, dynamos, transformers, &c. As the curves vary appreciably for different specimens of soft iron, it is important to be quite sure that the curve used refers to the particular variety in question.

It will be seen that, in the case of soft iron of average quality, saturation commences at about 15,000 lines per square centimetre, and may be regarded as complete at about 18,000, whereas cast iron is saturated when $B = 5000$, and nickel at a much lower value.

The method of measuring B, H, and μ, described in Experiment 236, is theoretically perfect, but is unfortunately very tedious in practice, because a ring has to be specially made and wound for each kind of iron that has to be investigated. It is therefore often convenient to modify it in such a way that the *same* magnetising coil may always be used. Now, we have shown that if the iron is in the form of a long rod, magnetised by a long solenoid, the value of H at the middle of the rod will be very nearly the same as if the iron were in the form of a ring.

Exp. 237. Make a magnetising solenoid by winding three or four layers of 20-gauge double cotton-covered wire on a paper or cardboard tube, not less than 50 centimetres in length and about 1 centimetre internal diameter. Cut off about five lengths of soft iron wire, say 20-gauge, each about twice as long as the solenoid, and tie them together in several places with thread. Cover about 3 inches at the middle of the bundle with thin paraffined paper, and wind on it about 50 turns of 30-gauge cotton-covered wire in one layer. Insert the iron into the solenoid, so that the test coil is in the middle, and bring out the ends carefully. This arrangement replaces the ring used in the previous experiments, and is to be connected up exactly in the same way, *i.e.* the ends of the magnetising solenoid will be connected to the reverser at *ab*; one end of the test coil wound on the iron wire will be joined up to the secondary of the standard solenoid and the other to the galvanometer. The method of experiment and the details of calculation are unaltered, except that the total area of cross section of iron must be found by calculating the section of each wire, and adding the results together, the diameter being measured with a micrometer gauge.

(It may be pointed out that if a specially designed standard solenoid is not available for calibrating the galvanometer, the magnetising solenoid itself may be used for that purpose, first removing the iron core, and winding a small secondary around it at the middle.)

The Magnetic Circuit.—In order to appreciate the usefulness of such curves as those drawn on pp. 403 and 404, it is necessary to look at the subject from another point of view.

As lines of magnetic force always form close curves, however much they branch out in some part of their path, we may regard them as forming complete circuits. It is not of the same nature as a current circuit, for there is no suggestion of a flow round the circuit; yet similar ideas may be applied to it, especially when the path of the lines is entirely, or almost entirely, inside iron.

Consider an iron ring inside a uniformly wound magnetising coil, similar to that mentioned in Experiment 236. The shape of the ring need not be circular, although that is the simplest case.

From p. 401 we have

$$H = \frac{1 \cdot 257 CN}{l}, \text{ and}$$

$$B = \mu H = \frac{1 \cdot 257 CN . \mu}{l}$$

Let A be the area of section of the iron, then total lines in iron (which we shall call the "Flux") is given by

Total lines $= \text{Flux} = B \times A = \frac{1 \cdot 257 CN \mu A}{l}$, which may be written in the form

$$\text{Flux} = \frac{1 \cdot 257 CN}{\frac{l}{A\mu}}$$

Now the numerator is really $4\pi \frac{C}{10} N = 4\pi i N$, and we have already found this to be the magnetic P.D. between the ends of the solenoid (see pp. 375 and 380).

Also, the denominator has the same form as a resistance, if we regard μ as being a kind of specific conductivity for lines of force. Now, μ is not a conductivity, and the expression $\frac{l}{A\mu}$ is not a resistance, but there is a mathematical similarity of form, which makes it possible to apply the ideas embodied in Ohm's law to a magnetic circuit. To avoid suggesting that the denominator is a resistance, it is called the magnetic *reluctance* of the circuit. The numerator is often termed "the magnetomotive force," and hence we have generally

$$\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Magnetic reluctance}}$$

If the magnetic circuit is not the same at all points, the magnetic reluctances of each part may be calculated separately, and the results added together to obtain the total reluctance, just as resistances are summed in current circuits. It is, however, usually more convenient to apply the expression as a whole to each part of the circuit in turn, as we apply Ohm's law to a *part* of a current circuit as well as to the whole.

Example 1.—The mean length of an iron ring is 62 centimetres. Find the ampere-turns required to produce 12,000 lines per square centimetre within it—assuming the permeability curve to be available.

$$\text{Generally,} \quad B = \frac{1 \cdot 257 CN}{l} \times \mu$$

and on reference to the curve, we find, say, that when $B = 12{,}000$, $\mu = 1500$. It is probably more convenient in the majority of problems to write this equation as follows :—

$$CN = \frac{B.l}{1 \cdot 257\mu}$$

$$i.e. \quad CN = \frac{\cdot 8B.l}{\mu} \text{ as the reciprocal of } 1 \cdot 257 = \cdot 8 \text{ nearly}$$

$$\text{Whence } CN = \text{ampere turns} = \frac{\cdot 8 \times 12{,}000 \times 62}{1500} = 397 \text{ nearly.}$$

Hence, the required degree of magnetisation will be produced with a uniform winding of any number of turns provided that the product of the current and the turns is 397. It will be noticed that the *thickness* of the iron is immaterial. Whatever the area of section may be, the value of B is unaltered. The *flux*, however, does depend upon the area, and equals B × area of section.

Example 2.—Find the number of ampere-turns required to produce the same value of B when a gap 1 millimetre in width is made in the iron ring mentioned in Example 1.

This gap is so small that to a first approximation we may consider the field uniform within it, and neglect the fringing out of the lines which must occur at the edges.

It is now best to think of the total ampere-turns required as being divisible into two parts, one part being required to produce the flux in the iron, and the other in the gap.

From Example 1 we have, therefore,

$$CN = \frac{\cdot 8B.l}{\mu} \text{ (in iron)} + \frac{\cdot 8B.l}{\mu} \text{ (in air gap).}$$

In the iron ring this is practically what it was before; for the only difference will be the putting of 61·9 for l instead of 62. It is, therefore, unnecesary to repeat that calculation.

Applying it to the air gap, for which $\mu = 1$, we have

$$CN = \frac{\cdot 8 \times 12{,}000 \times \cdot 1}{1} = 960$$

$$\therefore \quad \text{total } CN = \text{total ampere-turns} = 397 + 960 = 1357.$$

This example brings out, in a striking way, the enormous importance of permeability, and it also serves to illustrate the meaning of the statements made at the beginning of this chapter from yet another point of view.

For suppose we used this ring in Experiment 236. We should measure the values of B quite correctly, but if we obtained the values of H in the same way, then, when $B = 12{,}000$, we should have $H = \frac{1{\cdot}257CN}{l} = \frac{1{\cdot}257 \times 1357}{l}$ whereas its true value for the iron would be $\frac{1{\cdot}257 \times 397}{l}$.

We can also understand why approximately correct results are obtained when very long rods are used, as in Experiment 237. The expression

$$CN = \frac{{\cdot}8B.l}{\mu} \text{ (for iron)} + \frac{{\cdot}8B.l}{\mu} \text{ (for air gap)}$$

still applies, but now the "air gap" is the whole return path of the lines in air, and the longer the bar the farther away the lines will be thrown out into the surrounding space, and the greater will be the effective cross section available. As a result, in spite of the extra length of the air path, the average value of B in it is so small that the second term becomes almost negligible.

It will be seen that this line of thought leads to exactly the same conclusions as those based upon the conception of the demagnetising effect of free poles, and serves to illustrate the statement that the idea of poles is mainly a convenient mathematical convention. In the majority of practical applications, the magnetic circuit point of view is almost exclusively employed, but, for purposes of research, it often happens that a theory based on the properties of poles is most useful.

We have now to develop such a theory, but before doing so another experiment may be given to illustrate the properties of a magnetic circuit.

Exp. 238.—Connect an electromagnet to a current reverser and a single cell, if necessary putting a resistance in the circuit so that the magnet is only weakly excited. Wrap one or two turns of wire around the iron base and connect to a galvanometer as shown in Fig. 292. Shunt the galvanometer at first to avoid possible injury. Reverse the current and note the throw; divide this by 2, as it is due to all the lines in the iron cutting the test coil *twice*. Now put on an iron keeper, and notice the large throw produced. This means that the number of lines of force passing through the iron must have increased. Confirm this by again reversing the current. Taking the throws as proportional to the flux in the iron, it will be found that the flux is much greater—perhaps ten times greater—when the keeper is on than when it is off. As the ampere-turns have remained constant, this must be due to the improvement in the magnetic circuit, *i.e.* to the decrease in reluctance obtained by putting on the keeper.

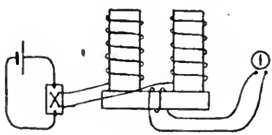


FIG. 292.

Intensity of Magnetisation and Susceptibility.—It has already been stated that, for any magnet, the term *moment* represents a perfectly definite quantity, directly measurable without ambiguity, although we cannot say as much for the term *magnet pole.* Now instead of thinking about the lines of force a magnet possesses, and measuring its strength in terms of their number, we may think of its *magnetic moment per unit of volume.* This is known as the *intensity of magnetisation*, which we shall denote by I. Hence, by definition,

$$I = \frac{\text{Moment}}{\text{Volume}} \qquad (1)$$

For our purpose it will be sufficient to consider the special case of a long magnet of length l and sectional area A the poles being regarded as located at the ends.

$$\text{Then } I = \frac{M}{V} = \frac{m \times l}{A \times l} = \frac{m}{A} = \frac{\text{strength of pole}}{\text{Area}}$$

or I is the number of unit poles per square centimetre of end surface.

Now suppose that the rod is of soft iron, magnetised by a long solenoid as in Experiment 237. For each value of H there will be some value of I, which we may measure experimentally by means of a magnetometer (see p. 146), and then a curve may be plotted for I and H. It will be almost identical in general *shape* with the B and H curve, and, for some purposes, will represent the facts equally well. We may write the relation between the two quantities in the form

$$I = k.H$$

where k is known as the *susceptibility* of the iron. It is, therefore, a measure of the magnetic quality of a substance from the "polar" point of view, just as μ is a similar measure from the "lines of force" point of view.

To find a relation between these quantities, we may refer to Fig. 288, from which it will be seen that the total flux inside the iron must consist of two distinct parts—(a) a uniform field due to the original magnetising force H, *which would exist whether the iron were present or not*, and which is numerically equal to H; (b) a very much stronger field, not necessarily uniform, contributed by the iron itself, which may be regarded as passing through the iron and emerging at the poles. The "magnetic induction" (B) inside the iron is the *sum* of the two (*i.e.* the *total* flux) divided by the area of section.

Now, the number of lines of force *per square centimetre* due to the iron is $4\pi I$ (for there are I unit poles per square centimetre of section, and each unit pole has 4π lines). By some writers these are termed "lines of maguetisation."

$$\therefore \quad B = H + 4\pi I^{1}$$

$$\text{Also} \quad I = kH$$

$$\therefore \quad B = H + 4\pi kH$$

$$= (1 + 4\pi k)H$$

$$\text{But by definition} \quad B = \mu H$$

$$\therefore \quad \mu = 1 + 4\pi k.$$

If our reasoning is correct, it follows that, when an iron rod is subjected to a magnetising force, which is raised to an extremely high value, B ought to gradually increase without limit because the term $\mu_v H$ increases without limit. On the other hand, I is the contribution of the iron itself, and should reach a limit when the iron is "saturated." Recent measurements with very intense magnetising forces have shown that this is really the case.

Magnetometer Method of obtaining Magnetisation Curves.—We give this method not only because it is especially useful for investigating the effects of residual magnetism, but also because it is an instance of the application of ideas previously outlined.

The solenoid already referred to may be used, but it will be better to make one specially for the purpose by winding from 6 to 10 layers of 22-gauge copper wire on a brass tube of about $\frac{1}{8}$ inch internal diameter and 70 centimetres in length. The number of turns and the length must be accurately known. An iron wire can then be magnetised beyond the point of saturation with a current of less than 1 ampere, which, if a suitable ammeter is not available, is readily measurable on a tangent galvanometer. A new piece of apparatus is required, viz. a coil of wire of any convenient size on a hollow circular frame (like a tangent galvanometer coil) and known as a "compensating coil."

Exp. 239. Set up the reflecting magnetometer described on p. 146. About 6 inches behind it, clamp the solenoid firmly by some non-magnetic support[2] in a vertical position, with its lower end 2 or 3 inches *below* the level of the magnetometer needle. About 10 or 12 inches in front of the magnetometer, place the compensating coil with its plane at right angles to that containing the solenoid and mirror, so that the light passes through it to the scale. The coil should be arranged to permit of adjustment by moving it slightly backwards or forwards. Connect up, as shown in Fig. 293, so that the *same* current flows round both coils. The compensating coil is used to produce a

---

[1] Although this expression is usually written in the form given, it will be seen later (Chapter XXXIV.) that it is physically incorrect, because B and H are quantities of different dimensions. In fact, it should be written $B = \mu_v H + 4\pi I$, where μ_v is the permeability of a vacuum, but it would be pedantic to do so.

[2] Obviously all *iron* fittings must be avoided in any experiments with a magnetometer.

field at the magnetometer needle equal and opposite to that produced there by the solenoid, so that for all strengths of current, the effect of the latter field is neutralised and the deflections obtained in the experiment are due to the iron only. The first step is to obtain this adjustment. Switch on a small current, and make sure that the deflection due to the compensating coil alone is in the opposite direction to that due to the solenoid alone. If not, reverse the connections of one of them. Then move the compensating coil backwards or forwards until the deflection is zero. It is very convenient to complete the adjustment by shunting this coil with a suitable resistance (usually a piece of platinoid wire). Increase the current to the maximum value intended to be used, and thus make sure that the adjustment is exact.

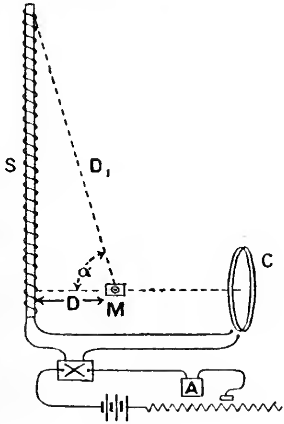


FIG. 293.

There is one difficulty still to be overcome. When the iron is placed in position, it will be subjected to the magnetising influence of the vertical component of the earth's field, as well as to that of the solenoid, and the softer the iron used, the more conspicuous and troublesome is the effect. There are various ways of dealing with it. One of the best is to have another independent winding on the solenoid, in which, during the experiment, is maintained a steady current from an independent source of a strength sufficient to produce a field equal and opposite to the vertical component. As our purpose is to demonstrate the method rather than to secure great accuracy in detail, we shall refer the student to works specially devoted to electrical measurements, and shall neglect the correction.

Let us suppose that the object of the experiment is to investigate the magnetic properties of steel. Cut off a piece of steel piano wire, about 50 centimetres long, *i.e.* rather shorter than the solenoid, and solder to one end about 15 centimetres of a stouter brass wire, springy enough to move easily up or down in the solenoid and yet to retain its position without slipping. Insert this in the solenoid. If it is not already accidently magnetised, there will be practically no deflection. Switch on a very small current, just enough to produce a readable deflection. Move the wire up or down until the deflection is greatest. Then the pole in the wire (which is not necessarily at the end) is in the correct position with reference to the needle, and the wire must not be moved again during the experiment. Increase the current by successive small steps, at each step reading the current and the deflection, until the maximum current to be used is reached. It will be known when saturation is obtained, because after that point is reached the deflection will scarcely be altered by any increase of current. Do not disturb the apparatus—it will be required for the next experiment—but measure the distance of the scale from the mirror, and also the distance of each pole in the iron from the mirror, marked D and D_1, in Fig. 293. The position of the lower pole is known, for it is on the same level as the mirror, and hence its distance from the top of the brass wire can be measured. Having located this pole, the position of the other can be found by symmetry.

If it is merely desired to show the *shape* of the magnetisation curve, without knowing actual values, plot a curve with current (which is proportional to H) for abscissæ, and the observed deflections (proportional to I) for ordinates.

If absolute values are required, the reasoning is as follows (although, as we are concerned both with H, our symbol for the magnetising force, and also with H, the symbol for the horizontal component of the earth's field, we shall denote the latter by H_e) :—

Let H_e = Horizontal component of earth's field.
A = Area of section of steel wire.
D, D_1 = Distance of poles from magnetometer needle (marked in Fig. 293).
α = Angle between lines marked D and D_1.
s = Distance of mirror from scale.
m = Pole strength of wire for any given current.
F = Field produced by wire at needle.
θ = Angle of deflection of mirror.
d = Deflection in scale divisions (for any given current).

$$\text{Then } F = \frac{m}{D^2} - \frac{m}{D_1^2} \cos \alpha, \text{ but } \cos \alpha = \frac{D}{D_1}$$

$$\therefore \quad F = \frac{m}{D^2} - \frac{mD}{D_1^3}$$

$$\text{Also } F = H_e \tan \theta$$

$$\therefore \quad m\left(\frac{1}{D^2} - \frac{D}{D_1^3}\right) = H_e \tan \theta$$

And as $m = IA$

$$\text{we have } I = \frac{H_e}{A\left(\frac{1}{D^2} - \frac{D}{D_1^3}\right)} \times \tan \theta = (\text{a constant}) \times \tan \theta$$

Again, if the deflection d and the scale distance s are measured in the same unit, then $\tan 2\theta = \frac{d}{s}$, and as θ is small, we may put $\tan \theta = \frac{d}{2s}$ approximately.

Hence, I can be calculated in terms of a constant multiplied by the observed deflection d, and as the values of the magnetising force H can be found in the form of another constant multiplied by the current, exactly as was done in the previous experiment, we obtain data from which a curve can be plotted with I for ordinates and H for abscissæ, and another curve with k for ordinates and I for abscissæ.

Although it would scarcely be the object of the experiment, it is also possible to obtain values of B from the data, for we have

$$B = H + 4\pi I \qquad \text{(p. 410)}$$
$$\text{Also } I = (\text{a constant}) \times \tan \theta$$
$$\therefore \quad B = H + 4\pi \times (\text{a constant}) \times \tan \theta$$
$$= H + (\text{a constant}) \tan \theta.$$

Cycles of Magnetisation.—**Exp. 240.** Repeat Experiment 239 until the maximum current is reached. Then, without interrupting it, gradually decrease it step by step, at each step reading the current and the deflection, until the current is zero. There will still be a large deflection, due to the residual magnetism. Reverse the current, again take the readings as it is gradually increased to its former maximum value. Then decrease again, still taking the readings, until the current is zero. Reverse, and continue readings until the maximum value is again reached. The whole cycle of operations may be repeated with advantage so that greater accuracy is secured.

To find the shape of the curve, plot deflections as ordinates against current as abscissæ. If the result is fairly satisfactory, the values of I and H may be calculated and plotted in the same way. Fig. 294 gives the result obtained in a certain experiment with steel wire, and the dotted line curve indicates, for the sake of comparison, the shape that would be obtained with soft iron. It will be seen that the values of H are plotted to the right and left of the origin respectively, according to the direction of the current, and the corresponding values of B above and below.

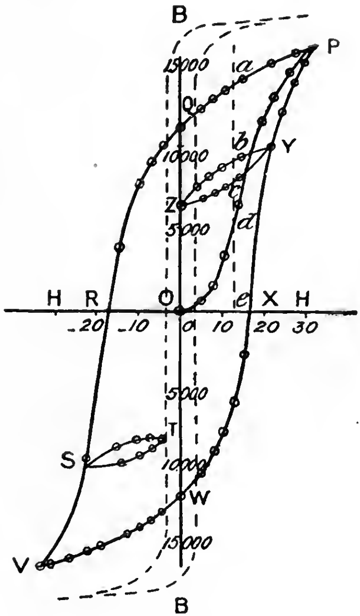


FIG. 294.

The branch OP is the first part obtained in the experiment just described. From P to Q the current is decreasing, and OQ is a measure of the residual magnetism when the current is reduced to zero. From P to R the current is increasing in the opposite direction, and the iron is not demagnetised until this has reached the value corresponding to R. Still increasing the current, the point V is reached, and by again decreasing and reversing the portion of the curve, VWXYP is obtained. The initial path OA is never again traversed unless the iron is first completely demagnetised. This is best done by heating it to redness, or (if that is undesirable, as affecting its state), by placing it in a solenoid carrying an alternating current which is gradually decreased to zero.

Such curves may also be obtained by ballistic methods—modifications of those already given—but as the details and precautions are rather more complicated, space does not permit of their description here.

It will be noticed that there is more than one value of B corresponding to any given value of H. In order to bring this out more clearly, when the point S was reached, the current was gradually reduced nearly to zero, thus tracing the lower part of the loop ST. Then it was increased again to its old value, thus tracing the upper part of the loop and again reaching S, the normal sequence of operations already described then being resumed. Exactly the same thing was done at the stage corresponding to Y, in this way obtaining the loop YZ, and evidently any number of loops could have been superposed on the main curve. Consider the points *a*, *b*, *c*, *d*, *e*, for each of which H has the same value. There are five values of μ corresponding to this value of H, and obviously there might be many more. It may, therefore, be asked—What meaning is to be attached to this term if its numerical value is so indefinite? Without giving a complete answer, it may be remarked that the use of μ in calculations is entirely in connection with soft iron, which readily loses its residual magnetism, so that if a soft iron wire be tapped or vibrated while we are carrying out the cycle of operations referred to above, the two branches of the curve tend to coalesce into one definite curve, which, under most practical conditions, is *the* magnetisation curve. The ballistic methods mentioned previously naturally give this mean curve. It is, however, necessary to keep in view the fact that the numerical value of μ *does* depend upon the way the iron is treated, owing to the tendency of the iron to persist in the state in which it happens to be. To this tendency, the term **hysteresis** has been applied, and the closed curves in Fig. 294 are known as hysteresis curves.

Loss of Energy due to Hysteresis.—If no such lag existed, *i.e.* if the curves of increasing and decreasing magnetisation coincided, so that for any value of H there was only one value of B, the energy expended in magnetising the iron would be completely returned to the circuit during demagnetisation. But the energy is not wholly returned, and the difference appears as heat in the iron.[1]

It can be shown that the energy lost in this way per complete cycle is proportional to the area of the figure enclosed by the curves. In the course of a B, H curve, if the area is reckoned in terms of B and H and divided by 4π, the result is the loss in ergs per cubic centimetre per cycle.

If the I, H curve be plotted, then the area expressed in terms of I and H will give the loss in ergs without dividing by 4π. In other words, the area included by the BH curve is 4π times the area included by the IH curve. This loss may be as much as 200,000 ergs per cubic centimetre in magnet steel. It is very important in connection with alternating currents (see Chapter XXVI.).

"Coercive Force and Retentivity."—Hopkinson first gave these terms a real meaning by defining OR, Fig. 294, as **coercive force**, and OQ as **retentivity**. Hence, retentivity is measured by the "residual" or "remanent" magnetism, which persists when the magnetising force is removed. The figure shows that it may be

[1] The heat is never sufficiently great to produce any marked rise of temperature. In Experiment 220 there is a certain amount of heat produced by the hysteresis, but the effect is mainly due to eddy currents, as stated.

greater in soft iron than in steel, but it is understood that the substance is not to be exposed to shock or vibration. Coercive force or coercivity is measured by the inverse magnetising force required to destroy this residual magnetism, and is much greater in steel than in soft iron.

Practical Measurement of Hysteresis Loss.—The determination of hysteresis losses can be very satisfactorily accomplished by the magnetometer (or ballistic) method, but only at the expense of much time and labour. It is, however, necessary to know the amount of such loss (under given conditions) in the sheet iron used for armatures and for transformer cores, so that, in practice, more rapid methods are required.

Ewing's Hysteresis Tester.—One of the earliest and most convenient arrangements for this purpose is due to Professor Ewing, and is shown diagrammatically in Fig. 295. A permanent steel magnet, M, is pivoted at P, so that it oscillates in a plane at right angles to the plane of the paper. At the top is fixed a long pointer moving over a divided scale. At the bottom is a weight, W, which can be raised or lowered to adjust the restoring force acting when the magnet is slightly deviated from its position of rest; and also a vane, D, moving in oil to make the motion more "dead-beat." Strips are cut from the sheet iron to be tested and made up into a bundle about 3 inches long and $\frac{5}{8}$ inch square. This bundle is clamped together and fixed to an arm, B, so that it can be rotated between the poles of the magnet by means of a hand wheel. When this is rotated at sufficient speed (the exact speed is immaterial), it will tend to drag the magnet after it, and as a result the latter will take up a definite position, and the corresponding deflection can be read off on the scale. The greater the tendency of the induced magnetism to persist, the greater will be this deflection, and it is, therefore, a measure of the hysteresis loss. The makers supply with each instrument a standard specimen of iron, for which the loss is known for a given value of B. This sample is placed in the holder and rotated in the same way—the reading thus obtained being used to calibrate the scale. Each bundle to be tested is made up as nearly as possible to the size and weight of the standard specimen.

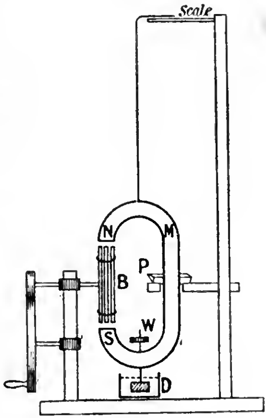


FIG. 295.

The magnetic cycle due to the rotation of iron in a constant magnetic field is exactly similar to the cycle occurring in an armature core, but is not strictly similar to the magnetic cycle performed when an iron core is magnetised by means of an alternating current, as in the case of a transformer. In the latter case, the actual iron loss is better measured by means of a wattmeter, and the principle of such methods will be indicated in connection with that instrument (see p. 566).

Relation between Hysteresis Loss and Magnetic Induction.—It is obvious, from Fig. 294, that the loss during a cycle depends on the maximum value of B attained during that cycle. A graph drawn connecting these quantities shows that they are not proportional, the loss increasing at a greater rate than B. No theoretical connection has been obtained between them, but Steinmitz has shown that if W be the loss in ergs per cubic centimetre per cycle, the experimental results are represented by an equation of the form

$$W = a \times B^{1.6}_{(max.)}$$

where a is a number, which is constant for a given sample of material. For good soft iron a is about ·001; for hard steel about ·025; and for the new silicon and aluminium alloys (see p. 423) it may be as low as ·0007.

Ewing's Theory of Magnetisation.—The loss of energy due to hysteresis has often been vaguely assigned to a kind of molecular friction, such that work must be done in moving the particles of the iron into new positions. Ewing experimented on a large model, which contained a great number of small pivoted magnets placed very close together and surrounded by a magnetising coil. The results obtained strongly support the conclusion, that it is quite unnecessary to postulate anything akin to friction between the particles in order to explain hysteresis, and his theory may be briefly outlined as follows. The ultimate magnetic particles in a piece of unmagnetised iron, left to itself, tend to arrange themselves in groups —not necessarily alike in configuration—each group being stable for small disturbances and producing no external magnetic field. When the iron is subjected to a magnetising force of gradually increasing strength, the first effect is to modify the configuration of each group, although its stability or general form, as a whole, is not destroyed. This corresponds to the short initial portion of the magnetisation curve, where B is slowly increasing and μ small.

As H increases, the individual members of each group tend to point along the field, except in so far as their directions are still modified by their influence on each other, and as a result some groups become unstable and break up. All the groups will not break up for the same value of H, but when instability begins, this action will go on somewhat rapidly. This corresponds to the steep portion of the BH curve, where a small increase in H produces a large increase in B. When most of the groups have been broken up, the third stage is reached, in which the iron is said to be "saturated." In this stage an increase of H merely tends to weaken the effect of their mutual influence, and produces more and more perfect alignment with the field. Suppose that H is now gradually diminished. Some groups of particles will tend to persist in their arrangement, being more or less in equilibrium under the action of their mutual forces. Therefore, when H is reduced to zero, there will be some residual magnetism, and H will have to be reversed in order to break up the new groupings. It is also evident that reducing H does not make the iron pass through each of its previous states for a given value of H.

The loss of energy is explained as being due to the fact that, whenever a particle becomes unstable and suddenly swings round into some new position of equilibrium, it acquires energy of motion, and it oscillates about its new position of equilibrium until that energy is gradually converted into heat.

Hence, the hysteresis loss is greatest in the second stage of magnetisation, and relatively small in the third.

Up to this point, we have assumed that the field varies in strength in order to produce the changes in magnetisation. There is, however, another method of procedure, for a piece of iron may be rotated in a field of uniform strength. This case occurs in practice in dynamo armatures. Evidently, during one complete revolution, the iron passes through one complete cycle of magnetisation. Let us suppose the strength of the field to be very great, so great, indeed, that the mutual magnetic forces between the particles becomes comparatively negligible in comparison. Then each particle always points along the field in the right direction, irrespective of the rotation of the iron as a mass, and no energy will be lost in its taking up new positions. Thus the theory leads to the conclusion that there should be little hysteresis loss, when iron is rotated in very strong fields—a conclusion which has been confirmed by direct experiment.

Attraction between a Magnet Pole and its Armature.— Let a magnetic field exist between two iron surfaces, each of area A, and distance d apart, and let us assume that the field is perfectly uniform, and of strength B, over this area. It will not be uniform, especially at the edges, but the departure from uniformity becomes smaller as d, compared with A, becomes smaller. (It will be remembered that similar assumptions were made in discussing a kindred theorem in statics; see p. 52.)

A unit pole, placed anywhere in the gap, will experience a force of H dynes, where $H=\frac{B}{\mu}$ and μ is unity. Consider one of the iron surfaces. Its pole strength is $\frac{BA}{4\pi}$, and if such a pole were placed in the field, the force on it would be $\frac{BHA}{4\pi}$ dynes.

But, in this case, the pole forms the boundary of the field, and the force is, therefore, only half as great (for the reasoning given on p. 93 for electrostatic fields applies to magnetic fields also).

$$\therefore \quad \text{Attraction} = \frac{BHA}{8\pi} \text{ dynes.}$$

As $H=\frac{B}{\mu}$, we have

$$\text{Attraction} = \frac{B^2A}{8\pi\mu} \text{ dynes } \left(\text{or } \frac{H^2A\mu}{8\pi} \text{ dynes}\right)$$

where μ is unity in all practical cases.

The result is independent of the distance between the surfaces, and, therefore, the above expression gives, very approximately, the pull between two iron surfaces in good contact.

Potential Energy of Uniform Field.—We may, also, apply it to find the energy of a uniform magnetic field. For, suppose that the surfaces are in contact, and are then separated through a distance, d, the field still remaining uniform.

The work done = force × distance $= \frac{BHA}{8\pi} \times d$ ergs, and as a result, a magnetic field has been formed in space, whose volume is $A \times d$ cubic centimetres,

$\therefore$ the energy of the field per unit volume $= \frac{BH}{8\pi} = \frac{B^2}{8\pi\mu}$ ergs per cubic centimetre.

Example.—A horse-shoe electromagnet with a core and keeper forged from 1 inch square iron is excited by 300 ampere-turns. Let the joints between the pole faces and the keeper be scraped so as to make a perfect fit, and assume that the permeability of the metal at the induction produced by 300 ampere-turns is 1500. The length of the magnetic circuit is 16 inches. Find the total flux through core and keeper, and the force required to tear the keeper off.

(City and Guilds of London Institute, 1893.)

(The chief difficulty in this example is due to the fact that all the dimensions are given in inches, whereas it is necessary to use centimetres in the calculation.)

$$\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Magnetic reluctance}} = \frac{1{\cdot}257\ \text{CN}}{\frac{l}{A\mu}}$$

$$= \frac{1{\cdot}257 \times 300 \times (2.54)^2 \times 1500}{16 \times 2{\cdot}54}$$

$$= 8{\cdot}9 \times 10^4 \text{ lines of magnetic force.}$$

$$\text{Also } B = \frac{\text{Flux}}{\text{Area}} = \frac{8{\cdot}9 \times 10^4}{(2{\cdot}54)^2} = 1{\cdot}2 \times 10^4 \text{ lines per square centimetre.}$$

Now, attraction for *each* pole is $\frac{B^2A}{8\pi}$ dynes,

$$\therefore \text{Total pull} = 2 \times \frac{B^2A}{8\pi} = \frac{2 \times (1{\cdot}2 \times 10^4)^2 \times (2{\cdot}54)^2}{8\pi}$$

$$= 74 \times 10^6 \text{ dynes} = 160 \text{ lbs. weight nearly.}$$

Various methods based on this principle have been devised for measuring B in terms of the pull required to detach two surfaces of iron. The uncertainty as to the actual area of contact and other difficulties detract from their accuracy, and it does not seem necessary to describe them.

Refraction of Lines of Magnetic Force.—The argument given on p. 99 for lines of electric force applies equally well to lines of magnetic force, provided that we write "unit pole" in place of "unit charge," and B, μ, H in place of F, K, U respectively.

Consider a magnetic field passing from a medium of permeability μ_1 into another of permeability μ_2. Fig. 67, p. 100, will represent this case, if we suppose that the above changes are made in the lettering.

Then, we have, as before :—

(1) The normal components of the field (B) must be the same in each medium.

(2) The tangential components of the force (H) must be the same in each medium.

The first condition requires that

$$B_1 \cos \theta_1 = B_2 \cos \theta_2 \qquad (1)$$

and the second condition, that

$$H_1 \sin \theta_1 = H_2 \sin \theta_2 \qquad (2)$$

$$\text{But } B_1 = \mu_1 H_1 \text{ and } B_2 = \mu_2 H_2$$

$$\therefore \mu_1 H_1 \cos \theta_1 = \mu_2 H_2 \cos \theta_2 \qquad (3)$$

$$\therefore \frac{(2)}{(3)} \qquad \frac{1}{\mu_1} \tan \theta_1 = \frac{1}{\mu_2} \tan \theta_2$$

$$\text{or } \frac{\mu_1}{\mu_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

Hence, lines passing (say) from air into iron are bent away from the normal, and when passing into a diamagnetic substance (for which $\mu < 1$) they are bent towards the normal.

If μ_1 refers to air, and μ_2 to soft iron of good quality, as, for instance, in Fig. 287, p. 398, then μ_2 is so large compared with μ_1 that the ratio is practically zero, in which case $\theta_1 = 0$, *i.e.* the line is incident normally.

Diamagnetism.—Faraday, in 1845, by aid of powerful electromagnets, demonstrated that *all* bodies are acted on by magnetic influence—some being attracted, others repelled. When experimenting with solids, he suspended small bars of various substances, *m*, between the poles (Fig. 296), and he found that some of them set themselves *axially*, *i.e.* in a line joining the poles. These substances were, therefore, attracted by the poles of the magnets, and to these he gave the name **paramagnetic**. Others, however, were repelled by the poles of the magnet into a position at right angles to the line joining the poles, *i.e.* *equatorially*, and to these he gave the name **diamagnetic**.

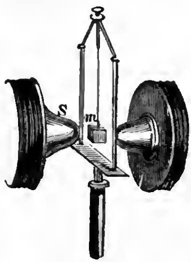


FIG. 296.

When liquids, contained in thin glass tubes, were suspended between the poles, they behaved similarly. Nearly all liquids are paramagnetic, and the tubes, therefore, set themselves axially; a few, however, are diamagnetic—notably blood, water, and alcohol—and the tubes, therefore, set themselves equatorially. The action on liquids may be observed in the following manner with very powerful electromagnets. The liquid is placed in a watch-glass, and rests on the poles S, Q (Fig. 297). If the liquid is diamagnetic, it is repelled from the poles and forms a little heap between them, A. If it is paramagnetic, it rises in a little heap over each pole, B. These changes, however, are so exceedingly small that it is very difficult to detect their existence.

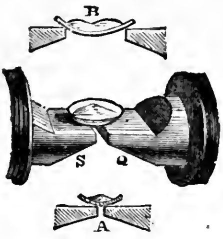


Fig. 297.

In experimenting with gases, Faraday caused it to be mixed with a small quantity of a visible gas or vapour, and then to ascend between the two poles of the magnet. He found that if the gas was paramagnetic, it spread out like a flame from an ordinary gas-burner between the poles; while, if it was diamagnetic, it spread out across them.

The following table gives the chief substances arranged in two classes:—

Paramagnetic.	Diamagnetic.
Iron.	Bismuth.
Nickel.	Phosphorus.
Cobalt.	Antimony.
Manganese.	Mercury.
Chromium.	Zinc.
Cerium.	Lead.
Platinum.	Tin.
Oxygen.	Copper.
Titanium.	Silver.
Palladium.	Gold.
Osmium.	Sulphur.
Many salts and ores of the above metals.	Selenium.
	Water.
	Alcohol.
	Air.
	Hydrogen.

It must be kept in mind that the behaviour of a feebly magnetic substance depends to some extent on the magnetic properties of the surrounding medium. Just in the same way as a balloon filled with a light gas rises through the air, because the force of gravity attracts the gas less than it attracts the atmosphere, so, if we suspend a paramagnetic body in a medium which is more strongly paramagnetic than the body, it behaves as though it were diamagnetic. Thus, if we suspend a weak solution of ferric chloride in air, it is paramagnetic;

if it be suspended in a strong solution of the same substance, it appears to be diamagnetic.

Oxygen is remarkable as a paramagnetic gas, and this property is still more evident in the liquid state. An experiment, first performed many years ago by Professor Dewar, shows this fact in a striking way.

He placed a vessel, containing oxygen in the liquid form, between the poles of an electromagnet (which happened to be the identical magnet with which Faraday first discovered the magnetic properties of oxygen gas). Upon exciting the magnet, the liquid oxygen sprang upwards, and adhered to the sides of the glass nearest the poles, until, owing to evaporation, it gradually disappeared.

The magnetic properties of oxygen and other feebly paramagnetic substances are, however, almost infinitesimally small in comparison with those of iron. For instance, the permeability of liquid oxygen is only 1·0041. Diamagnetic properties are relatively even feebler.

Nature of Diamagnetism.—A diamagnetic substance may be defined as a body, which, when placed in a magnetic field, acquires an induced polarity of the *opposite* kind to that induced in iron under similar conditions. Hence, while a rod of a paramagnetic substance tends to move into the strongest part of a magnetic field, and to set itself axially so that as many lines of force as possible may pass through it; a diamagnetic rod behaves in just the opposite way—it tends to move into the weakest part of the field, and to set itself so that as few lines of force as possible may pass through it.

We can express these facts in terms of μ and k by means of the relation $\mu = 1 + 4\pi k$ (already given on p. 410). For air, $\mu = 1$, and, therefore, $k = 0$.

For a paramagnetic body, μ is greater than unity, and k is greater than zero, both being positive.

For a diamagnetic body, the reversal of the induced polarity makes k negative, and, therefore, μ is less than unity, but as k is always very small, μ never becomes negative. For instance, the susceptibility of bismuth (the most powerfully diamagnetic substance known) is only $-\frac{1}{400,000}$.

Without attempting any complete discussion of the subject, it may be pointed out that diamagnetism is a property which all bodies might naturally be expected to possess. For consider Experiment 222, p. 358, in which we found that a copper disc suddenly brought into a magnetic field develops a polarity opposite to that of the field in virtue of an induced current produced in it. This current dies away almost instantaneously, because its energy is converted into heat by ohmic resistance; but suppose that a similar induced current is produced in the atoms of copper themselves, and that each atom may be regarded as a circuit of perfect conductivity; then the induced currents in the atoms would persist as long as the body remained in the field, and consequently it would behave like a diamagnetic substance. When removed from the field, an equal and opposite induced E.M.F. would destroy those currents, and leave the body in its original state. (According to modern ideas, we should not regard the current as something flowing *inside* the atom, but rather as being due to the orbital revolution of electrons *outside* it, much as planets rotate round the sun. But this does not affect the argument given above).

Considerable light has been thrown on the whole subject of magnetism by some comparatively recent researches of the late M. Curie (better known by his work on radium).

Langevin has embodied Curie's results in a general theory of great importance, in which the old classification (paramagnetic and diamagnetic) is departed from, and all bodies are ranked in one of three groups :—

(1) *Diamagnetic.*—These bodies develop a polarity opposite to that produced in iron. The susceptibility, k, is a constant, *independent of the magnetising force* H, and also independent of the temperature.[1]

(2) *Paramagnetic.*—Under this head are placed a number of very feebly magnetic substances, whose polarity is the same as that of iron. The susceptibility, k, is still independent of H, but *is inversely proportional to the absolute temperature.*

(3) *Ferromagnetic.*—This class includes a limited number of bodies (practically iron, cobalt, and nickel), of which iron may be taken as the type. Their polarity is the same as, but enormously stronger than, that of group 2, and k is not a constant. It varies largely with both the magnetising force and the temperature, although the variation does not follow any simple law.

As regards the effect of temperature, it has already been stated that iron becomes non-magnetic at a temperature—known as the critical temperature—a little above redness (about 750° C.). Curie, however, showed that in reality the iron at that temperature began to change into the paramagnetic state as defined in (2), k being very small, independent of H, and varying inversely as the absolute temperature.

According to Langevin's theory, all atoms are regarded as being surrounded by electrons in motion. If the orbits of these electrons are not substantially in the same plane, the magnetic moment *may* be zero, in which case the body is *diamagnetic*. If the atom has a magnetic moment, but the conditions are such that it is not appreciably affected by neighbouring atoms, then it is *paramagnetic*. If, however, the atoms do exert directive influences upon each other, the body is *ferromagnetic*.

[1] Bismuth shows exceptional properties. In the solid state it is the most strongly diamagnetic substance known, whereas after fusion it becomes the weakest.

Magnetic Alloys.—Heusler has recently discovered that certain alloys of manganese, aluminium, and copper are ferromagnetic to an extent almost equal to that of nickel or cobalt or cast iron of poor quality. These alloys differ from iron in not showing the "Kerr" effect (p. 498).

This property is also shown more or less strongly in quite a large number of alloys, in which the aluminium or the copper is replaced by another element, but they *all* contain manganese.

Manganese itself, as usually obtained, is only feebly paramagnetic, but when fused in an electric furnace it becomes ferromagnetic, having enormous coercive force and small retentivity, and not becoming "saturated" even under the influence of an extremely high magnetising force. When alloyed with iron in certain proportions it yields the well-known non-magnetic manganese steels. Its properties are somewhat peculiar, and have not yet been fully investigated.

Iron Alloys.—Barrett discovered that the addition of aluminium or silicon to iron in certain proportions yielded a material whose permeability for fairly small magnetising forces exceeded that of pure iron. For strong magnetising forces, however, it drops below that of pure iron. Such alloys are now very largely used for armature and transformer stampings, for they give increased permeability with small eddy current losses. This is due to the fact that, being alloys, they have naturally a specific resistance higher than that of pure soft iron, and therefore the eddy currents are smaller for a given E.M.F.

Professor B. Hopkinson has recently completed a very extensive examination of alloys under exceedingly strong magnetising forces. Under such conditions, no alloy has been found to be more magnetic than pure iron, but some were found to be more magnetic than the value corresponding to the sum of the effects of their constituents. Such a result, taken in conjunction with the extraordinary properties of Heusler alloys, is very suggestive. If it should be found that the property of magnetism depends to some extent upon the *arrangement* of the atoms, and not entirely upon their nature, then there remains the possibility of producing a body more magnetic than iron from non-magnetic materials. The opinion has, in fact, been recently expressed, that, whereas diamagnetism and paramagnetism are both purely atomic properties, ferromagnetism is not so dependent upon atomic properties as upon crystalline structure. The effect of temperature upon the three states may be regarded as supporting this view. In the first two states, the laws connecting temperature and magnetism are very simple; whereas in the third, the relation is extremely complex. For instance, under *weak* magnetising forces, the permeability of soft iron increases with a rise in temperature—at first slowly, but at about 700° C. it suddenly rises to a very high value, and then at the critical temperature of about 750° C., even more suddenly falls again practically to unity. Under stronger magnetising forces, the permeability merely falls with a rise of temperature, until it becomes unity at 750° C., without any such sudden increase in value just below that temperature.

Note.—Professor Kamerlingh Onnes has just published the results of important researches made at temperatures closely approaching absolute zero (already alluded to on p. 265), and he has found that at these extremely low temperatures, paramagnetic and diamagnetic substances deviate considerably from Curie's law (which states that their susceptibilities vary inversely as the absolute temperature).

EXERCISE XX

1. What is meant by *magnetic hysteresis?* How would you determine it for a specimen of iron given in the form of a rod?

(B. of E , Stage III., 1910.)

2. Describe one good method of determining the permeability of a specimen of iron. (Lond. Univ. B.Sc., 1903.)

3. Define magnetic induction B and magnetising force H, and give an account of an experimental method of determining their relation for a specimen of soft iron. (Lond. Univ. B.Sc. Internal, 1909.)

4. Show in what features a magnetic circuit is analogous to an electric circuit. In what respects does the analogy fail ?
(Lond. Univ. B.Sc. Internal, 1907.)

5. Calculate the number of ampere-turns of excitation required to magnetise, up to 14,000 lines per square centimetre, a soft iron ring, 28 inches in mean diameter, made of round iron 1 inch thick. [Assume permeability $=800$.] (C. and G., 1894.)

6. A ring-shaped electromagnet has an air-gap of 6 millimetres long and 20 square centimetres in area, the mean length of the core being 50 centimetres, and its cross-section 10 square centimetres. Calculate, approximately, the ampere-turns required to produce a field of strength, $H=5000$, in the air-gap. [Assume permeability of iron as 1800.] (C. and G., 1908.)

7. Two bars, each 5 square centimetres in cross-section, have their faced ends touching within a solenoid excited so as to produce a magnetic induction of 15,000 lines per square centimetre through the joint. Give, in pounds, the force requisite to pull them apart. (C. and G., 1894.)

8. Explain what is meant by residual magnetism, coercive force, permeability.

Draw a curve showing the manner in which the magnetism induced in a soft iron rod varies as the magnetising field is taken through a cycle, and state in a general way how from this diagram you would obtain the residual magnetism, coercive force, and permeability of the iron. (B. of E., 1911.)

CHAPTER XXVI*

INTRODUCTION TO THE THEORY OF ALTERNATING CURRENTS

The conception of an alternating E.M.F. or current has already been introduced (p. 353).

Such E.M.F.'s and currents as occur in practice may be regarded as strictly *periodic*, i.e. the maximum and minimum values recur after equal intervals of time, and all the half cycles are exactly alike. The wave form, however, is not necessarily of the simple and symmetrical shape shown in Fig. 270, p. 353. Now, the only shape of curve which can be readily treated mathematically is that known as the *sine curve*,[1] and, as a rule, the results obtained in this chapter apply only to curves of this kind. This is not such an absurd procedure as at first sight it may appear to be, partly because the wave form of many alternating generators approximates closely enough to the ideal curve to make our calculations practically useful, and partly because, in more advanced work, there are other ways of meeting the difficulty. At the same time, the fact that we are making certain initial assumptions must be kept in mind when comparing theory with practice.

What is implied by the statement, that an alternating E.M.F. or current follows a sine law, will be best understood by taking a particular case.

Consider a narrow coil, which may be either circular or rectangular in shape, rotating in a *perfectly uniform field*. (If this field is due to the earth, the above condition is satisfied. We are not, however, concerned with its origin, but unless it is perfectly uniform, the induced E.M.F. will not follow a sine law.)

For the sake of definiteness, we may think of the coil as being wound on the outside of a non-magnetic cylinder (serving merely as a carrier), which rotates between the poles of a magnet as shown in Fig. 298, it being assumed that the field within the space swept through by the rotating coil can be represented by equidistant and parallel straight lines. As already stated elsewhere, each turn of the coil is equivalent to two active conductors—perpendicular to the plane of the paper—joined in series by two inactive conductors, which do not cut the lines of force; and, as all the active conductors

[1] It may be advisable for the student to consult some work which treats of harmonic motion.

behave alike (provided that the coil is narrow enough to enable us to regard them as being sensibly in the same position), we may simplify the argument by considering only one of them, *i.e.* one half of a single turn.

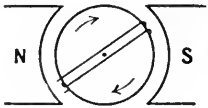


FIG. 298.

This case is shown in Fig. 299, where P and Q represent two consecutive positions of the rotating conductor, seen end-on, in a uniform field parallel to the plane of the paper and perpendicular to AB.

We have now to find the value of the E.M.F. induced in the conductor at any given instant. A simple application of the calculus would give us the solution in a few lines, but experience shows that students eventually acquire a more thorough grasp of the subject if at the outset they deduce their results from first principles.

Let the conductor rotate (say) clockwise, making n revolutions per second in a circle of radius r. Its linear velocity is, therefore, $2\pi nr$ centimetres per second, and its position at any given instant may be conveniently defined in terms of the angle θ, measured from A, one of the positions of zero E.M.F. Let Z be the number of lines of force passing through a cross-section of the cylinder taken through AB. Then the conductor cuts Z lines of force in moving from A to B, or from B to A. (Its length, perpendicular to the plane of the paper, need not be considered; it is really taken into account in defining Z.)

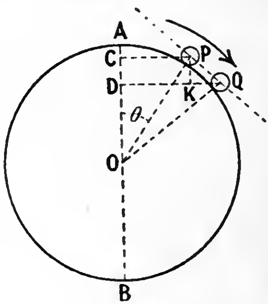


FIG. 299.

Suppose that it moves from P to a new position, Q, very close to P, in t seconds. We shall find (1) the *average* E.M.F. during this time, and then (2) deduce its value when t is so small that P and Q coincide. *This will be the instantaneous E.M.F. at P.*

(1) Let e be the *average* E.M.F. during the motion from P to Q. Then $e = \dfrac{\text{lines cut} \times 1}{t}$, and the lines cut are those lying between C and D, so that we have

$$\text{lines cut} = \frac{\text{CD}}{\text{AB}} \times \text{Z} = \frac{\text{CD}}{2r} \times \text{Z}$$

$$\therefore\ e = \frac{\text{CD} \times \text{Z}}{2r \times t} \text{ (absolute units).}$$

Now, as t is the time required to move through the arc PQ with velocity $2\pi rn$ centimetres per second,

$$\therefore\ t = \frac{\text{space}}{\text{velocity}} = \frac{\text{arc PQ}}{2\pi rn}$$

and as CD = PK,

$$\therefore\ e = \frac{PK \times Z}{2r \times \dfrac{\text{arc PQ}}{2\pi rn}} = \pi nZ\frac{PK}{\text{arc PQ}}$$

As the arc PQ is by assumption exceedingly small, it will not differ appreciably from the straight line PQ, so that we may write

$$e = \pi nZ \sin PQK.$$

(2) We have now to find what this becomes as t approaches zero, *i.e.* when, in the limit, P and Q coincide.

Evidently the straight line PQ becomes the tangent at P, and the angle PQK (Fig. 299) becomes the angle between this tangent and CP.

This is shown in Fig. 300, where HK is the tangent at P, and the angle PQK becomes the angle HPC. But by the geometry of the figure, angle HPC = θ,

$$\therefore\ \underset{\text{(instantaneous)}}{e} = \pi nZ \sin\theta \text{ (absolute units).}$$

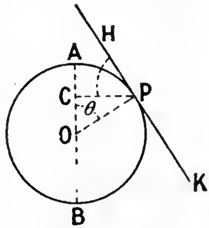


FIG. 300.

If there is another conductor, diametrically opposite, connected up to form one turn of a coil, then

$$\underset{\text{(inst.)}}{e} = 2\pi nZ \sin\theta,$$

and if there be N turns in a coil so narrow that θ may be taken as defining its mean position, then

$$\underset{\text{(inst.)}}{e} = 2\pi nZN \sin\theta.$$[1]

We see that the maximum value of this expression is reached when $\theta = 90°$, for then $\sin\theta = 1$,

$$\therefore\ \underset{\text{(maximum)}}{e} = 2\pi nZN.$$

Writing e_m for this value, we have

$$\underset{\text{(inst.)}}{e} = e_m \sin\theta.$$

Again, in Fig. 300, $\frac{PC}{OP} = \sin\theta$, or $PC = r \sin\theta$, hence, if the length of r be taken to represent the maximum value of the E.M.F.,

[1] It will be noticed that the coil (or conductor) moves through $2\pi n$ radians in 1 second, and therefore through $2\pi nt$ radians in t seconds. Hence, if t is the time required to move from A to P, we have $\theta = 2\pi nt$. This is often written pt, where $p = 2\pi n$.

then PC represents, to the same scale, the instantaneous value. We see that at A and B, PC = 0, and becomes equal to r when $\theta = 90°$ or 270°. In one complete revolution, the induced E.M.F. will pass through one complete cycle, and the frequency is (in this case) equal to the number of revolutions per second.

As in Fig. 270, mark off along a horizontal line any convenient length to represent 2π radians or 360°. On this plot as ordinates to any convenient scale the values of PC during one complete revolution—*above* the horizontal axis during the motion from A to B, and *below* it (to indicate the change in the direction of the induced E.M.F.) during the motion from B to A. The continuous curve drawn to connect the summits of these ordinates is known as a sine curve, and we shall assume that such a curve may be taken to represent any alternating E.M.F. or current. If the maximum value be known, the curve can be plotted by using a table of sines. For instance, when $\theta = 30°$, the instantaneous value is $e_m \sin 30°$, *i.e.* $\frac{1}{2}e_m$; when $\theta = 45°$, it is $\frac{e_m}{\sqrt{2}}$; for $\theta = 60°$, it is $\frac{\sqrt{3}}{2}e_m$; these values being evidently repeated for angles $\pi + 30$, $\pi + 45$, and $\pi + 60$.

Virtual Values.—We have now to define precisely what is meant by the terms "volt" and "ampere" when applied to alternating circuits. If we could deal with instantaneous values, no difficulty would arise; but in practice, we are obliged to read steady deflections on instruments calibrated by using direct currents, such deflections being the resultant of impulses which vary in value throughout each alternation. It would seem natural to suppose that these steady readings represent average values, but such is not the case, for, although the average value of a current (or of an E.M.F.), whose maximum is 100, works out at 63·7, it reads 70·7 on a suitable instrument; and this, moreover, represents its actual value as regards power.

As already stated on p. 353, the average value is the height of OA, Fig. 270, of a rectangle of the same area as the figure included by the curve,

$$\text{or,} \quad \underset{\text{(average)}}{e} = \frac{1}{\pi}\int_0^\pi e_m \sin\theta d\theta = \frac{e_m}{\pi}\int_0^\pi \sin\theta d\theta$$

$$= \frac{2}{\pi} \times e_m$$

It is not difficult to understand why this is not the value given by the instruments, for, taking the case of a *hot-wire* voltmeter or ammeter, the heating effect is given by C^2R or by $\frac{E^2}{R}$, and is, therefore, proportional to the square of the current or of the E.M.F. at any instant, and if the construction is such that the deflection is proportional to the heating effect, equal distances along the scale will not represent equal increments of average current or E.M.F., but equal increments of the average value of their squares. The scale is, however, graduated by means of direct currents, and so its reading indicates the value of the direct current (or E.M.F.), which produces the same heating effect, and this amounts to taking the square root of the average or mean value of the squares. It is these "square root of mean square" values which are implied by the term volt or ampere, when used in connection with alternating currents; they are also known as "virtual" volts or amperes, in order to distinguish them from the true averages, with which, as a rule, we are not concerned.

We may show this mathematically as follows:—

$$\underset{\text{(average value of square)}}{e^2} = \frac{1}{\pi}\int_o^\pi e_m^2 \sin^2 \theta d\theta$$

$$= \frac{e_m^2}{\pi}\int_o^\pi \sin^2 \theta d\theta$$

$$= \frac{e_m^2}{2\pi}\int_o^\pi (1 - \cos 2\theta) d\theta$$

$$= \frac{e_m^2}{2}$$

$$\therefore \quad \underset{\text{(virtual)}}{e} = \sqrt{\text{average value of square}} = \sqrt{\frac{e_m^2}{2}} = \frac{e_m}{\sqrt{2}}$$

For instance, if the pressure of alternating mains supplying a town is 100 volts, it implies that the maximum value of the E.M.F. is $\sqrt{2} \times 100 = 141{\cdot}4$ volts.

For the sake of definiteness, it will be convenient to consider what occurs when an alternating E.M.F. is applied to a coil of wire, without an iron core, such as the electromagnet coil already used in many experiments.

Let us suppose that the resistance of the coil is 10 ohms, and that an impressed E.M.F. of 100 volts sends 4 amperes through it. In this simple case, the *only* way in which power *can* be expended is as heat in the copper, which is $C^2R = 4^2 \times 10 = 160$ watts. Thus the current and the power correspond to an effective E.M.F. of 40 volts.

We shall call the voltage of the mains connected to the coil (*i.e.* the reading of a voltmeter connected across its terminals), the *impressed* E.M.F., and denote it by E, and the volts required by Ohm's law, the *resultant* E.M.F., and denote it by e_r. Hence, apparent power $=$ EC, and true power $= e_r$C.

The ratio $\dfrac{\text{true power}}{\text{apparent power}}$ is known as the "power factor," which evidently, in the case before us, is $\dfrac{e_r}{E} = \dfrac{40}{100} = {\cdot}4$.

Now, we know that as the current in the coil rises in strength, the magnetic field, formed in the space around it, also increases in strength, and we may regard the field as being "in step" or "in phase" with the current, *i.e.* they reach their maximum and mimimum values respectively at the same instants.[1] This field produces a self-induced E.M.F., already denoted by e_s, which is greatest when the rate of cutting is greatest, and *vice versâ*. A little consideration will show that this means it will be zero when the field (and the current) is at a maximum, and will be greatest when the field (and current) is passing through zero to reverse its direction. Thus, if the full line curve in Fig. 301 represents the current, e_s will pass through zero at the points A and B. Now, there are two possible ways of drawing the curve through these points, and in order to decide whether the maximum value of e_s lies above or below AB, we must remember that whilst a current is *decreasing* in strength, the induced E.M.F. is in the *same* direction. Hence, the curve of e_s must be on the same side of the axis of abscissæ as the decreasing current from M to L, and, therefore, it has the position shown by the dotted line. It will be understood that no meaning is to be attached to the *amplitude* of these curves; this diagram merely shows their phase relation to each other, which is expressed by saying that *e_s always lags $\frac{1}{4}$ of a period, or* 90° *in phase behind the current which produces it.*

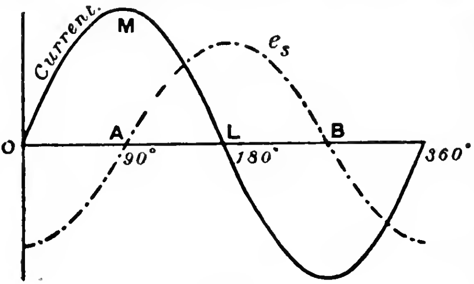


FIG. 301.

It is, however, important to remember that it is not really the rate of change of current which produces e_s, but the rate of change of flux, *i.e.* the rate of change in the number of lines of force passing through the interior of the coil. It will be found later that, whilst e_s always lags 90° behind the flux, it may lag more than 90° behind the current in the circuit. This is due to the fact that a part only of that current may be effective in producing the flux.

Applying these ideas to the case in question, let us consider what occurs when an alternating E.M.F. of E volts is applied to the coil. (It may be remarked that we are not concerned here with what occurs in the first fraction of a second after switching on.)

We know that an alternating current and flux will be established, and that the latter will call into existence a certain E.M.F. of self-

[1] As these terms "in step" and "in phase" are frequently used, the student should pay particular attention to this definition.

induction, e_s. If, for the sake of argument, we assume the current to be in step with E at starting, e_s must be 90° behind C, and therefore 90° behind E. Hence, there are two distinct E.M.F.'s acting on the circuit, and the current will be determined by, and will be in step with, the *resultant* E.M.F. (e_r). Now, the resultant of two curves, such as those drawn in Fig. 302, may be obtained graphically by taking the algebraic sum of the ordinates, which evidently is represented by a third curve lying between the two component curves. But the current depends on this third curve, and is in step with it, and therefore immediately begins to lag behind E, also as e_s must keep 90° behind C, that lags with it until some steady state is reached, for the current cannot lag more than 90° (for reasons that will be seen later).

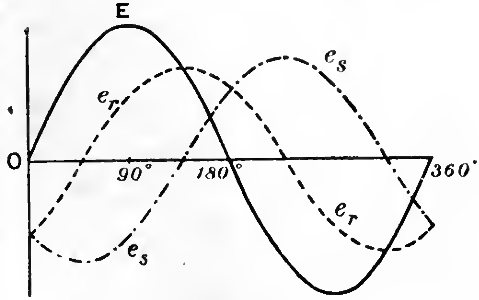


Fig. 302.

Hence, we have finally (1) the self-induced E.M.F., e_s, lagging *more* than 90° behind E; (2) the resultant E.M F., e_r, lagging *less* than 90° behind E, and smaller than E in magnitude; and (3) a current C in step with e_r and determined by Ohm's law as $\frac{e_r}{R}$.

Hence, the effect of self-induction has been to make the current smaller than it would otherwise be, and also to make it lag behind the impressed E.M.F. in phase, so that the true power in watts is no longer given by the product EC.

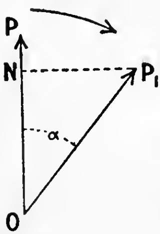


Fig. 303.

Vector Diagrams.—Currents and electromotive forces are not really vectors, but, when they follow a sine law, the properties of a sine curve make it possible to represent them graphically, and such methods are exceedingly useful both in theory and in practice.

Let OP (Fig. 303) be a line, whose length represents the *maximum* value of an impressed E.M.F., and let it rotate clockwise[1] about O, making one revolution in one complete alternation. Then, in any position, its projection $P_1N = OP \sin \alpha$, which

[1] The International Electro-Technical Commission (Turin, 1910) has recommended the general adoption of the *anti-clockwise* direction for vector diagrams. We should have accepted this recommendation but for the fact that the blocks used in this chapter were engraved before the Commission met.

therefore represents the *instantaneous* value of the E.M.F. at the instant it reaches that position.

Similarly, the maximum value of the current can be represented by another rotating line making an angle with OP equal to the angle of phase-difference; and the E.M.F. of self-induction by a third line, which must lag behind the current line by 90°. Then the projections of these three rotating lines will indicate the instantaneous values of the three quantities in their correct phase-relation. If the diagram is drawn to scale, it will be found, by completing the parallelogram, that E and e_s have a resultant e_r along the line of current, of such a magnitude that $e_r = CR$.

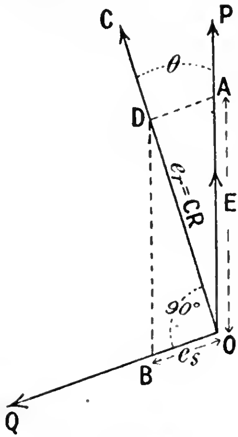


FIG. 304.

We thus obtain Fig. 304, in which a length OA to represent E is marked off along OP. The current acts along OC, lagging behind E by an angle θ. Then e_s must act along OQ, which makes an angle of 90° with OC, and if OB represents its value on the same scale as E, then OD is the resultant E.M.F., e_r.

Hence, the maximum values of E, e_s, and e_r can be represented by the three sides of a right-angled triangle, derived from the above figure, and this triangle is so frequently required, that we draw it in any convenient position without always thinking of the method by which it was obtained.

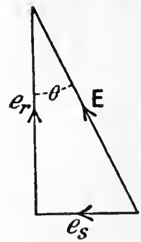


FIG. 305.

From Fig. 305, we perceive that if the impressed E.M.F. is a constant (as is often the case), then the greater e_s becomes, the greater will be the angle of lag, and the smaller will be the value of e_r (and, therefore, of the current).

When $e_s = 0$, then $e_r = E$, and there is no lag, as is the case in non-inductive circuits.

Evidently $E^2 = e_r^2 + e_s^2$, and although this relation has been obtained for maximum values, it also holds good for *virtual* values, because we can draw another triangle with the same angles and with its sides $\frac{E}{\sqrt{2}}$, $\frac{e_r}{\sqrt{2}}$, $\frac{e_s}{\sqrt{2}}$ to represent such virtual values.

Of course this relation does not apply to instantaneous values; for these we have simply $e_r = E \pm e_s$.

In the example given on p. 429, we have $E = 100$ volts, and $e_r = 40$ volts. Now

$$e_s = \sqrt{E^2 - e_r^2} = \sqrt{100^2 - 40^2} = \sqrt{8400} = 91{\cdot}6 \text{ volts.}$$

Also from Fig. 305, $\cos\theta = \frac{e_r}{E} = \frac{40}{100} = \cdot 4$, which, from mathematical tables, gives $\theta = 66° \; 25'$ nearly.

It will be noticed that $e_r = E\cos\theta$, and as $C = \frac{e_r}{R}$, we have

$$C = \frac{E\cos\theta}{R} = \frac{\sqrt{E^2 - e_s{}^2}}{R}$$

Again, it will be seen that:—

$$\text{True power} = e_r \times C = EC\cos\theta.$$

This result is especially important, because (although we have deduced these results from the consideration of a special case, and may consequently have to extend their meaning at times) under all circumstances the power supplied to an alternating circuit is $EC\cos\theta$.

Value of Self-induced Electromotive Force in Terms of Current.—The expression $C = \frac{\sqrt{E^2 - e_s{}^2}}{R}$ is inconvenient in its present form, because e_s is also a function of the current. To find a value for e_s, we make use of the equation

$$\underset{\text{(instantaneous)}}{e} = 2\pi nZN\sin\theta$$

This was obtained for a moving coil, which cut a stationary field according to a sine law, but it holds good for a stationary coil cut by a moving field, provided that the rate of cutting follows a sine law; and it therefore applies in the present instance, n being now the frequency of the alternating E.M.F.

$$\therefore \quad \underset{\text{(max.)}}{e_s} = 2\pi nZN$$

$$\text{and} \quad \underset{\text{(virtual volts)}}{e_s} = \frac{2\pi nZN}{\sqrt{2} \times 10^8}$$

Again, on p. 376 we obtained the equation, in absolute units,

$$Li = ZN,$$

which referred to a *steady* current of strength i being stopped or started, and therefore when applied to an alternating current, i must be the maximum value. For our purpose we must express this result in henries and virtual amperes.

$$\text{Now} \quad \underset{\text{(absolute)}}{L} = \underset{\text{(henries)}}{L} \times 10^9$$

$$\text{and} \quad \underset{\text{(max.)}}{i} = \underset{\text{(virtual)}}{i} \times \sqrt{2} = \frac{C_v}{10} \times \sqrt{2}$$

$$\therefore \quad \underset{\text{(abs.)}}{ZN} = \underset{\text{(max.)}}{L \times i} = \underset{\text{(henries)}}{L} \times 10^9 \times \frac{C_v}{10} \times \sqrt{2}$$

$$\therefore \quad \underset{\text{(virtual volts)}}{e_s} = \frac{2\pi n}{\sqrt{2} \times 10^8} \times L \times 10^9 \times \frac{C_v}{10} \times \sqrt{2}$$

Therefore, we have in practical units :—

$$e_s = 2\pi n\mathbf{LC}.$$

Substituting this value in

$$C = \frac{\sqrt{E^2 - e_s{}^2}}{R}$$

and simplifying, we have

$$C = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}}$$

from which we see that, when $L = 0$, $C = \frac{E}{R}$ as usual.

This expression is often convenient for use, but it is misleading unless the real nature of the facts is kept in mind. It suggests, for instance, that the decrease in current strength in an inductive circuit is due to a kind of increased resistance, and disguises the fact that it is really due to the presence of another E.M.F. Neither does it indicate that the current is out of step with the impressed E.M.F. It does, however, make clear the importance of **frequency**. When n is small, the second term in the denominator will also be comparatively small; when n is very great indeed, this term may become very large; and thus we find that, for currents of extremely high frequency, even straight copper bars behave as if they possessed considerable resistance, and a few turns of thick copper wire will "choke" powerfully.

The denominator of the last equation, $\sqrt{R^2 + (2\pi nL)^2}$, is generally known as the **impedance** of the circuit, where impedance is simply a useful term to express the *apparent* resistance of the circuit, *i.e.* the ratio $\frac{E}{C}$, when E is in volts and C in amperes. This ratio in the case of *steady* currents becomes, of course, the ohmic resistance.

The quantity $2\pi nL$ is often termed the **reactance.**

The relation between the **resistance**, the **reactance**, and the **impedance** of a circuit is easily understood by deriving a new triangle from Fig. 305. If each side be divided by current strength, C, we shall evidently obtain a *similar* triangle with sides

$$\frac{E}{C} = \sqrt{R^2 + (2\pi nL)^2}, \frac{e_r}{C} = R, \frac{e_s}{C} = 2\pi nL$$

(Fig. 306), and we, therefore, have

$$\tan\,\theta = \frac{2\pi nL}{R}$$

where θ is, as before, the angle of lag.

This tells us that, *when* L *is constant*, θ depends upon the *frequency* of the impressed E.M.F., but not at all on its numerical value, nor on the current strength.

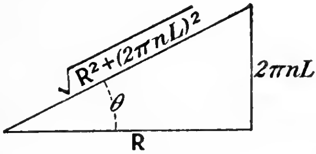


FIG. 306.

But we know that, when iron is present, L is not constant, because it depends on the permeability, μ, which in its turn varies with the state of magnetisation. At present, we are assuming the absence of iron, in which case L has a definite meaning, and can be approximately measured in a very simple way by means of a voltmeter and ammeter.

For instance, in our typical example, we have

$$C = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}}$$

$$\text{or } 4 = \frac{100}{\sqrt{10^2 + (2\pi nL)^2}}$$

$$\therefore \quad \sqrt{10^2 + (2\pi nL)^2} = 25$$

from which $\quad 2\pi nL = 23$ nearly,

$$\text{whence} \quad L = \frac{23}{2\pi \times 50} = \cdot073 \text{ henry.}$$

Example I. — A coil of wire, Fig. 307, having inductance $= \cdot01$ henry and negligible resistance, is placed in series with a non-inductive resistance of 4 ohms. If the impressed E.M.F. is 120 volts, find (1) the current; (2) the P.D.'s across the coil and across the non-inductive resistance; (3) the phase-relation between the current and the E.M.F.; (4) the power expended. (Frequency = 50.)

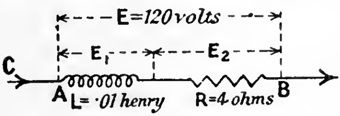


FIG. 307.

(1) Applying the previous equation to the coil and the resistance taken together, we have

$$C = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}} = \frac{120}{\sqrt{4^2 + (2\pi \times 50 \times \cdot01)^2}} = \frac{120}{\sqrt{25\cdot6}} = 24 \text{ amperes nearly}$$

(2) Applying the equation to the coil only, we have

$$C = \frac{E_1}{\sqrt{0 + (2\pi nL)^2}}$$

$$\therefore \quad 24 = \frac{E_1}{2\pi nL} = \frac{E_1}{3\cdot1}$$

$$\text{whence} \quad E_1 = 24 \times 3\cdot1 = 74\cdot4 \text{ volts.}$$

(Notice that, in this case, E_1 is numerically equal to e_s.)

Again, for the non-inductive resistance,

$$C=\frac{E_2}{R}, \text{ or } 24=\frac{E_2}{4}$$

$$\therefore \quad E_2=96 \text{ volts.}$$

Hence we see that E is not the sum of E_1 and E_2, as would be the case with a direct current (and as it actually is for the *instantaneous* values of alternating E.M.F.'s).

(3) The current lags behind the impressed E.M.F. of 120 volts by an angle θ, such that $\tan\theta=\frac{2\pi nL}{R}=\frac{3\cdot1}{}=\cdot77$,

$$i.e. \quad \theta=37\cdot7^\circ.$$

Also, it lags behind E_1 by 90°, for the tangent of the lag angle is $\frac{2\pi nL}{R}$, and $R=0$ for the coil.[1]

Evidently this is a limiting value, for an actual coil must have some resistance (although it can be made as small as we please), and hence θ will never quite reach 90°.

Finally, the current must be in step with E_2.

Hence we can represent the state of affairs by drawing a line OE (Fig. 308), on which we mark off a length to represent 120 volts. The current and e_s will be marked off along a line OC, making an angle of 37·7° with OE.

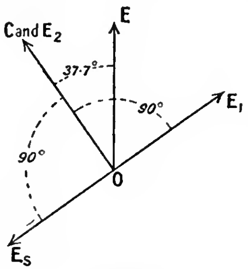


FIG. 308.

E_1 must be set off 90° in advance of the current, and e_s is exactly equal and opposite to it.

It will be seen that the three E.M.F.'s bear the same numerical relation to each other as they would have done had we considered the coil to have both inductance = ·01 henry and resistance = 4 ohms.

(4) By the conditions of the question, the power expended can only be the C^2r heating effect in the non-inductive resistance, which is $24\times24\times4$ $=2304$ watts. It will be found that this is identical with EC cos θ, which in *all* cases gives the power supplied to the circuit.

Example 2.—What would be the result if the resistance of 4 ohms was equally divided between the coil and the non-inductive resistance?

In the first place, such a change would not affect the equation for current, which would, therefore, be 24 amperes; nor would it

[1] Or we may say—because $R=0$, no energy is converted into heat, and there is no other source of loss; hence, no power is expended in the coil. Therefore, $E_1C\cos\theta=0$, or $\theta=90°$.

affect the angle between the current and the impressed E.M.F. of 120 volts.

E_1 would be altered in value, for we should have

$$C = \frac{E_1}{\sqrt{R^2 + (2\pi n L)^2}}, \text{ or } 24 = \frac{E_1}{\sqrt{2^2 + (3 \cdot 1)^2}} = \frac{E_1}{\sqrt{13 \cdot 6}}$$

$$i.e. \quad E_1 = 24 \times 3 \cdot 7 \text{ volts} = 88 \cdot 8 \text{ volts}$$

and the current would lag behind this by θ_1, where

$$\tan \theta_1 = \frac{2\pi n L}{2} = \frac{3 \cdot 1}{2} = 1 \cdot 55$$

$$i.e. \quad \theta_1 = 57° \text{ nearly.}$$

Again, E_2 would be $2 \times 24 = 48$ volts, in step with the current, whilst the power is evidently unaltered.

The figure can now be drawn as before, except that E_1 will be greater than e_s, and the angle between them will be less than 180°. Or the following method may be adopted, if we notice that there is now a resultant E.M.F. (e_r) of 48 volts in the coil itself, which

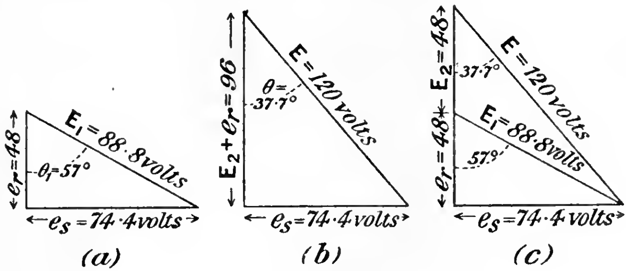


FIG. 309.

must be in step with the current, and, therefore, also in step with E_2. Hence, the resultant voltage is 48 volts for the coil alone, and 96 volts for the coil and resistance taken together.

Considering the coil alone, we obtain (*a*), Fig. 309. Considering coil and resistance together, we obtain (*b*). Superposing the two diagrams, we obtain (*c*).

Effect of Iron.—The results we have already obtained hold good only when iron is absent. When it is present, care must be taken in using them, and, without attempting an exhaustive discussion of the subject, it is desirable to point out certain effects caused by its presence.

In the first place, μ is a variable depending on the current strength, and hence L is some complicated function of the current. The result is that, even if the impressed E.M.F. follows a sine law, the current (and the self-induced E.M.F.) will not do so. Secondly, some energy is expended in the iron on account of hysteresis and eddy currents. Now, this energy is expended *outside the circuit*, although it must have been derived *from* the circuit, and we have repeatedly found that this only happens when a current is flowing *against* an E.M.F. As there are only two E.M.F.'s acting in the circuit—the impressed E.M.F., E, and the self-induced E.M.F., e_s—we infer that e_s must have a component in opposition to the current.

To a first approximation, the actual state of affairs is very simple, and is shown in Fig. 310. The current, C, lags behind E by some amount θ, and the total power taken from the circuit is, as always, EC cos θ.

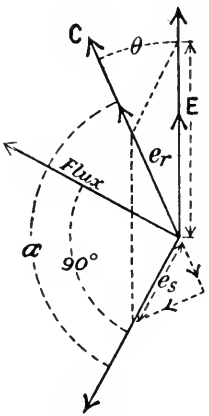


FIG. 310.

But e_s now lags *more* than 90° behind the current, and, as shown by the dotted lines, this means that it can be resolved into a component in direct opposition to the current and another at right angles to it. In fact, if α is the angle between e_s and the current, e_sC cos α represents the power expended in the iron, and if we obtain the resultant E.M.F., e_r, by completing the parallelogram, then $e_r \times$ C is the power expended as C^2r heat, which is equal to the *difference* of the quantities EC cos θ and e_sC cos α. (Of course, cos α is negative in sign, but this need only be taken to indicate the distinction between power taken in and power expended as work.)

Now e_s *must* lag 90° behind the alternating flux which produces it, and hence it follows that this flux is no longer in step with the current. This apparently paradoxical result may be at first somewhat difficult to understand. It will become clearer when the student has met with other instances of the kind, and he will then find that the actual current may conveniently be regarded as the vector sum of two components, viz. (1) the magnetising current, C_m, as required by the reluctance of the magnetic circuit, and calculable, in simple cases, by the methods already given on p. 407. The magnetising current is alone effective in producing the flux, with which it is strictly in step; although as it makes an angle of 90° with e_s, it does not represent external power; (2) an *iron loss* current, C_h, in step with the applied E.M.F., such that EC_h represents the power expended in the iron in hysteresis and eddy currents.

These matters are so important in their practical applications, that it is desirable to consider a few special cases.

CASE I.—Let an E.M.F., E, be impressed upon a circuit (or part of a circuit), which has self-induction, but has neither resistance nor iron losses. (This is approximately the case of a coil of wire of exceedingly low resistance, without an iron core).

The assumptions made imply that no power whatever is expended, even when the current or the flux is exceedingly great. Hence, the current must make an angle of 90° with both E and e_s, and, as there is no resistance, there is also no resultant E.M.F.

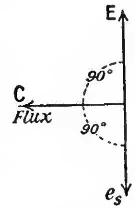


FIG. 311.

We, therefore, draw e_s (Fig. 311) equal and opposite to E, and put C at right angles to both, but lagging behind E and in advance of e_s. The magnitude of C, which is all magnetising current, is determined by the relation $e_s = 2\pi n LC$.

CASE II.—Let the circuit have iron losses and self-induction, but no resistance.

There is still no resultant voltage, but power is taken from the source, and expended outside the circuit. Hence we obtain Fig. 312, where E and e_s are still equal and opposite, but C lags less than 90° behind E. Then EC cos θ is the power taken from the source, and e_sC cos α is the power expended in the iron, and these are obviously equal. But the flux is out of step with the actual current, which is resolvable into magnetising and power components as already shown.

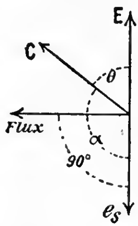


FIG. 312.

This is approximately the case of an ordinary choking coil, which has an iron core, but which has as small a resistance as possible. Such a coil is often said to afford a means of regulating current strength without wasting power, but we see that this statement ignores the iron losses. It is, however, much less wasteful than a mere resistance would be.

CASE III.—Let the circuit have self-induction and resistance, but no iron losses.

This was the case first considered in obtaining various formulæ, but the student will see, from the preceding argument, that it necessarily implies the existence of a resultant voltage and an angle of 90° between e_s and C.

We have also dealt in Fig. 310 with the case of the circuit having iron losses in addition. We see that it involves modifying the figure in Case II. to give a resultant E.M.F.

Transformers.—We have already shown that the work a current is capable of doing is measured by the product of its E.M.F. and its current-strength (*i.e.* using practical units, the product of its volts and amperes). We may have one current of 50 amperes at 1000 volts, and another current of 1000 amperes at 50 volts, and in both cases the energy expended to produce them and the work they are capable of doing will be the same, although one form may be more convenient for a particular purpose than the other. For instance, if we wish to transmit electrical energy from one place to another at some distance from it—say, to light a town five miles off—although it is more economical as regards transmission to keep the current small and the E.M.F. as high as possible (because a thinner and less expensive wire may be used to convey it, and because the loss of energy in a wire increases with the square of the current), it cannot, in that form, be used with advantage, as for distribution to consumers it is more convenient to employ as many amperes as possible at an E.M.F. rarely exceeding 100 or 200 volts.

Now, just as we can exchange a sovereign for smaller coins, or small coins for a sovereign, the money in each form having the same value, so we can exchange one current for another of different amperage and voltage, both having the same energy, with just a slight unavoidable loss in the transformation. An instrument to do this is known as a *transformer*, which is really a modified induction coil, with a closed magnetic circuit, *i.e.* the path of the lines is wholly within iron; the primary and secondary coils being arranged to ensure that, as far as possible, all the lines produced by the primary shall cut the secondary—an ideal state never actually reached in practice.

The first transformer was invented by Faraday in 1831, and consisted of two separate coils wound on a soft iron ring (Fig. 313).

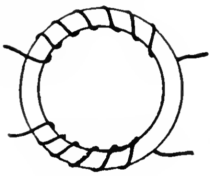


Fig. 313.

He wound 72 feet of bare copper wire (of about 18 B.W.G.) in three helices, one over the other, and insulated by layers of calico. The ends of each helix were brought out so that the coils could be used either together or separately. On another part of the ring, he wound 60 feet of wire in two helices, which were then connected in series with a galvanometer. When the three helices were joined in series, and connected with a battery so that they formed the primary, there was a sudden deflection of the galvanometer needle in the secondary. The needle soon came to rest, but when the circuit was broken, it was suddenly deflected in the opposite direction.

If a steady current be sent through one coil, the inductive effect on the second coil will be at making and breaking only; but if an alternating current be used, a current is induced in the other at each alternation, without using a contact-breaker.

An ordinary induction coil is a special type of transformer designed to give a very high secondary voltage. Hence the secondary must have many turns. Commercial transformers may be required either to raise or lower the primary voltage, *i.e.* either to transform "upwards" or "downwards," and in the latter case, as will be shown, it is the primary which has the greater number of turns.

For details of construction and descriptions of the various types of commercial transformers, we must refer the student to works on electrical engineering; in these pages it will be sufficient to regard them as affording instructive object-lessons to which the principles we have already outlined may be applied.

For the sake of definiteness, it will be convenient to take as an example an actual (but very small) transformer, made to work on 100 volt alternating supply mains at frequency 50, and to give out about 50 volts and a proportionately greater current. (This transformer was made by some students, and all necessary dimensions were taken during the process.) It consists of two long, almost rectangular coils, separately wound and insulated, placed side by side,[1] and then interlaced with iron stampings. Fig. 314 shows its general appearance; Fig. 315 its section in which the path of the lines of force is indicated.

One coil consists of 107 turns of 16-gauge copper wire, and has a

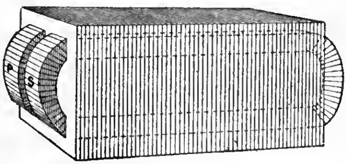


FIG. 314.

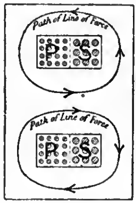


FIG. 315.

resistance of ·66 ohm. The other has 53 turns of 13-gauge wire, with a resistance of ·16 ohm.

The mean length of the magnetic circuit, *i.e.* the length marked with arrows in Fig. 315, is 25·4 centimetres; and its sectional area, *i.e.* the nett section of iron core at right angles to the path of the lines, is 122 square centimetres.

In a particular experiment, (1) the coil of 107 turns was used as the primary and connected to mains at a pressure of 104 volts, *the secondary being on open circuit.* It was then found that a current of ·27 ampere passed through the primary, and that a voltmeter connected across the secondary read 51 volts.

This illustrates the powerful "choking" effect under such circumstances; for we know, by Ohm's law, that a steady E.M.F. of 104 volts acting on a resistance of ·66 ohm would produce a current of $\frac{104}{\cdot 66} = 156$ amperes, which would have at once burnt out the

[1] It is more usual to place one coil *inside* the other, as such an arrangement tends to diminish "magnetic leakage."

insulation. We infer that the current must, therefore, lag considerably, and that e_s must be nearly equal to E.

(2) When the secondary circuit was closed through an adjustable resistance, and a current taken from it, the primary current automatically increased—always keeping at roughly half the strength of the secondary current—the P.D. at the secondary terminals meanwhile slightly decreasing. The practical limit of the current which may be taken out is determined by the rise in temperature of the conductors, *i.e.* upon their thickness. In this case it was about 5 or 6 amperes for the primary and twice as much for the secondary.

The power obtained from the secondary will always be less than the power supplied to the primary, the difference representing the waste in C^2r heat in both coils, and the loss due to eddy currents and hysteresis in the core; but in well-designed transformers of fair size, these losses can be made relatively small and an efficiency of considerably over 90 per cent. easily obtained.

Theory of Transformer.—Let us now consider what occurs when an E.M.F. of E volts is applied to a primary of N_1 turns, the secondary of N_2 turns being on open circuit.

Evidently an alternating flux is produced, which, as it appears and disappears, cuts the turns of both coils at the same instant. (We shall assume, as before, that none of the lines miss the secondary.) In the primary it produces a self-induced E.M.F., which we have hitherto denoted by e_s, but, for convenience in this case, we shall write e_1, where

$$\underset{(\text{max.})}{e_1} = 2\pi n Z N_1 \text{ absolute units.}$$

$$\text{or} \quad e_1 = \frac{2\pi n Z N_1}{\sqrt{2} \times 10^8} = \frac{4{\cdot}45 n Z N_1}{10^8} \text{ virtual volts.} \qquad (1)$$

From Fig. 312, p. 439, we know that this E.M.F. must be nearly equal in magnitude to E and in almost exactly opposite phase.

In the secondary, the same flux produces an induced E.M.F., e_2, such that

$$e_2 = \frac{4{\cdot}45 n Z N_2}{10^8} \text{ virtual volts.} \qquad (2)$$

This must be in exactly the same direction and phase as e_1, for the secondary conductors are cut by the same flux in the same direction at the same instant.

From equations (1) and (2) we notice that $\frac{e_1}{e_2} = \frac{N_1}{N_2}$, and as e_1 is very nearly equal to E, we have approximately $\frac{E}{e_2} = \frac{N_1}{N_2}$, a very important result. (The student can verify this statement by noticing that, in the example given on p. 441, $\frac{104}{51} = \frac{107}{53}$ nearly.)

Value of B in the Iron Core.—In the secondary, we have seen that $e_2 = 51$ volts, and that the frequency $= 50$.

$$\text{Now}\quad e_2 = \frac{4{\cdot}45 \times n \times ZN_2}{10^8}$$

$$\text{or}\quad 51 = \frac{4{\cdot}45 \times 50 \times Z \times 53}{10^8}$$

whence $Z = 43 \times 10^4$ lines of force.

Again, $Z = B \times$ area of section of iron; and area $= 122$ sq. cm.

$$\text{whence}\quad B = \frac{43 \times 10^4}{122} = 3500 \text{ lines per sq. cm. nearly.}$$

It will be noticed, from the manner in which equation (2) was originally obtained, that this is the *maximum* value of B.

Explanation of the term "Load."—When the secondary of a transformer is on open circuit, there is "no load" upon it. When it is closed, it is said to be "loaded." This does not mean that it is short-circuited, but closed through, say, a load of lamps. If the external portion of the secondary circuit is such that no magnetism is produced therein, the load is said to be "non-inductive." As a matter of fact, however, a real load is never absolutely non-inductive, although when it consists of, say, a number of incandescent lamps, it may be treated as non-inductive in ordinary calculations. If the secondary circuit contains coils of wire with iron cores, *e.g.* if alternating motors are being driven by a current from a transformer, then the load is said to be "inductive."

In all cases, the number of watts expended in the secondary circuit is the measure of the load. Hence, when this is inductive, the load may be small, although the current output is large.

Calculation of "No Load" Current, *i.e.* the current in the primary when the secondary is in open circuit.—This consists of two components—(1) the magnetising current, C_m, and (2) the iron loss current, C_h. The former may be obtained from the expression given on p. 407 for direct currents—

$$CN = \frac{{\cdot}8Bl}{\mu}$$

Hence, if we apply it to an alternating magnetic circuit, using the *maximum* value of B, we shall obtain the *maximum* value of C, but we require the virtual value, or

$$\underset{\text{(max.)}}{C_m} = \frac{{\cdot}8Bl}{\mu N_1}$$

$$\underset{\text{(virtual)}}{C_m} = \frac{8Bl}{\sqrt{2}.\mu N_1}$$

In this expression, $B = 3500$; $l = 25{\cdot}4$ centimetres; $N_1 = 107$;

and by reference to the permeability curve for this kind of iron, it is found that when $B = 3500$, $\mu = 3200$.

$$\therefore \quad C_m = \frac{\cdot 8 \times 3500 \times 25\cdot 4}{1\cdot 414 \times 3200 \times 107} = \cdot 147 \text{ ampere.}$$

This current is small because the magnetic reluctance is low. If part of the path of the lines were in air, as in a spark coil, the magnetising current would be enormously greater.

To determine C_h, the actual power supplied to the primary (with secondary on open circuit) was measured by a wattmeter (see p. 565), and was found to amount to 18 watts. This loss must be due entirely to hysteresis and eddy currents, for the copper loss ($C^2 r$) is $(\cdot 27)^2 \times \cdot 66 = \cdot 048$ watt, and is negligible.

This component of the current is in step with the impressed E.M.F., and therefore $EC_h = 18$,

$$\textit{i.e.} \quad C_h = \frac{18}{104} = \cdot 17 \text{ ampere.}$$

Now C_m is almost exactly 90° behind the impressed E.M.F., and

$$\therefore \text{ no load current} = \sqrt{C_m{}^2 + C_h{}^2} = \sqrt{\cdot 147^2 + \cdot 17^2} = \cdot 23 \text{ ampere,}$$

which, for a rough calculation, agrees, within experimental error, with the value actually read on an ammeter.

The angle of lag is found by putting $EC \cos \theta = 18$, and using the *observed* value of the current,

$$\text{or } \cos \theta = \frac{18}{104 \times \cdot 27} = \cdot 64,$$

$$\textit{i.e.}\ \theta = 50°.$$

Were it not for the iron losses, this would have been practically 90°.

Effect of a Secondary Load.—Suppose the secondary circuit to be closed on a *non-inductive* resistance, such that the secondary current is 12 amperes. The iron core is now subjected to the magnetising influence of $12 \times 53 = 636$ secondary ampere-turns, and (as we know that the primary current must be at least 6 amperes) about the same number of primary ampere-turns, whereas previously the apparent ampere-turns were $\cdot 27 \times 107 = 29$. We know that the secondary current must be nearly in exact opposition to the primary current, and hence the question arises as to what is the resultant effect of these nearly equal and opposite sets of ampere-turns. The answer is indicated by noticing that, as the secondary current increases, the secondary volts (the secondary load being non-inductive) remain nearly constant, falling very little more than is naturally due to internal resistance. As the secondary resistance is ·16 ohm, and as 1·92 volts are required to send 12 amperes through it, even if e_r remained constant, we should expect the P.D. at the secondary terminals to drop by that amount (*i.e.* 1·92 volts). (Really there is another cause of drop due to the fact that some of the flux *misses* the secondary winding, an effect known as "magnetic leakage.")

Hence, as the P.D. at the secondary terminals does not fall more than we should expect from the above causes, we infer that *the flux remains constant at all loads*—a most important result, from which it follows that the magnetising current and the iron losses are constant at all loads.

We can now predict, from principles of energy, what the value of the new primary current must be. Suppose that the P.D. at the secondary terminals is 48 volts, when the current is 12 amperes. This current is practically in step with the watts, when the secondary load is non-inductive. Then we have

Watts given out by secondary	$= 48 \times 12$	$= 576$
,, lost as heat in ,,	$= 12^2 \times {\cdot}16$	$= 23$
,, ,, ,, primary	$= 6^2 \times {\cdot}66$ (approx.)	$= 24$
		623

In the last line, we have *estimated* the primary current at 6 amperes—it cannot be far from that value, and any slight error is negligible. To find its actual value, we notice that 623 watts must be supplied in addition to those required at no load, and this means a current in step with E, such that $104 \times C = 623$, or $C = 6$ amperes. To this must be added the iron loss current, shown above to be unaltered, of ·17 ampere, or 6·17 amperes in all.

Finally, we have to compound this with the magnetising current of ·147 ampere, the actual primary current being, therefore, $\sqrt{(6{\cdot}17)^2 + ({\cdot}147)^2}$. We see that the second term is negligible, and that, therefore, the current must, for all practical purposes, be in step with the impressed E.M.F.

Efficiency of the Transformer at this particular load will be given by

$$\text{Efficiency} = \frac{\text{Watts given out}}{\text{Watts supplied}} = \frac{\text{Output}}{\text{Output} + \text{losses}}$$

$$= \frac{48 \times 12}{(48 \times 12) + 24 + 23 + 18} = {\cdot}9, \text{ or } 90 \text{ per cent.}$$

It will be noticed that the efficiency is less at smaller loads. This is important, because, in practice, transformers usually have to be running continuously for most of the time on light loads. Now, the copper loss increases as the load increases, whereas the iron losses do not, and hence it is all important to keep these losses small, by exercising care in the selection of the iron and in the details of the design.

Automatic Adjustment of Primary Current to Load.—Our argument, up to the present, has depended largely upon the axiom that power given out must be balanced by power supplied, but it has not explained the manner in which the primary current adjusts itself to the load.

Space does not permit us to explain this fully. It may, however, be remarked that our previous results show that when an alternating E.M.F. is applied to an inductive circuit of *very small resistance*, the current rises in strength until the flux produced is great enough to establish an E.M.F. of self-induction, very nearly equal and opposite to the impressed E.M.F. This is the case in the primary when the secondary is on open circuit. When the latter circuit is closed (the load being non-inductive, and magnetic leakage being practically negligible), the first effect of the secondary ampere-turns is to weaken the flux, and therefore e_s in the primary. Hence, the primary current immediately rises until e_s is again practically equal to E, until the flux reaches its old value. In fact, the two opposed sets of primary and secondary ampere-turns are not quite in exactly opposite phases, and they adjust themselves until there is a small resultant, which represents their effective magnetising value, and which is practically the same at all loads.

Effect of Condensers.—Case I.—Let a condenser of capacity K be connected, by wires of negligible resistance, to alternating mains of E.M.F., E, and frequency n. It will behave, to some extent, like a tank fed by a pipe connected to a large reservoir, in which the level of the water is alternately raised and lowered. If the level in the reservoir were constant, it would correspond to the case of direct currents, the tank (or condenser) merely filling (or charging) up until the water-level (or P.D. across it) became equal to that of the reservoir.

As it is, the water surges in and out of the tank, which acts, therefore, as if it had an *impedance* instead of an infinite resistance. We see that the P.D. between the condenser terminals must rise during the charging like a back E.M.F., until it becomes equal and opposite to the impressed E.M.F. at the instant the latter reaches its maximum value (the current being then zero). As the impressed E.M.F. falls in strength, this condenser E.M.F. becomes operative in producing a current *against* it, thus discharging the condenser. This current reaches its maximum value at the instant the impressed E.M.F. is zero, and then, when the latter reverses its direction, again begins to charge up the condenser, but with the opposite polarity.

Evidently the current is out of step with the impressed volts, and, if we assume a negligible resistance in the connections *and no loss in the condenser*, there can be no expenditure of energy, and we can predict that the phase-difference must be 90°. We can also regard the condenser as possessing an E.M.F. of its own, measured by the P.D. between its terminals, just as a coil possesses a self-induced E.M.F. of its own. We shall denote this condenser E.M.F. by e_k.

In Fig. 316, let the curve E represent the impressed E.M.F. In this case, the condenser E.M.F., e_k, is equal and opposite to it.

During the first quarter period, while E is rising, the condenser is charging up, and therefore e_k rises in opposition to E, meanwhile the current impelled by E must be diminishing in strength, and reaches zero at the instant the condenser is fully charged, *i.e.* when E is a maximum. Hence, the current curve must pass through the points A and B. As there are two ways of drawing it through these points, we must select the right one by noticing that C is in the same direction as E, whilst E is increasing, and *against* E whilst E is decreasing (for then the condenser is *discharging*), the current being then impelled by e_k in its direction. We now see that the current lags 90° behind e_k, and *leads* 90° in advance of E. This latter fact is most important, for just as introducing self-induction into a current makes the current *lag* with respect to the impressed E.M.F., so the introduction of capacity makes the current *lead* with respect to it. (Really *some* loss occurs in the dielectric, for it becomes warm. This is an effect analogous to hysteresis, and means that the lead is actually less than 90°).

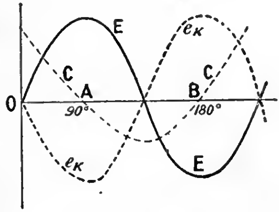


FIG. 316.

We have now to find an expression for the current strength. As in previous cases, this is most readily obtained by means of the calculus, but it is instructive to deduce it from first principles.

Let T be the time of one complete alternation, so that $T = \frac{1}{n}$, where n is the frequency. The condenser charges up in $\frac{1}{4}$ of a period, *i.e.* in time $\frac{T}{4}$, and the quantity in it is Q_m when e_k is a maximum (which we shall denote by e_m),

$$\therefore \quad Q_m = e_m K \qquad \text{(i.)}$$

Now Q_m may be written $C_a \times \frac{T}{4}$, where C_a is the *average* current, and it has already been stated (p. 428), that for a sine curve :—

$$\text{Average} = \frac{2}{\pi} \times \text{maximum.}$$

$$\text{From (i.)} \quad C_a \times \frac{T}{4} = e_m K$$

$$\therefore \quad C_m \times \frac{2}{\pi} \times \frac{1}{4n} = e_m K$$

$$\text{or} \quad C_m = 2\pi n K e_m$$

This obviously holds good for virtual values, for to obtain them it is only necessary to divide each side by $\sqrt{2}$,
and, therefore, we have generally

$$C = 2\pi n K e_k$$

or, as $E = e_k$, we may write

$$C = 2\pi n K E$$

which shows that a condenser may be regarded as having an *impedance* of $\frac{1}{2\pi n K}$.

CASE II.—Let the condenser be in series with a non-inductive resistance, R, Fig. 317. We know e_k is 90° in advance of the current, and that the current is in advance of E by some angle θ. Also that there are two E.M.F.'s in the circuit, viz. E and e_k, which must have a resultant e_r in step with the current, such that $e_r = CR$. Hence, we obtain Fig. 318, from which we learn that

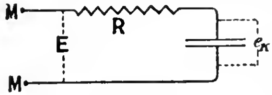


FIG. 317.

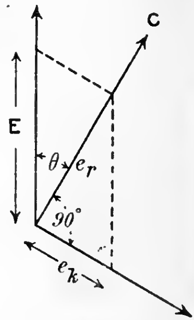


FIG. 318.

$$C = \frac{e_r}{R} = \frac{\sqrt{E^2 - e_k^2}}{R}$$

but $C = 2\pi n K e_k$

whence, eliminating e_k, we obtain

$$C = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi n K}\right)^2}}$$

The figure shows that this current is in advance of the impressed E.M.F. by an angle θ, such that $\tan\theta = \frac{1}{2\pi n K R}$.

CASE III.—Let the resistance, R, be *inductive*, having a self-inductance, L. In this case, the effect of L is to introduce a *lag* (θ_1), and that of K to produce a *lead* (θ_2) of current with respect to E. The two triangles of impedance are shown in Fig. 319. Combining the two, we obtain Fig. 320, and whether that is a lag or a lead depends on which is the greater, $2\pi n L$ or $\frac{1}{2\pi n K}$.

$$\text{Hence } C = \frac{E}{\sqrt{R^2 + \left(2\pi n L - \frac{1}{2\pi n K}\right)^2}}$$

From which we see that when $2\pi n L = \frac{1}{2\pi n K}$, the impedance reduces to R, and there is no lag. If R is small, this may mean an enormous current, although the resistance of the circuit is really infinite. Here we have an example of the general rule that inductance and capacity produce opposite effects, so that one of them can be used to neutralise the influence of the other.

The series arrangement just given is by no means the only method available, but it is impossible for us to discuss the question in detail.

It is, however, very desirable to obtain some idea of the real meaning of these results, and it may be pointed out that self-induction implies that energy is being stored up in the form of a *magnetic* field whilst the current is rising in strength, and is being paid back into the circuit whilst it is falling in strength. On the other hand, the presence of capacity implies that energy is being stored up in the form of an *electrostatic* field in the dielectric whilst the current is falling, and is being paid back whilst it is rising.

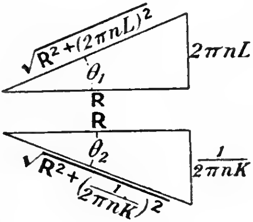


FIG. 319.

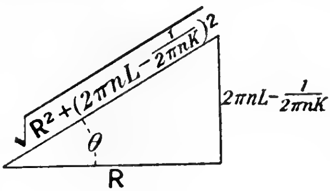


FIG. 320.

Hence, when both are present, energy is being stored up in one form whilst it is being paid back in the other, and this see-saw action may go on without making any further demand on the circuit (except to make good small losses due to heating, &c.) after the first few moments during which this state of affairs is being established. All actual circuits possess some self-induction and some capacity, and hence the ordinary form of Ohm's law is a limiting case, always applicable to direct current circuits when the current has reached a steady value, but only to alternating circuits when the above quantities are small enough to be negligible.

Example.—What E.M.F. will be required to drive 10 virtual amperes through a condenser, of which the resistance is 1200 megohms, and the capacity 22 microfarads, the frequency of supply being 80?

(City and Guilds, London.)

(Hitherto a condenser has been treated as having an *infinite* resistance, whereas it is obvious that it really has an extremely high one, which, although practically negligible in this question, must be taken into account. But it must not be supposed that the resistance of the condenser itself can be treated as if it were a

2 F

non-inductive resistance *in series* with the condenser, and therefore the equation we have given in Case II. above does not apply.)

In this example, the actual current of 10 amperes is made up of two components—

(1) A current (C_r) *in step* with the impressed E.M.F., E, and of magnitude $\frac{E}{R}$, where $R = 1200 \times 10^6$ ohms.

(2) A current (C_k), 90° in advance of E, and of magnitude $2\pi n KE$.

As these currents differ in phase by 90°, we must have

$$\sqrt{C_r^2 + C_k^2} = 10 \text{ amperes}$$

$$\text{or} \quad C_r^2 + C_k^2 = 100 \text{ amperes}$$

$$\therefore \quad \frac{E^2}{R^2} + (2\pi n KE)^2 = 100$$

$$\text{or } E = \frac{10}{\sqrt{\frac{1}{R^2} + (2\pi n K)^2}} = \frac{10}{\sqrt{\left(\frac{1}{1200 \times 10^6}\right)^2 + \left(2\pi \times 80 \times \frac{22}{10^6}\right)^2}}.$$

The first term in the denominator is obviously negligible, and we have, therefore,

$$E = \frac{10}{2 \times \pi \times 80 \times \frac{22}{10^6}} = \frac{10^7}{11{,}053} = 905 \text{ volts nearly.}$$

Power in an Alternating Current.—The measurement of alternating power by means of a wattmeter is discussed on p. 565, here it is merely necessary to point out how it varies during a complete cycle, when the current lags (or leads) with reference to the impressed E.M.F.

In Fig. 321, let E and C represent the curves of E.M.F. and current respectively for one complete cycle. Draw vertical dotted lines through the points where either of them is zero. We notice that at the instants corresponding to *a*, *b*, *c*, *d*, *e*, the *instantaneous* power is zero (because it is EC, and at these instants one of the factors is zero). Hence the power rises and falls periodically, its frequency being *twice* that of the current. Also, during the period of time *ab*, the current is flowing in the opposite direction to the impressed E.M.F. (of course, being impelled by a temporarily stronger E.M.F., e_s, not shown in the figure), which means that energy, supplied by the energy of the disappearing magnetic field, is being returned to the circuit.

During the interval *bc*, the current and the E.M.F. are in the *same* direction, which means that energy is being taken from the source, and, in this case, partly expended in storing up energy in the form of a magnetic field, *i.e.* the circuit *loses* energy.

The period *cd* is similar to *ab*, and *de* to *bc*, as far as energy is concerned. Hence, the effective power, EC cos θ, depends on the nett energy expended in the circuit during the cycle. If we draw a second curve to indicate the rise and fall of power, it will take the form shown at the bottom of Fig. 321. If there is no lag, the power expended in the circuit is EC; if the lag is 90°, evidently the periods marked G and L are equal, and on the whole no power is expended. If, on the other hand, the lag is greater than

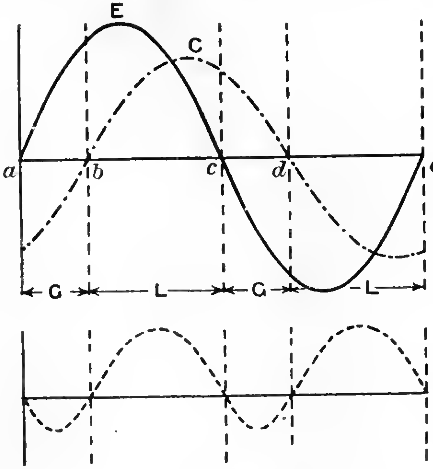


FIG. 321.

90°, evidently more power is gained by the circuit than is supplied from it, which is absurd in this case, although such a figure would correctly represent the state of affairs when E is the E.M.F. of an alternator working as a motor; the interpretation being that it is not now acting as a generator, and on the whole giving out energy to the circuit; but that it is acting as a motor, and on the whole *receiving* energy from the circuit, which it expends as mechanical work.

EXERCISE XXI

1. Find the impressed voltage required to send a virtual current of 50 amperes through a resistance of 2 ohms, if the self-inductance is ·005 henry and the periodicity is 63.

2. A 100 candle-power glow-lamp requires an alternating current of 6 amperes at 60 volts. The supply is at a pressure of 100 volts, at a frequency of 75 complete cycles per second. To give the lamp the right voltage, a

choking-coil is to be inserted. Calculate how much self-induction, in henries, this choking coil must have. (C. and G., 1893.)

3. Define impedance and reactance as applied to alternate current circuits, and draw a clock diagram showing the phase-relation between E.M.F. and current in a circuit whose impedance is 5 ohms and reactance 4 ohms. What is the resistance of the circuit? (C. and G., 1905.)

4. Calculate the number of alternating volts needed to drive an alternating current of 10 amperes through a choking-coil having a coefficient of self-induction of $\frac{1}{100}$ henry, the frequency being taken at 80 periods per second. (C. and G., 1896.)

5. A certain choking-coil of negligible resistance takes a current of 8 amperes if supplied at 100 volts, at 50 periods per second. A certain non-inductive resistance, under the same conditions, carries 10 amperes. If the two are transferred to a supply system working at 150 volts, at 40 periods per second, what total current will they take, (*a*) if joined in series, (*b*) if joined in parallel?

6. The core of a transformer contains 100 square centimetres nett sectional area of iron. Assume that it is to be used on a circuit in which the frequency is 75 complete periods a second, and that the primary currents supplied will be such as to raise the magnetic induction at each alternation to 5000 lines per square centimetre. Find how many turns there must be in the secondary coil if it is to produce 100 volts on open circuit. (C. and G., 1893.)

7. An alternate current transformer has 200 turns in its secondary winding, the induced E.M.F. in which is 102 volts, the frequency being 100 periods per second. Assume that B will not exceed 4000 lines per square centimetre. What must be the sectional area of the core?

(C. and G., 1894.)

8. Point out in what way the pressure at the secondary terminals of a good transformer depends on that at the primary terminals, and on the respective number of windings and their resistance. A transformer with a well-closed magnetic circuit is taking 10 amperes at 2050 volts. Its primary coil consists of 860 turns of wire having a total resistance of 5 ohms. Its secondary consists of 43 turns, with a resistance of $\frac{1}{80}$ of an ohm. Assuming that magnetic leakage, eddy currents, and hysteresis are negligibly small, calculate the pressure at the secondary terminals when 200 amperes are being supplied to the lamps. (C. and G., 1896.)

9. What is meant by the magnetising current of a transformer, and what is its phase-relation to the supply voltage? Find the value of the magnetising current of a transformer with closed iron circuit, and of which the data are as follows :—

Primary voltage 2200, primary turns 320, area of core 130 square inches, mean length of core 40 inches, $\mu=2000$, frequency$=50$. (C. and G., 1908.)

10. An alternating pressure of 1000 volts, frequency 50, is applied to a circuit containing a condenser of 4 microfarads capacity in series with a resistance of 600 ohms. Find the current in the circuit, and the P.D. across the condenser and the resistance respectively. On what assumptions does your result depend, and how far do you consider them to be justified?

CHAPTER XXVII *

MEASUREMENT OF INDUCTANCE

UNTIL recently, the measurement of the coefficients of self and mutual induction was of little practical importance. Electrical engineers seldom or never required such measurements, as they were mostly concerned with circuits containing iron, for which L and M have no definite values. The rapid development of wireless telegraphy has, however, altered this state of affairs. In its theory and practice, L and M are extremely important quantities, and it is becoming necessary for commercial purposes to measure their values with rapidity and accuracy.

We, therefore, describe a few well-known methods based upon the use of a ballistic galvanometer. As such methods depend upon the properties of varying currents, their theory cannot be readily expressed in terms of equations relating to steady currents, and the descriptions given must be regarded as explanations rather than proofs. (See also remarks on p. 460.)

Measurement of Self-Inductance.—(1) *By means of an alternating current of known frequency.* This method has already been mentioned (p. 435). Besides having the great advantages of convenience and readiness of application, it is often extremely useful when only moderate accuracy is required. For instance, the self-induction of the standard solenoid used in several previous experiments was measured in this way, and it was found that a P.D. of 26 virtual volts sent 7 amperes through it, the frequency being 50. The resistance of the solenoid was 3·54 ohms, and, therefore, we have

$$C = \frac{E}{\sqrt{R^2 + (2\pi n L)^2}}$$

$$7 = \frac{26}{\sqrt{(3{\cdot}54)^2 + (2\pi \times 50 \times L)^2}}$$

$$\therefore \quad (3{\cdot}54)^2 + (2\pi \times 50L)^2 = \left(\frac{26}{7}\right)^2$$

$$\text{or } 2\pi \times 50L = \left(\frac{26}{7}\right)^2 - (3{\cdot}54)^2$$

from which L = ·003 henry.

The next significant figure cannot be relied upon, for it would be altered by an extremely small change in the value of current or of E.M.F., and, moreover, the frequency is seldom *exactly* the nominal value when public supply mains are used. Again, the formula assumes that the current obeys a sine law, a condition which may be only approximately satisfied.

(2) *By comparison with a known capacity.*—The coil, or portion of a circuit whose self-inductance is to be measured, is placed in one arm of a Wheatstone bridge, as shown at P(L) in Fig. 322 (1), where P stands for the resistance in that arm. It is understood that the resistances in the other arms are non-inductive. Exact balance for steady currents is first obtained in the

usual way, *i.e.* there is no deflection when the galvanometer circuit is closed *after* the battery circuit is closed, which means that $P \times R_1 = S \times R$. (1)

Then the battery circuit is opened while the galvanometer circuit remains closed, the result being a momentary throw due to self-induction L. Let this be d scale divisions. Also determine the damping factor as usual. The next step is to calibrate the galvanometer by some method which will facilitate calculation. There are several ways of doing this; in the present case, the connections are altered as shown in Fig. 322 (2), a condenser of known capacity K and the galvanometer being connected between the points A and B. When the battery key is closed, a P.D. will exist between A and B, and then, on closing the galvanometer key, this becomes the P.D. between the condenser coatings, the charging throw being d_1 divisions. The damping factor should again be determined, because the circuit arrangements have been altered, and it cannot be regarded as necessarily the same in both cases.

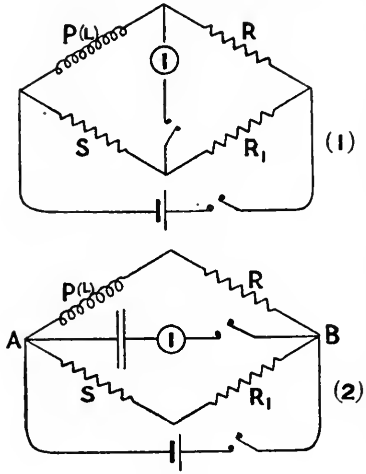


FIG. 322.

Theory of Method. — Let i be the steady current through the arm P when balance is obtained in the ordinary way. Then, the effect of opening the battery key (whilst the galvanometer key is closed) is equivalent to setting free a quantity Q in the arm P, such that

$$Q = \frac{Li}{\text{Resistance of circuit}}$$

This resistance is easily written down (being P + S + resistance of shunted galvanometer), hence, we have

$$Q = \frac{Li}{P + S + \dfrac{(R + R_1)g}{R + R_1 + g}}$$

Of this quantity a portion q passes through the galvanometer, such that

$$q = \frac{R + R_1}{R + R_1 + g} \times Q$$

also $q \propto d\left(1 + \frac{l}{2}\right)$, where d is the throw produced.

$$\therefore \quad q = \frac{Li}{P + S + \dfrac{(R + R_1)g}{R + R_1 + g}} \times \frac{R + R_1}{R + R_1 + g} \propto d\left(1 + \frac{l}{2}\right)$$

but from equation (1) $S = \frac{PR_1}{R}$, and by substitution and simplification we obtain

$$q = \frac{LiR}{P(R + R_1 + g) + Rg} \propto d\left(1 + \frac{l}{2}\right) \qquad (2)$$

In the second part of the experiment, let e be the P.D. between A and B, then

$$e = i\ (P + R), \text{ by Ohm's law.}$$

Also, if q_1 be the quantity charging the condenser and producing a throw d_1, we have

$$q_1 = c\mathrm{K} \propto d_1\left(1+\frac{l_1}{2}\right)$$

$$\therefore \quad q_1 = i(\mathrm{P}+\mathrm{R})\mathrm{K} \propto d_1\left(1+\frac{l}{2}\right) \qquad (3)$$

whence, dividing (2) by (3), we obtain

$$\frac{\mathrm{L}i\mathrm{R}}{\{\mathrm{P}(\mathrm{R}+\mathrm{R}_1+g)+\mathrm{R}g\}\{i(\mathrm{P}+\mathrm{R})\mathrm{K}\}} = \frac{d\left(1+\frac{l}{2}\right)}{d_1\left(1+\frac{l_1}{2}\right)}$$

from which i cancels out, and we have finally

$$\mathrm{L} = \frac{\{\mathrm{P}(\mathrm{R}+\mathrm{R}_1+g)+\mathrm{R}g\}(\mathrm{P}+\mathrm{R})\mathrm{K}}{\mathrm{R}} \times \frac{d\left(1+\frac{l}{2}\right)}{d_1\left(1+\frac{l_1}{2}\right)}$$

The self-inductance of an electromagnet coil (without iron) was measured in this way. R and R_1 were each 10 ohms, and in order to obtain *exact* balance for steady currents (which is the chief difficulty in such methods), an adjustable non-inductive resistance was placed in series with the coil and altered until balance was obtained, when S=12 ohms. Hence, P=12 ohms. The throw d was 242 divisions, and the logarithmic decrement l was found to be ·051. In the second part of the experiment, using a condenser of 1 microfarad capacity, the throw was 208 divisions, and l was ·059. The galvanometer resistance was 957 ohms, and hence,

$$\mathrm{L} = \frac{\{12(10+10+957)+10\times 957\}(12+10)\times\frac{1}{10^6}}{10} \times \frac{242\times 1{\cdot}025}{208\times 1{\cdot}029}$$

$= {\cdot}054$ henry.

(The alternating current method gave ·058 henry.)

(3) *Anderson's Method.*—This is a very good method, and, in practice, it would often be adopted in preference to any other.

The theoretical diagram is shown in Fig. 323, and the actual arrangement in Fig 324.

The Wheatstone bridge method, slightly modified, is again used, but in this case the throw due to self-induction is neutralised by an equal and opposite effect due to a known capacity, and hence it has the advantage of being a null method.

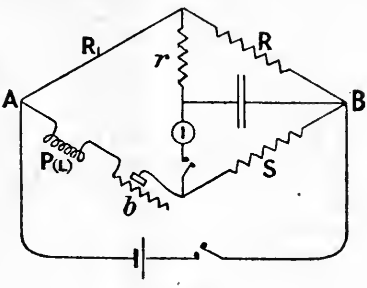


FIG. 323.

First obtain *exact* balance for steady currents. During this operation the resistance r can be cut out. The galvanometer should be shunted at first to avoid injury, and it will be necessary (as in the previous experiment) to introduce a small variable resistance in the arm P (such as a straight piece of bare wire indicated by b, Fig. 323), in order to obtain balance for steady currents. It will also be advisable to make R large and R_1 small.

This balance will not be disturbed by introducing the resistance r into the galvanometer circuit. Adjust its value until there is no throw when the galvanometer key is closed first. It may be necessary to find by trial the best value of K, if several condensers are available. As a rule, select a small capacity when measuring a small inductance, and *vice versâ*.

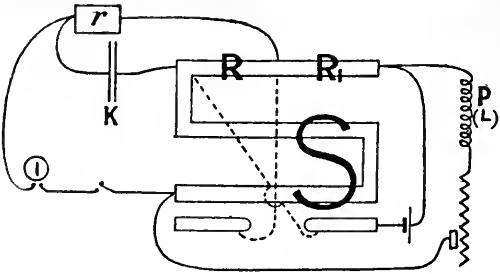


FIG. 324.

The theory of the method cannot be worked out in any simple way, but it leads to the following expression:

$$L = K(Sr + PR + Pr).$$

When this method was used to measure the self-induction of the standard solenoid (p. 395) the following values were obtained: $R = 1000$ ohms, $R_1 = 10$ ohms, $S = 359$ ohms, therefore P (including b) $= 3{\cdot}59$ ohms, $r = 28$ ohms, $K = \frac{1}{4}$ microfarad (obtained by putting two $\frac{1}{2}$ microfarad condensers in series).

$$\therefore \quad L = \frac{1}{4 \times 10^6}\{(359 \times 28) + (3{\cdot}59 \times 1000) + (3{\cdot}59 \times 10)\}$$

$$= {\cdot}0034 \text{ henry.}$$

Value of L by Calculation.—It has already been shown that for a *long* solenoid,

$$L = \frac{4\pi n^2 A\mu}{l \times 10^9} \text{ henries (approximately).}$$

In this case, $n = 869$ turns, $l = 136$ centimetres, diameter $= 8{\cdot}4$ centimetres. $\therefore A = \pi\,(4{\cdot}2)^2$.

$$\therefore \quad L = \frac{4\pi \times 869^2 \times \pi \times (4{\cdot}2)^2}{136 \times 10^9} = {\cdot}0038 \text{ henry.}$$

As the value arrived at from this formula is known to be an approximation (see p. 376), the general agreement is satisfactory.

Meaning of Result.—From the explanations already given (see p. 376), it will be seen that the statement that $L = {\cdot}0034$ henry for a certain coil means that when unit current (*i.e.* 10 amperes) is passing through it, a magnetic flux is linked with it such that the product of the flux × number of turns is ${\cdot}0034 \times 10^9$.

Measurement of Mutual Induction.—The coefficient of mutual induction has already been defined (p. 378). In practical electrical engineering, it is a quantity of little importance, except in connection with wireless telegraphy. Hence, we merely indicate two or three methods of general application.

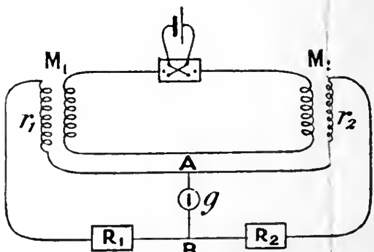


FIG. 325.

(1) *Comparison of Two Mutual Inductances.*—The two mutual inductances, M_1 and M_2, are arranged as shown in Fig. 325. The respective primary coils

are connected through a reversing key to a cell or battery; and the secondaries are placed in series with each other and two resistance boxes, R_1 and R_2, a galvanometer bridging across this circuit at AB. R_1 and R_2 are then adjusted until there is no galvanometer throw when the primary current is reversed. To obtain this result, it is necessary to connect up the secondaries the right way, *i.e.* so that the two induced E.M.F.'s are in series, a condition easily found by trial. Let i be the steady primary current, e_1 and e_2 the average value of the E.M.F.'s produced in the secondaries when this current is reversed. Then

$$\left.\begin{aligned} e_1 &= \frac{2M_1 i}{\text{time of reversal}} \\ e_2 &= \frac{2M_2 i}{\text{time of reversal}} \end{aligned}\right\}$$

The coefficient 2 would disappear if the circuit was merely broken. Reversal means a double cutting of the lines, and increases the sensitiveness of the test.

$$\therefore \frac{e_1}{e_2} = \frac{M_1}{M_2}$$

The arrangement of the secondary circuit is exactly similar to that of the case discussed on p. 226, where it was shown that the condition of A and B being at the same potential is

$$\frac{e_1}{e_2} = \frac{r_1 + R_1}{r_2 + R_2}$$

Hence, $$\frac{M_1}{M_2} = \frac{r_1 + R_1}{r_2 + R_2}$$

This method was adopted to compare the mutual inductance between the standard solenoid and its secondary, used in Experiment 232 (M_1), with that between one of the electromagnet coils and a small coil slipped inside it, used in Experiment 223 (M_2).

It is a matter of convenience as to which coil of each pair is made the primary — in this particular experiment, the coil of 1000 turns inside the solenoid was used in the one case, and the small coil inside the magnet coil in the other. Their resistances were 46·2 and 1·75 ohms respectively, therefore $r_1 = 46{\cdot}2$ and $r_2 = 1{\cdot}75$. Balance was obtained when $R_1 = 30$ ohms and $R_2 = 290$ ohms.

$$\therefore \frac{M_1}{M_2} = \frac{r_1 + R_1}{r_2 + R_2} = \frac{46{\cdot}2 + 30}{1{\cdot}75 + 290} = \frac{76{\cdot}2}{291{\cdot}75} = \frac{1}{3{\cdot}8}$$

(2) *Carey Foster's Method of measuring a Mutual Inductance.*—In this method the discharge due to mutual induction is balanced by an opposite discharge from a condenser of known capacity, the arrangement being shown in Fig. 326, where P and S are the primary and secondary coils of the mutual inductance to be measured, and R and R_1 are resistance boxes. If the infinity plug be taken out of the box R_1, thus rendering that part of the circuit inoperative, it will be found that a throw is produced when the battery key is open or closed, due to the condenser; and if the plug is replaced and the condenser circuit broken between A and D, another throw is produced under similar conditions, due to the mutual inductance. Hence, if the connections are such that these throws are in opposite directions, it is possible to adjust R and R_1 until they balance each

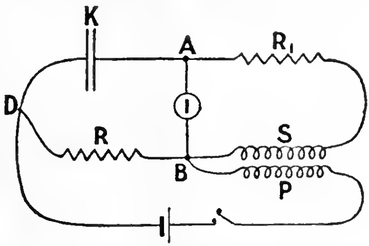


FIG. 326.

other, *i.e.* until no throw is produced when the key is opened or closed. A current reverser may be employed to increase the sensitiveness of the arrangement. When this condition is attained, A and B are at the same potential, and, therefore (if i be the steady current in R and P), the P.D. across the condenser = P.D. across DB = iR, and the quantity discharged from the condenser = $i.\mathrm{R} \times \mathrm{K}$ (when the circuit is broken).

Similarly the quantity produced by the mutual inductance is

$$\frac{\mathrm{M}i}{\mathrm{R}_1 + \mathrm{S}}$$

and the condition for balance is

$$\frac{\mathrm{M}i}{\mathrm{R}_1 + \mathrm{S}} = i\mathrm{RK}$$

$$\text{or } \mathrm{M} = \mathrm{KR}(\mathrm{R}_1 + \mathrm{S}),$$

where M will be in henries, if K is in farads and the resistance in ohms.

The mutual inductance between the electromagnet coil and its secondary (referred to as M_2 on p. 457) was measured by this method. For convenience, in this case, the small inner coil was used as the secondary, and the values obtained were :—

$$\mathrm{K} = 1 \text{ microfarad} = \frac{1}{10^{15}} \text{ absolute units of capacity, } \mathrm{S} = 1{\cdot}75 \text{ ohms,}$$

$$\mathrm{R} = 108 \text{ ohms, } \mathrm{R}_1 = 50 \text{ ohms.}$$

$$\text{Now } \mathrm{M} = \mathrm{KR}\ (\mathrm{R}_1 + \mathrm{S})$$

$$= \frac{1}{10^{15}} \times 108 \times 10^9\{(50 + 1{\cdot}75)10^9\}$$

$$= 5{,}589{,}000 \text{ absolute units}$$

$$\text{or } {\cdot}0056 \text{ henry.}$$

The mutual inductance between the standard solenoid and its secondary may be found approximately by calculation as explained on p. 378, from the equation :—

$$\mathrm{M} = \frac{4\pi n_1 \times n_2 \times \mathrm{A}\mu}{l}, \text{ where } \mu = 1$$

$$\text{In this case, } \mathrm{M} = \frac{4\pi \times 869 \times 1000 \times \pi \times (2{\cdot}53)^2}{136}$$

$$= 1{,}600{,}000 \text{ absolute units}$$

$$\text{or } {\cdot}0016 \text{ henry.}$$

The ratio of the mutual inductances is, therefore,

$$\frac{{\cdot}0016}{{\cdot}0056} = \frac{1}{3{\cdot}5}$$

which is a rough confirmation of the result obtained in the previous case.

Note on Choice of Galvanometer.—The two preceding methods are *null* methods, and it might be thought that a dead-beat galvanometer would be the most suitable — as it is certainly the most convenient — instrument to employ. But the state of "balance" does not necessarily exist at *each instant* during the process of "charge" or "discharge," and hence, it is desirable to employ an ordinary ballistic instrument of fairly long period.

Relation between M and L.—There is another very instructive method of measuring a coefficient of mutual inductance, which may be mentioned, although it is not recommended in practice. Each of the two circuits or coils, which together form the mutual inductance, has, when removed from the other, a definite self-inductance. Let these be L_1 and L_2 respectively. Let the two coils be arranged so as to form the mutual inductance in question and connected in series in such a way that their respective magnetic effects are in the same direction. Then the two coils constitute a single winding having also a definite self-inductance, which can be measured by any of the methods previously given. Let this be L_3. Next reverse the connections of one of the coils, so that their ampere-turns are in opposition, and again measure the joint self-inductance, L_4. The relation between these quantities will be most easily understood, if we consider the special case of two solenoids, one wound over the other, having the same length and practically the same diameter, the turns being n_1 and n_2 respectively.

$$\text{Then } L_3 = \frac{4\pi(n_1+n_2)^2 A\mu}{l}$$

$$L_4 = \frac{4\pi(n_1-n_2)^2 A\mu}{l}$$

$$\text{But } L_1 = \frac{4\pi n_1^2 A\mu}{l}, \text{ and } L_2 = \frac{4\pi n_2^2 A\mu}{l}$$

$$\text{and } M = \frac{4\pi n_1 n_2 A\mu}{l}$$

$$\therefore \quad L_3 = L_1 + L_2 + 2M$$

$$\text{and } L_4 = L_1 + L_2 - 2M$$

whence, by subtraction, $4M = L_3 - L_4$

$$\textit{i.e.}\ M = \frac{L_3 - L_4}{4}$$

Hence, M can be found from two measurements of an inductance, and although this result has been deduced from the particular case of two solenoidal coils, a little consideration will show that it holds generally, whatever may be the shape or arrangement of the circuit, because in the first arrangement, the inductances of the coils will become L_1+M, L_2+M; and in the second, L_1-M, L_2-M respectively.

$$\therefore \quad L_3 = L_1 + M + L_2 + M, \text{ and}$$

$$L_4 = L_1 - M + L_1 - M,$$

which are identical with equations already obtained.

Variable Standards of Inductance.—The previous results may evidently be applied to the construction of standards of self-induction, although for this purpose the solenoidal form is not convenient. If two coils (of the tangent galvanometer type, for example) are connected in series, and one is arranged to rotate inside the other, the nett value of L will vary with their relative positions, and may be read off on a fixed scale by means of a pointer attached to the movable coil. The actual values are usually obtained by calculation from the dimensions of the system; an operation of some difficulty when great accuracy is required. Such standards were first introduced by Ayrton and Perry many years ago. At the present time, standards based on mutual induction are preferred, as their working range is greater and the absolute values are more readily calculated. In practice they are equally useful, for M and L are expressed in the same units, and it is as easy to compare L with M as to compare two L's or two M's.

Vibration Galvanometers.—A student, who works through the previous methods (and possibly others given in more advanced treatises), will find them not only troublesome in many respects, but also particularly unsatisfactory in the case of *small* inductances. In the problems of wireless telegraphy, the accurate measurement of small inductances is of great importance, so that other methods, depending on the properties of alternating currents, are rapidly being developed, which, in all probability, will soon render obsolete those already described. The chief difficulty, hitherto met with, has been the want of a suitable galvanometer for alternating currents. Ayrton and Perry long ago made the first step in the right direction by introducing their *secohmmeter* (practically a rotating commutator, which made an ordinary galvanometer serviceable with alternating currents produced by rapidly reversing a direct current), but this instrument never came into general use. At the present time, the problem appears to be in a fair way of being solved by the use of vibration galvanometers. Wien and Rubens first made use of the principle, and, a few years ago, a very convenient form was brought out in this country by Campbell. This particular instrument is of the D'Arsonval type; a very narrow coil, carried by a bifilar suspension, being supported under adjustable tension between the poles of a permanent magnet.

When an alternating current is passed through the coil, the latter tends to oscillate with it, but the effect is almost negligibly small under ordinary conditions, and hence the chief feature of the instrument is an application of the principle of resonance. By varying the tension of the suspension, the moving system is "tuned," until its own natural period of vibration is equal to that of the alternating current in use at the moment, and then the spot of light on the scale widens out into a continuous band, the length of which is a measure of the current strength as long as the frequency is unchanged. When thus "tuned," the instrument is suitable for use in all kinds of null methods, such as those already described. It also lends itself to a very ready and exact comparison of inductances and capacities with a suitable variable standard of inductance, and thus the chief difficulties met with in ballistic methods disappear.

As vibration galvanometers are not yet in general use, space does not permit us to go further into details. It may be remarked, however, that the operation of "tuning" is equivalent to ensuring that the moving system is exactly at the end of its swing when the current changes its direction. Then, evidently the successive impulses act together, and a very feeble alternating current may produce a large cumulative effect. It might be thought that the amplitude would steadily increase without limit, but, as in other cases of the kind, it only increases until the energy dissipated in each swing becomes equal to the energy imparted to it during that swing.

CHAPTER XXVIII

THERMO-ELECTRICITY

The Seebeck Effect.—**Exp. 241.** Connect two copper wires to the terminals of a reflecting galvanometer, and complete the circuit by a piece of iron wire, twisting the junctions tightly together. Hold a lighted match to one junction, observe that the galvanometer is deflected, and note the direction. On removing the match, the spot of light gradually comes to rest. Now apply a match to the other junction, and notice that the deflection is in the opposite direction.

Compare these directions with that produced by putting the ends of the galvanometer wires on the tongue, as in Experiment 218, and thus show that the current passes from the copper to the iron through the *hot* junction.

Such currents—discovered by Seebeck in 1822, and called thermo-electric currents—are produced in any circuit formed of two dissimilar conductors, the only necessary condition being a difference in temperature between the two junctions. It is noteworthy that the effect is usually greater for poor conductors than for good conducting metals like copper, silver, or gold. This is clearly shown by the following list, in which the order is such that, if a circuit be made with any pair of substances, the current through the hot junction is from the one above to the one below, its strength increasing with their distance apart on the list.[1]

Bismuth	Gold
Platinum	Silver
Cobalt	Zinc
German silver	Iron
Lead	Antimony
Copper	

Evidently an E.M.F. is produced in the circuit, the magnitude of which depends both on the nature of the substances and on the difference of temperature between the junctions. It is usual to

[1] It must be understood that this sequence refers to comparatively small differences of temperature under ordinary circumstances. It is shown later that for wider ranges of temperature the direction of the current may be reversed. In fact, such a list has no meaning unless the temperature conditions are specified.

express certain important experimental results in the form of two laws, which may be stated as follows :—

1. **Law of Successive Temperatures.** — Suppose that ${}_{10}E_{20}$ is the E.M.F. in the circuit, when the junctions are at 10° C. and 20° C. respectively; and that ${}_{20}E_{30}$ is the E.M.F., when they are 20° and 30° respectively. Then the law states that the E.M.F., when the junctions are at 10° and 30°, will be the algebraic sum of the preceding values, or ${}_{10}E_{30} = {}_{10}E_{20} + {}_{20}E_{30}$, which may be generalised by writing

$$ {}_{t_1}E_{t_3} = {}_{t_1}E_{t_2} + {}_{t_2}E_{t_3} $$

2. **Law of Intermediate Contacts.**—This very important experimental law states that we may open any junction at a given temperature—say $t°$—and insert in the circuit any number of different conductors in series, *without altering the E.M.F. in the circuit*, provided that all these intermediate conductors are kept at the same temperature, $t°$, as that of the original junction.

This means, for instance, that we may solder two wires together to obtain good contact, and the result will be the same as if the bare wires are twisted together, although a thin layer of solder may everywhere prevent actual contact. It also makes it possible to insert a galvanometer in the circuit, and to use such an arrangement as that shown in Fig. 327.

Measurement of Thermo-electromotive Force.—This offers certain difficulties on account of the very small value of such E.M.F.s; for instance, the E.M.F. of a copper-iron circuit, when the junctions are kept at 0° C. and 100° C. respectively, is only about ·0013 volt.

The following arrangement is given on account of its simplicity. A more exact method is described later.

Exp. 242, *to measure the E.M.F. for copper and iron at different temperatures.* Connect up an ordinary potentiometer as shown in Fig. 327. The potentiometer, AB, is in circuit with a single Daniell's cell, D, and a resistance box, R. The thermo-couple is made by cutting off any convenient length, say 1 yard, of iron wire, and soldering to its ends two other pieces of copper wire, each of about the same length. It is then connected up as shown, one junction being kept in melting ice, and the other being placed in a vessel, which can be heated over a Bunsen burner.

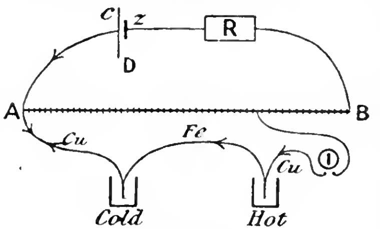


Fig. 327.

If it is intended to carry the measurements above 100° C., colza oil may be used instead of water up to about 200° C. The galvanometer should be a reflecting instrument, and it must be remembered that, as the E.M.F.'s to be measured are so small, bad contacts will lead to troublesome difficulties. Again, if we attempt to employ the ordinary potentiometer method described on p. 304, the P.D. between the ends of AB will have to be greater than the E.M.F. of the standard cell used for purposes of comparison, and then all the points of balance obtained with the thermo-couple will be so close to A as to be unreadable. We must, therefore, make the constant P.D. across AB only a little greater than the E.M.F.'s to be measured, and in this simple form of experiment, we dispense with a standard cell by making use of the fact that the E.M.F. of a Daniell's cell is practically 1·1 volt. Begin with the hot junction at about 10° C., and then increase R until the position of balance occurs at a convenient point on the potentiometer scale. It is desirable to make the *first* reading as large as is consistent with getting the *last* reading on the scale, *e.g.* if it is intended to work up to 100° C., the position of balance when the hot junction is at that temperature will roughly correspond to ten times as many divisions as for 10° C., hence, if there are 1000 divisions on the potentiometer scale, it will be convenient to start with about 80 divisions for a difference of temperature of 10°.

This adjustment being made, R remains constant during the experiment.

Increase the temperature of the hot junction to about 20° C., again obtain balance, and so on, obtaining readings for as many different temperatures as possible.

Let R be the number of ohms unplugged in the resistance box, r the resistance of *one* division of the potentiometer wire (found, if necessary, by a separate experiment), N the total number of divisions, and d the reading in scale divisions at any temperature t° C.

Then, as the resistance of cell and connecting wires is negligible compared with R,

$$\text{Current through potentiometer wire} = \frac{1 \cdot 1}{R + Nr} \text{ amperes.}$$

$$\text{Also, the P.D. across any one division of that wire} = Cr = \frac{1 \cdot 1}{R + Nr} \times r$$

= a constant (provided that the current through potentiometer wire does not alter during the experiment). Therefore, we have only to multiply this constant by the observed reading, d, to obtain the E.M.F. of the thermo-couple for that difference of temperature. It will be convenient to express the result in absolute units or in micro-volts (one millionth of a volt).

If the results of this experiment be plotted with the values of E for ordinates, and the temperatures of the hot junction for abscissæ, the graph obtained will not differ much from a straight line, but when the measurements are extended through a *wider range* of temperature, the E.M.F. passes through a maximum, then decreases in value to zero, and finally reverses its direction—the resulting curve being sensibly parabolic (see Fig. 331, p. 468). The temperatures at which these effects occur depend upon the substances used; in the case of copper and iron (one junction being kept at 0° C.), the maximum E.M.F. is obtained when the other is at 275° C., and reversal occurs at 550° C.

The reversal of the current is easily shown as follows:—

Exp. 243. Connect two copper wires to the terminals of a dead-beat reflecting galvanometer, and complete the circuit by means of a piece of iron wire. Gradually bring one of the junctions near a very small gas flame, approaching the coolest part of the flame at first. The deflection will be observed to increase to a maximum, then decrease, and finally increase again in the opposite direction. Remove the junction from the flame, and as it cools, watch the deflection, which will be found to pass through the same stages in the inverse order.

Peltier Effect.—About twelve years after Seebeck's discovery, Peltier discovered that, when a current is passed across the junction of two dissimilar substances, there is *either* a small absorption *or* a small evolution of heat *at the junction.* This effect is quite distinct from the ordinary Joule effect, which is *always* an evolution of heat, and is independent of the direction of the current. Moreover, the heat appears in all parts of the circuit, and is proportional to C^2R. On the other hand, the Peltier effect is always confined to the junctions, and is *reversible*, being either an absorption or an evolution of heat according to the direction of the current. Again, it is proportional to the current and not to its square, and it does not depend upon the ohmic resistance at all (except in so far as that modifies the current strength).

Peltier's Cross.—One of the methods used by Peltier to demonstrate this effect is shown in Fig. 328. Two rods of antimony and bismuth, A and B, are soldered together to form a cross, and connected up as shown to a cell and a galvanometer. The current from the cell does not pass through the galvanometer, and if the bars were of the same material, nothing would happen. As it is, the galvanometer shows a deflection, and if the cell current passes from bismuth to antimony, this deflection shows that the galvanometer current passes from antimony to bismuth *through the junction*, i.e. it indicates that the junction is *cooled.*

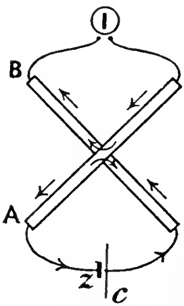


Fig. 328.

Mr. S. G. Starling has suggested another method of demonstrating the Peltier effect. A bar of bismuth is placed endwise between two bars of antimony, the surfaces being amalgamated to reduce the resistance due to bad contact. A coil of 36-gauge insulated copper wire is wound over each junction, and the arrangement protected from air currents by a surrounding tube. The two coils are then connected up in the arms of a Wheatstone bridge, and balance obtained. When a current is passed through the compound bar (1 ampere is said to be sufficient), one of the two coils is heated and the other is cooled by the Peltier effect; the consequent change in resistance disturbing the balance and producing a deflection of the galvanometer. The direction of this deflection indicates the existence of a cooling effect when the current flows from bismuth to antimony, and *vice versâ*.

The fact that the Peltier effect is *reversible* affords a clue to its nature. Consider a copper-iron circuit, Fig. 329. Energy is always absorbed when a current flows from copper to iron, whether that current is a Seebeck effect due to heating the junction, or whether it is obtained from some external source, and this indicates that it is flowing *up* a slope of potential.[1] When a current flows from iron to copper, energy is evolved in the form of an additional amount of heat, which means that the current is flowing *down* a slope of potential. Hence, we infer the existence of a P.D. between two unlike substances in contact; in the above case iron being positive to copper (at ordinary temperatures).

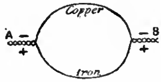


FIG. 329.

Here there is some danger of confusion of thought, because we are naturally inclined to think that when a current is flowing *down* a slope of potential, it is flowing in the *same* direction as an E.M.F.; and when it is flowing *up* such a slope, it is flowing *against* a back E.M.F. As a matter of fact, it is easy to see that at a part of the circuit *in which an E.M.F. is being generated*, a current in the same direction as the generated E.M.F. is flowing *up* a slope of potential, and *vice versâ*; whereas in conductors serving merely to complete the circuit, it is of necessity in the same direction as the E.M.F., and is also flowing *down* a potential slope. Consider an ordinary battery circuit. In the battery itself the current is flowing in the direction of the generated E.M.F., and also *up* a potential slope (as if pumped up to a higher level); a process which requires a supply of energy. In this case the energy is derived from chemical actions going on in the cells, such actions liberating less free heat than they would otherwise do. That is, energy is being *absorbed* where the current is flowing *up* a slope of potential. In the external circuit, the current is flowing in the direction of the E.M.F. (i.e. *down* a slope of potential), and is liberating, as C^2R heat, the previously acquired energy, which in this case cannot be liberated in any other form. Now in this part of the circuit, insert a back E.M.F. (say a cell of negligible internal resistance connected up the wrong way), and we may suppose

[1] We might express this in the language of electrostatics by saying that, when electric charges are moving in an electric field *against* the electric force, *i.e.* when work must be done upon them, they are moving *up* a slope of potential. When they move *with* the electric force, they are flowing *down* a slope of potential.

that the battery power is increased to keep the current unaltered in strength. A little consideration will show that the opposed cell is equivalent to a locally steep downward slope of potential, and, as a consequence, an extra amount of energy is liberated in it over and above the C^2R heat which is the same as before. But where the current flows downwards through the source of a back E.M.F., the energy evolved is not *necessarily* in the form of heat, and in this case it is expended in reversed chemical action, *i.e.* in decomposing some chemical compound, while in other cases—such as that of a motor—it may take the form of mechanical work.

Applying these considerations to the thermo-electric circuit shown in Fig. 329, we notice that when the junctions are at the same temperature, there are two equal and opposite Peltier E.M.F.'s, and hence no current flows. When this balance is upset by altering the temperature of one of the junctions (say by raising the temperature at A), we know that a current flows from copper to iron through the hot junction, energy being absorbed at A and given out at B, and we should naturally expect the former to be the greater, the difference between the two representing the energy available in the circuit. But experiment shows that, in this particular case, the energy evolved at B is *greater* than that absorbed at A, *i.e.* the Peltier effect at A is weaker than that at the colder junction B, and yet the current is flowing in the direction of the weaker E.M.F.

Thomson Effect.—Considerations such as these led the late Lord Kelvin (then Professor Thomson) to predict the existence of some other reversible heat effect in a circuit; a conclusion he soon verified experimentally by showing that a P.D., or slope of potential, existed between the hot and cold parts of one and the same substance, *e.g.* hot copper is positive to cold copper, and hot iron is negative to cold iron. Without going into details, the principle of the method will be understood from Fig. 330. AB is a bar of the metal in question, which by some external means is kept hot in the middle and cold at the ends. Intermediate points, P and Q, on the bar can be found, which are at the same temperatures. If there is any difference of potential between the hot and cold parts of the bar, a current flowing through it must, in one half, be flowing *up* the slope of potential, and, in the other half, *down* it. In the former case, energy will be absorbed and the temperature lowered; in the latter, energy will be liberated and the temperature raised. As a matter of fact, when a strong current is sent through the bar, it is found that the temperatures at P and Q are no longer equal: in the case of copper, that at Q is greater than that at P, when the current flows from P to Q; in the case of iron,

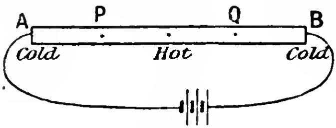


FIG. 330.

the higher temperature is at P. Silver, cadmium, zinc, and antimony behave like copper; platinum, bismuth, cobalt, nickel, and mercury like iron. In the latter metals, the Thomson effect is said to be "negative." Lead is noteworthy on account of the entire absence of the effect—at any rate, it is too small to be measurable.

Mr. S. G. Starling demonstrates these facts by a method similar in principle to that already given in the case of the Peltier effect. The two ends of a U-shaped iron rod are bent to dip into vessels of mercury, thus ensuring good contact and also keeping the ends cool. Over the middle portion of each limb a coil of fine insulated copper wire is wound, the two coils being placed in the arms of a Wheatstone bridge. The rod is then heated to redness at the bend, and the bridge adjusted until balance is obtained. When a current is passed through the bar (by means of the mercury cups), this balance is disturbed, and the direction of the ensuing deflection shows that the Thomson effect in iron is negative. The parts on which the coils are wound must be packed in asbestos wool in order to eliminate external disturbances.

We can now restate the general argument as follows: If, in any circuit made up of different materials, the temperature is the same at all points, the algebraic sum of the Peltier and Thomson effects is zero; but if such uniformity of temperature does not exist, then that sum may amount to an absorption of heat, in which case a thermo-current corresponding to the energy absorbed per second will flow round the circuit.

Coefficient of the Peltier Effect for Two given Substances.—This is usually defined as *the amount of energy, measured in ergs, absorbed or evolved when one absolute unit of current flows across the junction for one second.* We shall denote it by P.

Hence, if H units of heat are absorbed or evolved by a current i flowing for t seconds, then

$$H \times 41{\cdot}8 \times 10^6 = P \times i \times t \text{ ergs.}$$

Now this energy is also equal to $e \times i \times t$ ergs, where e is the P.D. between the substances at the junction, from which we see that P is also the value of e in absolute units, and there is often a distinct advantage, as regards clearness of thought, in regarding P as an E.M.F. rather than as energy.

Example.—The coefficient of the Peltier effect for copper-iron at 0° C. is 436,000. If a current of 40 amperes flows from copper to iron for one minute at this temperature, what happens at the junction?

We found by Experiment 241 that a thermo-current flows from copper to iron through the hot junction, hence we know that, when a battery current is sent in this direction through the junction, there is an absorption of heat.

$$\text{Now energy absorbed} = P.i.t \text{ ergs}$$

$$= 436{,}000 \times \frac{40}{10} \times 60 = 104{\cdot}64 \times 10^6 \text{ ergs.}$$

$$\therefore \quad \text{Heat units absorbed} = \frac{104{\cdot}64 \times 10^6}{41{\cdot}8 \times 10^6} = 2{\cdot}5 \text{ (gram.C°).}$$

Of course, this tells us nothing respecting the actual fall of temperature at the junction. That depends upon the size of the conductors, their specific heat, and the rate at which heat is carried away by conduction, and it is also affected by the production of ordinary C^2R heat. Obviously it will be very small.

Coefficient of the Thomson Effect.—Let one absolute unit of current flow for one second from a part of a substance at temperature T to another part at temperature T_1, where T_1 differs *very little* from T (this stipulation is made necessary by the fact that the effect varies with temperature). Then if s be the coefficient of the Thomson effect, the amount of energy absorbed or evolved is $s(T - T_1)$, more conveniently written $s.dT$, s being taken as positive for substances that behave like copper and negative for those that behave like iron.

Hence, when current i flows for t seconds, the energy absorbed or evolved $= i.t.s.dT$ ergs, from which it follows, as before, that $s.dT$ is the P.D. between the points in question.

When we are dealing with great differences of temperature, s varies from point to point along the conductor, and the total P.D. must be obtained by a summation, expressed by writing $\int_T^{T_1} s.dT$.

Consider again the circuit shown in Fig. 329, and suppose that the junctions are at T and T_1 respectively (T_1 being greater than T). A current is flowing, whose direction is from copper to iron through the hot junction, due to an E.M.F., which is the resultant of four components, viz. the two Peltier E.M.F.'s at the junctions, which oppose each other but which are unequal in magnitude; and the Thomson E.M.F.'s between hot and cold copper, and between hot and cold iron, which are in the same direction.

It will be noticed that energy is being absorbed both in the wires and in the hot junction, and is being given out at the cold junction. If E be the thermo-electromotive force in the circuit, as measured in Experiment 242, then

$$E = \int_T^{T_1} s_{Cu} \cdot dT - \int_T^{T_1} s_{Fe} \cdot dT + P_1 - P.$$

As the sign of s is negative for iron, the second term is numerically positive.

Neutral Point and Thermo-electric Heights.—As already stated, the curve obtained by experiments, such as the one described on p. 462, is sensibly parabolic, and for copper and iron has the shape shown in Fig. 331. Hence, when one junction is kept at 0° C., we know from the theory of parabolic curves that the relation between E.M.F. and temperature can be represented by an equation of the form

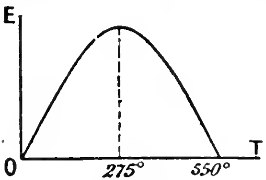


FIG. 331.

$$E = aT + bT^2 \quad *$$

where a and b are constants for a given pair of metals, and are opposite in sign. Solving the quadratic ($bT^2 + aT - E = 0$) we obtain

$$T = \frac{-a \pm \sqrt{a^2 + 4bE}}{2b}$$

* T is used for temperature to avoid confusion with t (time). It does not necessarily mean *absolute* temperature until so stated (on p. 476).

From this we learn that there are two values of T for any given value E, unless the quantity under the root sign becomes zero, in which case $T = -\frac{a}{2b}$. Hence, this is the temperature corresponding to the summit of the curve, which for copper-iron is 275° C. It is called the *neutral point*.

We also learn from the equation that E vanishes when $T = 0$, and when $bT^2 = -aT$, or when $T = -\frac{a}{b}$, *i.e.* when T is twice the temperature of the neutral point.

Thermo-electric Diagram.—These results are more simply obtained by differentiating the equation $E = aT + bT^2$, for

$$\frac{dE}{dT} = a + 2bT, \text{ which vanishes when } T = -\frac{a}{2b}.$$

Now, this is a linear equation in temperature, and hence if we plot a graph with the values of $\frac{dE}{dT}$ for ordinates, and temperatures for abscissæ, the result will be a straight line. With a few exceptions, this holds good generally. The values of $\frac{dE}{dT}$ are known as **thermo-electric heights**, for reasons which will become apparent later.

Because the Thomson effect is zero for lead, this has been taken as the standard metal, and in obtaining the curves of E.M.F. each metal has formed a couple with lead. Hence, when a value of the thermo-electric height for, say, copper at a given temperature is stated, it must be understood to refer to such a lead-copper couple.[1]

When the curves of thermo-electric height and temperature for different substances are plotted on the same sheet, the result is known as a thermo-electric diagram. Such a diagram is shown in Fig. 332 (taken from Carey Foster and Porter's *Electricity and Magnetism*), in which the ordinates represent thermo-electric heights in microvolts (millionths of a volt) per degree, and the abscissæ temperatures in centigrade degrees. When the direction of a current is *from* lead to the other substance through the hot junction, the E.M.F.'s are plotted *above* the base line, and *vice versâ*.

Consider one of the lines, shown separately for convenience in Fig. 333. As $\frac{dE}{dT} = a + 2bT$, it follows that a is the intercept on the

[1] It is usually assumed that the axis of the parabola in Fig. 331 is parallel to the axis of E.M.F., but since the above was written, it has been shown that the metals platinum, copper, cadmium, nickel, manganese, palladium, and aluminium give, when combined with lead, parabolas with *inclined* axes, the range of temperature being from −200° C. to +100° C. The graph of $\frac{dE}{dT}$ and temperature is then no longer a straight line.

axis, and $2b$ the tangent of the angle of slope. Also it is evident mathematically that E, the electromotive force in the circuit corresponding to junction-temperatures T and T_1, is proportional to the area of the figure ABT_1T.

This may also be seen by regarding $\frac{dE}{dT}$ as the E.M.F. in the circuit due to a unit difference of temperature of the junctions.

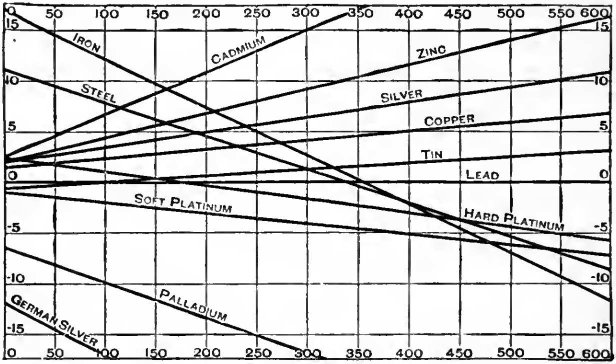


Fig. 332.

Then AT would give the E.M.F. when the junctions were at $T-\frac{1}{2}$, $T+\frac{1}{2}$ respectively; and similarly BT_1 would give the E.M.F. for a difference of 1° about the mean temperature T_1. Hence, when the temperatures are T and T_1 respectively, we may take the E.M.F. *per degree difference* of temperature as being the mean of these values, represented by the line CD in the figure. This must be multiplied by $T_1 - T$, the actual difference of temperature, and the result is the area of the trapezium ABT_1T as before. The value thus obtained is the E.M.F. when lead forms part of the couple, but the argument can easily be extended to other cases.

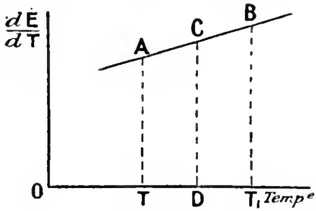


Fig. 333.

Suppose we require to find the E.M.F. due to a copper-iron couple, at junction-temperatures T and T_1, both being below 275°. Then from Fig. 334, it follows that the area ABT_1T represents the E.M.F. of an iron-lead couple, and DCT_1T that of a copper-lead couple. These areas both lie on the same side of the zero line, and the E.M.F. of the copper-iron couple is given by their difference, or is proportional to the area ABCD

The two lines intersect at the neutral point 275°, and evidently when one junction is at 0° C., the E.M.F. is a maximum when the other is at 275° C. Any area beyond the neutral point corresponds to a reversal of direction, and hence the circuit E.M.F. will not only be zero when one junction is at 0° C. and the other at 550° C., as already stated, *but also whenever the mean temperature of the junction is* 275° C.

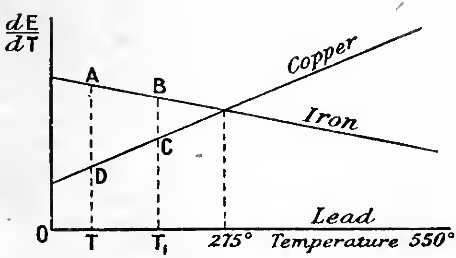


FIG. 334.

Hence, the rule for calculating the E.M.F. in a thermo-electric circuit may be stated as follows: *To find the E.M.F., multiply the difference of the thermo-electric heights at the* mean *temperature of the junctions by the difference of temperature.*

Example.—If the thermo-electric height of iron at any temperature $t°$ C. is given by $1734 - 4{\cdot}87t$, and that of copper by $136 + {\cdot}95t$; find the E.M.F. of a copper-iron couple when the junctions are at 20° C. and 100° C. respectively.

$$\text{Difference of thermo-electric heights} = (1734 - 4{\cdot}87t) - (136 + {\cdot}95t) = 1598 - 5{\cdot}82t$$

$$\text{and the mean temperature of the junctions is } \frac{20 + 100}{2} = 60°$$

$$\therefore \text{ Difference of thermo-electric heights} = 1598 - (5{\cdot}82 \times 60) = 1249$$

$$\text{also difference of temperatures} = 100 - 20 = 80$$

$$\therefore \text{E} = 1249 \times 80 = 99{,}920 \text{ absolute units} = \frac{99{,}920}{10^8} \text{ volts.}$$

By applying the principles of thermo-dynamics (see appendix to this chapter) it can also be shown that P, the coefficient of the Peltier effect at any temperature T, is AD × T, where T *is the absolute temperature of the junction.* (Thus far, all temperatures have been expressed in centigrade degrees.) From this it appears that the Peltier effect vanishes at the neutral point, *i.e.* at that temperature two different substances behave as though they are alike, there being *no* P.D. across the surfaces in contact.

It is there also shown that the value of $s\,d\text{T}$ for the Thomson effect, between any two *absolute* temperatures, T and T_1, is equal to the difference of the thermo-electric heights at T and T_1 multiplied by the mean temperature of the junction.

In fact, the slope of the lines in the thermo-electric diagram indicates the existence and nature of the Thomson effect. For example, the line for copper slopes upwards as the temperature rises, which means that hot copper is positive to cold copper; whereas the line for iron falls, telling us that hot iron is negative to cold iron. When there is no slope, there is no Thomson effect, as in the case of lead. Also, the method of plotting leads to the result that, when a current flows *upward* in the diagram, *i.e.* either along a rising line, as from cold to hot copper, or from any metal to another whose line at that temperature is *above* that of the first metal, then energy is *absorbed*, because a current is flowing *up* the slope of potential, and *vice versâ*. These results are easily remembered as being analogous to the gravitational case—bodies, when falling, evolving energy; bodies, when rising, acquiring energy.

Example.—Using the data given in the last example, again find the E.M.F. in a copper-iron circuit with the junctions at 20° C. and 100° C. by calculating the values of the Peltier and Thomson effects.

As already found, the difference of the thermo-electric heights for copper and iron is $1598 - 5{\cdot}82t$, where t is the temperature in centigrade degrees.

Now the Peltier E.M.F. is equal to this difference multiplied by the *absolute* temperature of the junction.

$$\therefore \text{ Peltier E.M.F. at hot junction} = \{1598 - (5{\cdot}82 \times 100)\} \times 373 = 378{,}968$$

$$\text{and reverse Peltier E.M.F. at cold junction} = (1598 - \overline{5{\cdot}82 \times 20}) \times 293 = 434{,}109$$

Again, the Thomson effect in each metal is equal to the difference of the thermo-electric heights multiplied by the mean absolute temperature of the junctions.

∴ Thomson's E.M.F. in iron

$$= \{(1734 - \overline{4{\cdot}87 \times 20}) - (1734 - \overline{4{\cdot}87 \times 100})\}(60 + 273)$$
$$= 129{,}737$$

and Thomson's E.M.F. in copper

$$= \{(136 + \overline{{\cdot}95 \times 100}) - (136 + \overline{{\cdot}95 \times 20})\}(60 + 273)$$
$$= 25{,}308$$

We have already found that the first, third, and fourth of these are in the same direction, the second being in the opposite direction; hence

$$\text{E.M.F.} = 378{,}968 - 434{,}109 + 129{,}737 + 25{,}308$$
$$= 99{,}904 \text{ absolute units,}$$

a result which agrees with the former value.

Practical Applications.—The earliest of these was the famous **thermopile**, invented by Melloni, much used in the past in connection with researches in radiant heat. It consists of a number of small bismuth-antimony couples connected in series (Fig. 335), and built up into a rectangular or a cubical form (Fig. 336). When the terminals are joined to a sensitive galvanometer (G, Fig. 337), a very slight difference of temperature between the two exposed faces of the pile (A and the opposite one inside the cone C) will produce a perceptible deflection, which for temperatures between 0° C. and 100° C. may be taken as proportional to the temperature-difference without serious error. The chief defect of the thermopile, in its original form, is due to its comparatively large mass, which means that it does not respond quickly to changes in temperature. This difficulty has recently been overcome in a form devised by Rubens, now almost exclusively used for purposes of research.

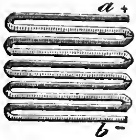


FIG. 335.

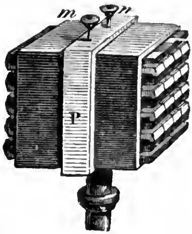


FIG. 336.

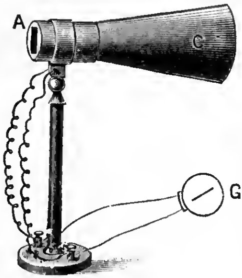


FIG. 337.

A thermopile of the ordinary type may be used to demonstrate the Peltier effect as follows:—

Exp. 244. Connect a battery of a few cells to a thermopile, and pass a current for a short time. Then quickly disconnect the battery, and join up the thermopile to a galvanometer. Notice that a deflection is produced, and that it gradually dies away.

The passage of the current heated one set of junctions and cooled the other, which thus left the pile in a state to produce a thermo-current. Had there been no Peltier effect, both sets of junctions would have been equally heated according to the C^2R law.

Measurement of Temperature.—A *single* thermo-couple is frequently used to measure temperature, its great advantage being due to its extremely small mass, and hence its ready response to temperature-changes.

It is necessary (*a*) to use materials that will not oxidise and will not melt at the highest temperature to be investigated, and (*b*) to avoid the temperature of the neutral point. For temperatures up to 1200° C., a wire of pure platinum joined to a wire of platinum-rhodium (or platinum-iridium) alloy, is largely used, and instruments made on these lines are supplied commercially with a galvanometer, calibrated so that the temperature of the junction can be obtained at once from the deflection. In such measurements the temperature of the cold junction, *i.e.* the wires connected to the galvanometer, should be kept constant, although when this is not the case, a simple correction may be applied, which is exact enough for most industrial purposes.

The most direct method of calibrating the galvanometer is carried out by taking a series of readings with the thermo-electric couple at a number of known temperatures between, say, 0° C. and 1000° C. It is now possible to obtain known temperatures with sufficient accuracy for this purpose.

Boys' Radio-Micrometer.—This is probably the most sensitive of all appliances for the detection of radiant heat. It resembles a galvanometer of the moving-coil type (Chapter XIX.), but the coil is replaced by a single loop of copper wire, carrying a mirror and suspended by a quartz fibre. The two ends of the loop below the polar gap carry a small bismuth-antimony couple soldered to a thin blackened disc of copper. The couple is heated by radiation falling upon it, and the current flows round the copper loop, which is then rotated by the field like the coil of the galvanometer in Fig. 228. When the junction is placed at the focus of a suitable parabolic mirror, it is said to detect the radiation from a candle flame at a distance of three miles.

The principle of the construction will be understood by referring to Fig. 231, p. 298, which shows the arrangement adopted in Duddell's thermo-galvanometer. If the "heater" be removed, and the thermo-couple warmed by radiation falling upon it from a distant source, the result is Professor Boys' instrument, of which the thermo-galvanometer is an adaptation.

Callendar's Radio-balance.—This is the latest instrument for the measurement of radiation. In its simplest form, the radiation is admitted through an aperture 2 millimetres across, and falls upon a copper disc having a diameter of 3 millimetres to which two thermo-junctions are attached to form a Peltier cross. One couple is connected to a sensitive galvanometer for indicating changes of temperature; the other is connected to a battery and an adjustable resistance, and included in the circuit is a suitable ammeter or potentiometer device for measuring accurately the current required to reduce the galvanometer deflection to zero. When the disc is heated by radiation, a thermo-current flows, and then the battery current is adjusted until this is exactly neutralised by the Peltier cooling effect. Hence, it has the advantages of a "null" method, and is especially suitable for absolute measurements.

Much greater sensitiveness is obtained in another form of the instrument, known as the *Cup radio-balance*, in which the radiation enters a small copper cylinder—thus absorbing practically the whole of it—and a pile of several couples in series is used in place of a single couple. Various devices are employed to eliminate external disturbances and the C^2R heating effect. This type is especially suitable for measuring the heat evolved by small quantities of radioactive bodies.

Thermo-electric Generators.—Many attempts have been made to apply the principles outlined in this chapter to the construction of commercially successful current generators. Such a generator, as compared with other devices for producing electrical energy, possesses several great advantages, *e.g.* (1) no consumption of material takes place in the pile itself; (2) but little attention is necessary, the action being easily set up by lighting a gas jet or other heating arrangement; (3) it can work continuously for long periods without detriment. Its disadvantages, which have hitherto proved fatal to extensive commercial use, are (1) inefficiency, only about 5 per cent. of the energy of the fuel consumed being actually utilised in producing the current; (2) the very low difference of potential produced at a junction, which necessitates the use of a large number of elements to obtain even a moderate E.M.F. The latter disadvantage is partially compensated by the low resistance, and for plating purposes, in which high E.M.F. is not required, a certain amount of success has been obtained.

It appears probable, however, taking everything into consideration, that further research may lead to important practical results.

APPENDIX TO CHAPTER XXVIII

A THERMO-ELECTRIC E.M.F. may be measured more accurately as follows: AB (Fig. 338) is an ordinary potentiometer wire, having, say, 1000 divisions and a *total* resistance of R ohms, through which a steady current is maintained by a Daniell's cell or an accumulator, and in whose circuit two ordinary resistance boxes, P and R_1, are included. Against this can be balanced either a standard cell or a thermo-electromotive force, by using the simple switch described in Experiment 178. A reflecting galvanometer is used to indicate balance, provided with a shunt to protect it during the preliminary adjustments, and another resistance box, K, in which about 20,000 ohms are unplugged, is included in the circuit merely to protect the standard cell. It does not affect in any way the position of balance.

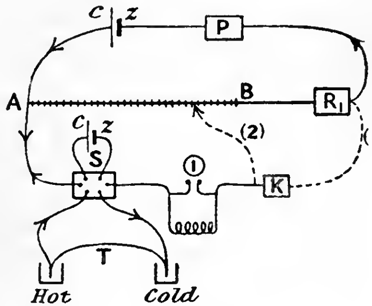


FIG. 338.

The principle of the method will be easily understood from the details of an actual experiment. The resistance of AB is 85 ohms (= R), and the E.M.F. of a Clark cell is taken as 1·434 volt; and it is desired to adjust matters so that the P.D. per scale division is $\frac{1}{100000}$ volt, *i.e.* the P.D. across AB has to be $\frac{1}{100}$ volt. A connection is made from K to R_1 as indicated by the dotted line (1)—that marked (2) is not wanted yet—so that the Clark cell can be balanced across the total resistances, $R+R_1$. The question then arises, What value must be given to R_1 to make the P.D. across AB $\frac{1}{100}$ volt, when the P.D. across $R+R_1$ is 1·434 volt?

Let C be the steady current in AB, then we must have:—

$$\frac{1}{100} = C \times 85$$

$$\text{and } 1{\cdot}434 = C(R_1+85)$$

$$\therefore \frac{1}{143{\cdot}4} = \frac{85}{R_1+85}$$

$$\text{or } R_1 = 12{,}104 \text{ ohms.}$$

Unplug 12,104 ohms in R_1. Then alter the value of P until the Clark cell is balanced across $R+R_1$ (removing the galvanometer shunt and cutting out K for the final adjustment). R_1 and P must not be touched again.

Remove connection (1), and replace by (2), and then begin the actual measurements of the thermo-electromotive force, by keeping one junction of T at 0° and gradually raising the temperature of the other. If balance is obtained at d divisions, then the thermo-E.M.F. is $\frac{1}{100000} \times d$ volts $= d \times 1000$ absolute units. If the limit of the scale is reached, the arrangement can be readjusted, making, say, the P.D. across AB = 1 volt, and, therefore, the P.D. per scale division $= \frac{1}{1000}$ volt.

Application of Thermo-dynamics.—Without attempting an exhaustive treatment of the subject, it may be remarked that, at the junctions of two different substances, there is a *reversible* transformation of work into heat, or heat into work, to which the second law of thermo-dynamics may be applied. On the other hand, superposed upon this is an irreversible change of work into heat due to the C^2R effect, to which that law does not apply. But as the one effect depends on the first power of the current, and the other upon its square, if the current is sufficiently small the C^2R heat is relatively a small quantity of the second order, and is, therefore, in comparison, negligible. This condition implies that the difference of temperature between the junctions must also be very small.

Consider a copper-iron circuit with junctions at temperatures T and $T+dT$, a small current i, therefore, flowing round the circuit. The quantity of heat, measured in ergs, which is absorbed in the wires in unit time by the Thomson effect is $(s_{Cu}-s_{Fe})dT \times i$; and the Peltier effect produces, at one junction, an evolution of heat $P \times i$, and at the other an absorption $P_1 \times i$.

But in any reversible process, the quantity of heat taken in, divided by the absolute temperature at which it is taken in, is equal to the heat given out, divided by the absolute temperature at which it is given out, or $\frac{Q}{T} = \frac{Q_1}{T_1}$.

Hence, $\frac{(s_{Cu}-s_{Fe})dT\times i}{T}+\frac{P_1 i}{T+dT}=\frac{P\times i}{T}$, where T is the absolute temperature. Now, as P_1 is only very slightly greater than P, it may be written $P+dP$, and i cancels out,

$$\text{or}\quad \frac{(s_{Cu}-s_{Fe})dT}{T}+\frac{P+dP}{T+dT}=\frac{P}{T}$$

$$\text{Hence,}\quad \frac{(s_{Cu}-s_{Fe})dT}{T}=\frac{P(T+dT)-T(P+dP)}{T(T+dT)}$$

$$\therefore\quad (s_{Cu}-s_{Fe})dT=\frac{PdT-TdP}{T+dT}$$

$$\text{or}\quad s_{Cu}-s_{Fe}=\frac{PdT}{dT(T+dT)}-\frac{TdP}{dT(T+dT)},$$ which in the limit

becomes
$$s_{Cu}-s_{Fe}=\frac{P}{T}-\frac{dP}{dT}$$

Now, on p. 468, we obtained the expression

$$E=\int_{T}^{T_1}(s_{Cu}-s_{Fe})dT+P_1-P$$

$$\therefore\quad \frac{dE}{dT}=(s_{Cu}-s_{Fe})+\frac{dP}{dT}$$

Substituting the value of $s_{Cu}-s_{Fe}$, we have

$$\frac{dE}{dT}=\frac{P}{T}=\text{thermo-electric height,}$$

or P for any two substances = the *difference* of T.E.H (each given with respect to lead) × absolute temperature.

$$\text{Again,}\quad \frac{d}{dT}\cdot\frac{P}{T}=\frac{T\frac{dP}{dT}-P}{T_2}=-\frac{1}{T}\left(\frac{P}{T}-\frac{dP}{dT}\right)$$

$$\text{But}\quad s_{Cu}-s_{Fe}=\frac{P}{T}-\frac{dP}{dT}$$

$$\therefore\quad s_{Cu}-s_{Fe}=-T\left(\frac{d}{dT}\cdot\frac{P}{T}\right)=-T\left(\frac{d}{dT}\cdot\frac{dE}{dT}\right)$$

$$=-T\frac{d}{dT}(a+2bT)$$

$$=-2bT$$

Although we have taken the case of copper-iron as an illustration, a similar result obviously holds good when one of the substances is lead, for which $s=0$, and hence we see that the absolute values of the Thomson coefficient can be obtained in terms of b. The details of such calculations are purposely omitted on account of the limitations of space. We are, however, now in a position to understand more fully the meaning of the thermo-electric diagram. The minus sign in the above expression depends on our convention as regards direction, and does not affect the magnitude of the quantity.

In Fig. 339, let AB be any two points on the line for copper, corresponding to absolute temperatures T and T_1.

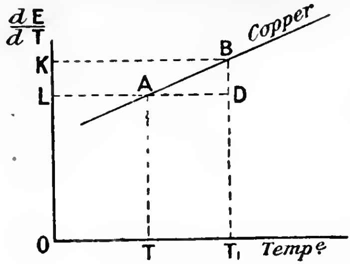


FIG. 339.

Then, because $\frac{dE}{dT}=a+2bT$, $2b$ is the tangent of the angle of slope, or

$$2b=\frac{BD}{AD}$$
$$=\frac{\text{Difference of T.E.H at T and } T_1}{T_1-T}$$

Also, $s_{Cu}=2bT$

$$\therefore \quad s_{Cu}=\frac{\text{Diff. of T.E.H}}{T_1-T}\times T$$

And the Thomson P.D. between T and T_1 will be

$$\int_T^{T_1} s dT=\frac{\text{Diff. of T.E.H}}{T_1-T}\int_T^{T_1} T dT$$
$$=\frac{\text{Diff. of T.E.H.}}{T_1-T}\times\frac{T_1^2-T^2}{2}$$
$$=\text{Diff. of T.E.H}\times\frac{T_1+T}{2}$$

which, it will be noticed, is represented by the area KBAL.

Now, consider again the diagram for copper and iron, as in Fig. 340. The Thomson E.M.F. in iron is represented by the area KABL, and in copper by the area MDCN, and we know that these are in the same direction. Again, the Peltier effect at the hot junction is, by the equation on p. 477, represented by the area LBDM, and this is known to be in the same direction as the Thomson effect. The sum of these will be given by the area KABDCN.

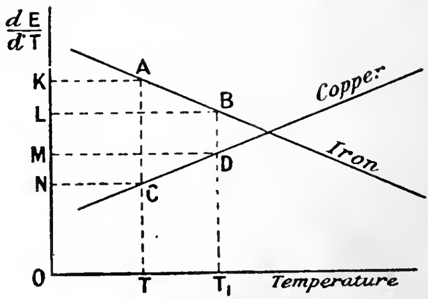


FIG. 340.

From this must be deducted the Peltier E.M.F. at the cold junction, known to be in the opposite direction, *i.e.* we must subtract the area KACN. This leaves the area ABDC to represent the actual nett E.M.F. in the circuit, as already stated on p. 470.

Volta Effect.—Volta discovered (by means of his condensing electroscope) that, when two rods or plates of copper and zinc are brought into contact, the copper becomes negatively, and the zinc positively charged. Other metals show similar effects, but in different degrees. This result implies the existence of a P.D. between copper and zinc (the copper being positive to the zinc) *before* contact, a flow then taking place which makes their potentials equal. This contact electrification was regarded by Volta as the source of the E.M.F. in a voltaic cell, a hypothesis which excited endless discussion for nearly one hundred years after his death.

Now, the Volta P.D. between copper and zinc is about ·8 volt, and if such a large difference of potential actually existed between these metals, reversible heating effects of a distinctly noticeable amount would be produced whenever a current passed across the junction of the two metals. But experiments reveal only the existence of a Peltier E.M.F. of almost negligibly smaller magnitude, and there is every reason to believe that this is the only true contact E.M.F. between the metals.

It is now usual to regard the Volta effect as being due to the immersion of the metals in air (oxygen), incipient chemical action tending to set up a P.D. between each metal and the surrounding air, this difference being the Volta E.M.F.

Apparently the question could be easily decided by measuring the Volta effect in a vacuum, but it is extremely difficult to get rid of the surface films of occluded gases, which vitiate the results of the experiments. By taking the most elaborate precautions, it has, however, been shown that the magnitude of the Volta E.M.F. is largely reduced in the absence of oxygen, a fact which sufficiently confirms the general accuracy of the explanation.

Exercise XXII

1. Two wires, one of copper, the other of iron, are twisted together at one end, the other ends being connected to a suitable galvanometer. Describe and explain the indications of the galvanometer as the iron-copper junction is gradually heated to bright redness. What becomes of the heat absorbed at the junction? (B. of E., 1908.)

2. Write a short account of the construction and practical applications of the thermo-electric junction. (B. of E., 1904.)

3. Explain how the metals can be arranged in a thermo-electric series, and the conditions under which such a series has a definite meaning. (B. of E., 1896.)

4. Two bars of bismuth, A and B, are attached to the extremities of a bar of antimony, and a current is passed from A to B. Is there any difference, and if so what, between the effects produced at the two junctions? How does the effect in each case depend on the strength of the current?

5. A ring is made, partly of iron and partly of copper wire, the junctions being A and B. If A be kept at 0° and B at 100°, a thermo-electric current is produced in the circuit. Similarly, a current is produced if A be at 100° and B at 200°. Have the currents in each case the same strength? Give reasons for your answer.

6. A thermopile is joined up in series with a Daniell's cell, and the current allowed to flow for a short time. The thermopile is then removed from the circuit, and connected to the terminals of a galvanometer, the needle of which is thereupon considerably deflected but gradually returns to its undisturbed position. Explain this. (B. of E., 1898.)

7. Under what circumstances can an electric current cool the conductor through which it passes? What will be the result if the direction of the current be reversed? (B. of E., 1894.)

8. Explain how thermo-electric currents can be utilised for measuring temperature. Illustrate your answer by describing the arrangements you would employ for measuring the temperature of the outside of a pipe which is conveying steam. (B. of E., 1910.)

9. What is meant by thermo-electric power, and how can the data for a diagram representing it be obtained? (Lond. Univ. B.Sc., 1903.)

10. Prove that the coefficient of the Peltier effect at a given junction is the product of the absolute temperature of the junction and the rate of change of the whole E.M.F. of the circuit with the temperature of that junction. (Lond. Univ. B.Sc., Honours, Internal, 1907.)

11. If the thermo-electric height of iron is $1734 - 4{\cdot}87t$, and of silver $214 + 1{\cdot}50t$, where t is the temperature in centigrade degrees, find the E.M.F. of a silver-iron couple when the junctions are at 16° C. and 180° C. respectively. Also find the temperature of the "neutral point."

CHAPTER XXIX *

PASSAGE OF A DISCHARGE THROUGH GASES

The phenomena attending the passage of a discharge through a gas differ in many important respects from those observed in the case of metallic or electrolytic conduction; for instance, one and the same gas may, according to circumstances, act like an excellent insulator, a poor insulator, or a true, if feeble, conductor.

At atmospheric pressure, most gases act like good insulators of small dielectric strength, *i.e.* they insulate well, but are somewhat readily pierced by a spark. At greater pressures, they are still better insulators, and are much less readily pierced by a spark.

If the pressure is reduced, the dielectric strength decreases, passing through a minimum value (at a pressure which depends upon the nature of the gas, but which is of the order of a millimetre of mercury), thence increasing again until, when extremely rarefied, the gas becomes totally non-conducting.

Again, by the action of certain agencies (X-rays, radium rays, &c.), a gas at ordinary pressure may be brought into a state in which it possesses true conductivity, *i.e.* instead of requiring an applied P.D. above a certain minimum value to ensure a discharge—which then occurs more or less suddenly as a spark—a small P.D. can send a current through it, which, up to a certain point, obeys Ohm's law. In this state the gas is said to be *ionised.*

The very existence of the phenomena discussed in the first section of this book shows that air at ordinary pressure is a non-conductor.[1] This fact and the increase of insulating power with increased pressure indicate that some real difficulty attends the passage of a charge from a solid body to the gas particles immediately in contact with it. The same difficulty is strikingly met with in the case of liquids, *e.g.* electrified water gives off unelectrified vapour, and so does boiling and electrified mercury.

On the other hand, it has been shown (see p. 82) that a gas *can* be electrified—*e.g.* by point discharges—and that it can be pierced by a spark, both effects requiring a certain high strength of the

[1] Recent work connected with radio-activity has shown that air always possesses a certain (although very feeble) conductivity. This is why a charged body, no matter how perfectly insulated, in time loses its charge; an effect which, until the discovery of radio-activity, was always ascribed to imperfect insulation. See p. 105.

electric field at the junction of solid and gas, and suggesting some disruptive process quite distinct from ordinary conduction. (It should be noticed that when a spark passes through a gas, it produces a temporary path of very small resistance, and is, in fact, equivalent to suddenly joining the spark terminals by a good conductor. As Faraday first observed, it may be difficult to get the first spark to pass, but it is then easier for others to follow.)

Spark Discharge at Ordinary Pressure.—The earliest measurements of the relation between sparking voltage and spark length were made many years ago by Lord Kelvin by means of the (then newly invented) absolute electrometer, and his work brought out the fact that the dielectric strength of air was relatively much greater for very short sparks than for longer ones. To a considerable extent, however, the sparking voltage depends upon the experimental conditions (*e.g.* upon the size and shape of the terminals), and thus the question is somewhat complicated.

The order of magnitude of these quantities is indicated in the following table, which refers to brass balls two centimetres in diameter in air at ordinary pressure. It will be seen that the voltage does not increase proportionally to the length of spark :—

Spark Length in Centimetres.	P.D. in Volts.
·1	4,700
·5	17,500
1·0	31,300
2·0	47,400
3·0	57,500
4·0	64,200
5·0	69,800

The chief facts may be briefly summarised as follows :—

(1) In gases at ordinary pressures, the relation between P.D. and spark length (except in the case of *very* short sparks) is practically linear, and may be expressed in the form

$$E = a + bl,$$

where a and b are constants depending upon the nature of the gas, and l is the spark length.

(2) As the spark length decreases, this expression holds good until (in the case of air), l is about $\frac{1}{4000}$ inch, and E is about 350 volts. This is known as the critical spark length, and for it E has its minimum value.

(3) For still shorter sparks, E increases up to a certain point, and then the law again abruptly changes, and for extremely short sparks of microscopic length, the expression becomes $E = ml$, where m is a constant.[1]

[1] The peculiar properties of microscopic spark gaps have been applied recently by S. G. Brown in the construction of a very remarkable telephone relay.

(4) The minimum sparking potential is the same for the same gas at all pressures, but the corresponding sparking distance is nearly inversely proportional to the pressure, and hence arises the greatly increased length of spark in vacuum tubes.

(5) For extremely low pressures, however, this relation ceases to hold good, and the discharge tube finally becomes non-conducting.

Discharge in Rarefied Gases.—The phenomena met with in gases under reduced pressure are usually demonstrated by means of exhausted glass tubes, of extremely varied shapes and sizes, provided with metal electrodes (generally of aluminium, because this metal disintegrates least under the action of the discharge). The electrodes are joined to platinum wires sealed through the glass. Such a tube, suitable for examining the effects that occur in the initial stages of an exhaustion, is shown in Fig. 341.

If the tube, before exhaustion, is connected to the secondary of an induction coil in action, no effect is produced until the pressure falls to about half an inch of mercury, and then a faint glow becomes visible near the terminals. As exhaustion proceeds, this luminosity

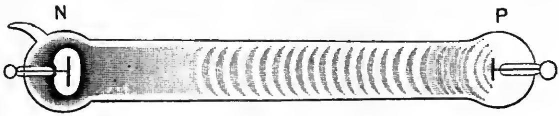


FIG. 341.

gradually increases, and long thin undulatory threads of light extend from terminal to terminal, until gradually expanding and combining, they form a straight luminous column—red in the case of air—which begins at the anode and reaches to within a short distance of the cathode. This is known as *the positive column.* The cathode is surrounded by a very striking bluish glow—known as *the negative glow* or *the cathode glow*—separated from the positive column by a comparatively non-luminous space, known as the *Faraday dark space.* With increasing exhaustion, this space gradually lengthens, the negative glow increases in size and brilliancy up to a certain point, and the positive column begins to split up into thin distinct discs or slices, known as *striæ* (always having their concave sides towards the anode) which gradually increase in size and move further apart. Meanwhile, the negative glow has separated from the surface of the cathode, leaving a sharply defined space, known as the *Crookes' dark space.* The colour of the positive column depends on the gas used, and to some extent on the nature of the discharge. In air or nitrogen, it is red; in hydrogen, blue or red; in carbon dioxide, white; but all these colours decrease in brilliancy as the exhaustion proceeds. They are best seen in narrow tubes, such as are used for spectrum analysis.

The distribution of potential along the tube is far from uniform. It behaves as if nearly the whole of the resistance was concentrated in the comparatively small region between the cathode and the negative glow. This "cathode fall of potential" is about 300 volts, and appears to be closely related to the minimum sparking potential already referred to.

With further exhaustion, the tube begins to decrease in brilliancy, both dark spaces increasing in size, the negative glow becoming more nebulous and ill-defined, and the column of striæ receding until, unless the tube is very long, only a few indistinct traces are left near the positive terminal, and these at last disappear.

Meanwhile new phenomena have been gradually coming into existence. When the Crookes' dark space is large enough to reach the walls of the tube, the glass becomes brilliantly luminous near the cathode with a phosphorescent light, which is yellowish-green if the tube is made of German (or soda) glass, and blue if made of English (or lead) glass.

At an exhaustion of something like $\frac{1}{10^6}$ of an atmosphere, the Crookes' dark space practically fills the whole of a tube of moderate size, and within this dark space, substances like lime, chalk, alumina, diamond, ruby, and many salts and minerals shine with a phosphorescent glow of various colours, the emitted light in certain cases giving a characteristic spectrum.

If the exhaustion is still continued, the discharge has an increasing difficulty in passing, until at last the tube refuses to conduct.

Cathode Rays.—The phosphorescence produced on the glass near the cathode, at rather high exhaustion, appears to have been first noticed by Plücker, and afterwards studied by Hittorf and Goldstein. The last two observers found that the phosphorescence was due to something emitted from the cathode, which could be stopped by obstacles. They attributed the effect to some peculiar kind of wave motion emanating from the cathode, which Goldstein termed *kathodenstrahlen* or *cathode rays*, and the term still survives, although the implied meaning has been modified. The subject was then very thoroughly investigated by Crookes, who not only systematised the results obtained by previous observers, but also largely extended our knowledge concerning the facts and properties of these rays.

We may briefly summarise his results, and those of later workers, as follows:—

(1) The cathode rays always leave the cathode normally to its surface and are negatively electrified.

(2) They behave as if possessed of inertia, being capable of deflecting obstacles and of turning light wheels.

(3) The substances struck by them are heated, and the temperature may be sufficiently high to melt glass or platinum.

(4) A very large number of crystals, minerals, and salts phosphoresce brilliantly under their impact. In the case of salts, this effect is profoundly influenced by the presence of small quantities of an impurity. Specimens prepared with ordinary care may phosphoresce brilliantly, although when extreme pains are taken to ensure purity, they may refuse to phosphoresce at all.

(5) In certain cases, *e.g.* the haloid salts of the alkalies, a change of colour is produced, which is apparently due to a change in chemical composition.

(6) They are deflected both by a magnetic field and by an electric field.

(7) Their production and properties are quite independent of the nature of the gas used in the tube.

(8) A substance struck by them emits an entirely new kind of radiation, generally known as the X-rays. This effect was discovered by Röntgen in 1895, many years after the discovery of the preceding properties.

Nature of Cathode Rays.—Properties (2) and (5) are especially suggestive. They are not of the kind found to be associated with wave motion, but are exactly such as might be expected to belong to a stream of electrified particles projected from the cathode. Crookes strongly advocated the latter view, and as the state of the particles could not well be defined as solid, liquid, or gaseous, he used the term *radiant matter* to denote the peculiar conditions holding good in the cathode stream. On the other hand, the continental school of physicists almost universally supported Goldstein's theory of wave motion, and the question remained undecided for many years. The remarkable independence of the nature of the gas, mentioned in result (7), was particularly difficult to account for on the particle theory, for one would naturally expect to find some difference in the behaviour of atoms of varying mass. Certain experiments by Lenard, which showed that the cathode stream could pass through a very thin aluminium window into the open air, were also regarded as strongly supporting the hypothesis of wave motion.

Magnetic Effect of a Moving Charge.—According to the ideas of Faraday and Maxwell, a moving charge should be equivalent to a current, and should produce by its motion a moving field. This fact was first demonstrated by Rowland in 1876. By rotating a charged disc with great speed, he obtained a deflection of a compass-needle, similar to that produced by a current moving in a circular path. In such an experiment, various disturbing influences have to be eliminated, and in recent years its accuracy has been questioned. Further investigation, however, has completely established its validity.

It follows that a stream of electrified particles moving from the cathode should behave like a flexible current and should be acted upon by a magnetic field exactly like any other current, *i.e.* the particles should move at right angles to the field in order to obtain the maximum effect, and should then be acted upon by a force at right angles to the field and *to their own direction* *of motion.* The latter condition implies that the actual direction of the force changes as the deflection changes, being always directed to the centre of curvature of the path. Hence, that path is approximately circular for *small* deflections.

The nature of this effect has been illustrated by means of Experiment 148, p. 243, in which a flexible conductor carrying a current was found to wrap itself round a bar magnet, *i.e.* round the magnetic field inside the magnet. Similarly, an electrified particle, moving at right angles to a magnetic field, tends to wrap itself round the lines of force, and its path may become a portion of a circle or a spiral according to circumstances.

Velocity of Cathode Rays.—The elementary theory of the subject may be outlined as follows:—

Let a mass, m, carrying a charge, e, move with a velocity, v, at right angles to the lines of force in a magnetic field of strength B. Then the quantity e is conveyed through a distance l in t seconds, where $l=vt$. Now, by definition, the equivalent current strength, i, is given by $i=\frac{Q}{t}=\frac{e}{t}=\frac{ev}{l}$.

$$\text{Again,}\quad \text{Force}=B\times i\times l \quad \text{(see p. 373)}$$

$$\therefore\quad \text{Force}=B\times\frac{ev}{l}\times l=B.e.v \text{ dynes.} \qquad (1)$$

Let r be the radius of curvature of the path at any instant, then the effect of this force will be to produce an acceleration, a, directed towards the centre.

Now we know by ordinary mechanics that for motion in a circle

$$a=\frac{v^2}{r}$$

$$\text{and}\quad \text{force}=\text{mass}\times\text{acceleration}$$

$$\therefore\quad B.e.v=m\times\frac{v^2}{r}$$

$$\text{or}\quad \frac{B}{v}\cdot\frac{e}{m}=\frac{1}{r} \qquad (2)$$

This equation contains three unknown quantities, e, v, and m (B and r being measurable experimentally). Various devices have been employed to obtain other relations between these quantities, but here it will be sufficient to indicate the principle of what is probably the most satisfactory method.

Action of an Electrostatic Field.—Let us suppose that the moving particle is subjected to the action of an electric field, the direction of motion being at right angles to the field. The particle then experiences an electrostatic force tending to move it *along* the field. This force differs from the preceding one in several respects: (1) its direction is invariable, and is independent of the direction of the moving particle; (2) it does not depend upon the velocity of the particle, being merely $U\times e_s$, where U is the electric force on unit charge, and e_s the strength of the charge in *static units.* As a result of (1), the path is not circular or spiral around the lines of force. In fact, it resembles that of a body moving horizontally with uniform velocity under the influence of gravity, in both cases the path being parabolic. But if we restrict the argument to quite small curvatures, no serious error will be made if we treat the path as a circular arc.

Then we obtain, by a repetition of the former argument,

$$\text{force}=\text{mass}\times\text{acceleration}$$

$$Ue_s=m\times\frac{v^2}{r}$$

$$\text{or}\quad \frac{U}{v^2}\cdot\frac{e_s}{m}=\frac{1}{r} \qquad (3)$$

This deflection, due to an electric field, is much more difficult to obtain experimentally than the magnetic deflection, for the magnetic lines freely penetrate the tube, and are thus readily brought to bear upon the cathode stream, whereas the electric lines are apt to be diverted or partially cut off on account of the residual gas acting as a semi-conductor. Sir J. J. Thomson succeeded, where other experimenters had failed, by using an extremely high vacuum. His method, carried out in 1897, consisted in subjecting a narrow pencil of the cathode stream to the simultaneous action of the two kinds of field, so arranged that the deflection produced by the one tended to neutralise that produced by the other.

For balance, *i.e.* no deflection, we have

$$\frac{B.e \text{ (magnetic)}}{v.m} = \frac{Ue \text{ (static)}}{mv^2}$$

$$\text{from which} \quad v = \frac{U.e \text{ (static)}}{B.e \text{ (magnetic)}}$$

In this equation the *same* quantity of charge must be expressed in static units in the numerator and in magnetic units in the denominator. Now the static unit of quantity is very much the smaller of the two units, and hence the numerical value of the charge is greater in the numerator than in the denominator. Again, on p. 581 it is shown that 3×10^{10} static units are equal to 1 magnetic unit of quantity; and hence, in the above case,

$$e \text{ (static)} = e \text{ (magnetic)} \times 3 \times 10^{10}.$$

The final result is therefore

$$v = \frac{U \times 3 \times 10^{10}}{B} = \frac{U}{B} \times 3 \times 10^{10} \text{ centimetres per second.} \qquad (4)$$

Knowing v, the ratio $\frac{e}{m}$ can be found from the curvature produced when one field acts alone.

The results obtained by Sir J. J. Thomson were

$$v = \text{from } 2{\cdot}8 \text{ to } 3{\cdot}6 \times 10^9 \text{ centimetres per second;}$$

$$\frac{e}{m} = 7{\cdot}7 \times 10^6 \text{ (where } e \text{ is in magnetic units)}$$ [1]

and these values were found to be independent of the nature of the gas.

Hence, the velocity of the cathode particles is about $\frac{1}{10}$th the velocity of light—enormously greater than any other velocity known at the time.

Value of the Ratio of Charge to Mass.—The value of the ratio $\frac{e}{m}$ can be determined by several quite independent methods. For instance, it appears in the theory of the Zeeman effect (p. 499), and it can also be measured by experiments on radio-active bodies, which are essentially of the same kind as those just described. All methods agree in assigning to $\frac{e}{m}$ a value of about 10^7 (e being in magnetic units).

Now, in electrolysis, we are also dealing with charged particles, but the ratio $\frac{e}{m}$ is not constant. For example, 1 gram of hydrogen carries 96,000 coulombs, or 9600 magnetic units,

$$\therefore \text{ for hydrogen } \frac{e}{m} = \frac{9600}{1} = 10^4 \text{ in round figures.}$$

---

[1] Later experiments have given

$$\frac{e}{m} = 1{\cdot}77 \times 10^7 \text{ magnetic units.}$$

Again, 1 gram of sodium carries $\frac{96,000}{23}$ coulombs,

$$\therefore \quad \text{for sodium } \frac{e}{m} = \frac{9600}{23} \text{ or } \frac{10^4}{23}$$

The greatest value is evidently obtained in the case of hydrogen, but even this is very much less than 10^7.

Hence, if the particles which constitute the cathode rays are as large as the atoms of hydrogen, the charge they carry must be much larger than the charge carried by those atoms during electrolysis. On the other hand, the fact that *all* monad atoms, during electrolysis, carry the *same* charge, is strong presumptive evidence that some natural electrical unit is involved. But, if a particle in the cathode stream be assumed to carry this typical monad charge, its mass must be much smaller than that of the hydrogen atom—a somewhat startling conclusion.

The question could only be decided by measuring either e or m independently, and this was eventually accomplished by Sir J. J. Thomson in 1898. Space does not permit us to give the details of this extremely difficult and brilliant research. His results, since confirmed in various ways, show that the charge carried by each particle is really identical with that carried by the monad atom in electrolysis, its value being, as already stated on p. 328, about $1{\cdot}57 \times 10^{-20}$ magnetic units, or $4{\cdot}68 \times 10^{-10}$ static units.

Hence, if $\frac{e}{m} = 1{\cdot}77 \times 10^7$ for cathode particle,

and $\frac{e}{m_h} = 10^4$ in electrolyis, where m_h is the mass of the hydrogen atom,

$$\frac{m}{m_h} = \frac{10^4}{1{\cdot}7 \times 10^7} = \frac{1}{1770}$$

i.e. the mass of the cathode ray particle is about $\frac{1}{1700}$ of the mass of the hydrogen atom.

Electrons.—It appears, therefore, that in the cathode stream we are not dealing with ordinary matter. Moreover, the same charge, associated with the same mass, is found to be concerned in many other phenomena, and there is good experimental evidence for believing that the apparent mass is purely electrical in origin. In fact, the cathode rays appear to consist of independent particles of negative electricity, now known as *electrons*.

Negative electricity, therefore, is a real entity, *i.e.* it exists independently of matter, and what we call a current must in all cases involve the flow of electrons round the circuit.

The questions at once arise: What is the nature of positive electricity? Do positive electrons also exist, or does a positive charge consist of ordinary atoms minus electrons? As yet it is impossible to give a definite answer, and until that can be done many other extremely important questions must remain in abeyance.

Positively charged particles can certainly be obtained in many different ways, and the ratio $\frac{e}{m}$ has often been determined for them. This value always approximates to 10^4, a result which indicates that either the charge is smaller than that met with in electrolysis—an improbable supposition—or else that the particles themselves are of atomic dimensions. There is no experimental evidence, which even suggests the existence of positively charged bodies as small as the negative electrons.

Canal Rays.—This term is due to Goldstein, who discovered that when holes were made in the cathode, luminous beams were formed extending from the holes *on the side remote from the anode.* These were ultimately found to consist chiefly of positively charged particles, for which $\frac{e}{m} = 10^4$, moving from the cathode with a velocity of about 2×10^8 centimetres per second.

During recent years, these rays have been very carefully studied by Sir J. J. Thomson, and his work, although yet unfinished, shows that several kinds of positively charged particles exist in vacuum tubes. Some of these possess $\frac{e}{m}$ ratios corresponding to the kind of ions actually present, and, therefore, depending upon the gas used in the tube, but *there are also others for which* $\frac{e}{m} = 10^4$, *and which are quite independent of the nature of the gas.*

This last result is extremely suggestive, and at the time of writing it seems possible, but by no means certain, that positively charged bodies may exist, which are not merely electrified particles of ordinary matter. As to their actual nature, nothing can be surmised at present. It may, however, be remarked, that positively charged particles are thrown off by radium (α particles). These, also, have the ratio $\frac{e}{m} = 10^4$ (nearly), and have proved to be atoms of *helium.*

At the same time, there are no reasons for thinking that these are identical with the bodies found in the canal rays.

Röntgen or X-Rays.—Röntgen's epoch-making discovery deserves special notice. In 1895, he discovered that any substance struck by the cathode rays—in the first instance, the glass walls of a highly exhausted tube—gave off a totally new kind of radiation, invisible to the eye, but capable of affecting photographic plates and of exciting phosphorescence in certain bodies, of which the various platinocyanides, and scheelite (native calcium tungstate) are among the most important. The new radiation was especially remarkable for its power of penetrating ordinary matter more or less readily; the relative transparency of different materials being roughly proportional to their density, but being otherwise quite independent of their physical nature. It could not be refracted, it did not yield interference phenomena, and it was incapable of regular reflection. Neither was it deflected by either a magnetic or an electric field.

Its nature has given rise to much discussion, and although it is generally regarded as being some form of wave motion in the ether, it is by no means certain that this is the case, in fact recent research appears to disprove it.

The best form of tube for producing the X-rays is one in which the cathode stream is brought to a point focus by means of a cup-shaped cathode, and then made to impinge on the anode, which is a metal plate inclined to their direction at an angle of 45°. The X-rays then emanate from practically a point source, and consequently yield sharp shadows on photographic plates or on phosphorescent screens. Their application in surgery is too well known to require attention here.

Ionising Power of X-Rays.—Perhaps the most important and characteristic property of the Röntgen rays is their power of producing temporary conductivity in gases; a property most readily demonstrated by the fact that an electroscope will not retain a charge near a tube in action.

According to modern ideas, such conductivity depends upon the presence of charged particles or ions, and is essentially of the same character as that met with in electrolysis.

Under ordinary circumstances, gases contain very few free ions, and hence are excellent non-conductors, but they may be "ionised" by various agencies (see p. 494), and they then acquire a relatively feeble but true conductivity. Until the discovery of the electron, there was some difficulty in conceiving the nature of the process, for, although ordinary gaseous molecules might be regarded as being resolved into their constituent atoms—charged positively and negatively respectively—this explanation could not be applied to monatomic gases, which are also readily capable of being ionised. If, however, we regard ionisation as due to the detachment of one or more electrons, the difficulty largely disappears.

Suppose that two metal plates are placed a short distance apart and connected in series with a battery and a very sensitive galvanometer.[1] Let the radiation from an X-ray tube traverse the space between the plates. If now a gradually increasing P.D. be applied to the plates, the current at first obeys Ohm's law, *i.e.* is proportional to the P.D., but its rate of increase gradually diminishes, and after a time the current remains constant in strength, although the P.D. is still increasing. In this stage, the current is said to be "saturated." When the applied P.D. reaches a very high value (which for air at ordinary pressure is about 30,000 volts per centimetre), another stage is reached in which the current somewhat suddenly begins to increase again with very great rapidity.

[1] Ordinary galvanometers are scarcely sensitive enough for this purpose, and another method of measuring these extremely small currents is described later (see p. 491).

Again, when the potential difference is sufficient to produce saturation, the current (although the P.D. remains constant) is increased by increasing the distance between the plates—a behaviour exactly opposite to that shown by liquid or metallic conductors.

These results are easily explained, if we suppose that the X-rays are producing a certain number of ions in the gas, which tend to recombine. The number of ions, therefore, increases until a state of balance is reached, exactly equal quantities then being formed and being recombined per second.

If now a P.D. be applied to the gas, oppositely-directed processions of charged particles will move towards the plates, the actions going on being similar to those already explained in the case of electrolysis. Increasing the P.D. will increase the number of ions arriving at each plate per second, and as long as there is an abundance of such ions available, Ohm's law is satisfied. It is, however, evident that no more ions can arrive at a plate per second than are produced by the ionising agency in the same time, and when this state of affairs is reached, further increases in the applied P.D. cannot increase the current, and we get the stage of saturation. If, however, the P.D. be raised to some high value, depending on the pressure, the velocities acquired by the moving ions may become sufficiently great to enable them to produce fresh ions by collisions with neutral atoms or molecules, and then the current rapidly increases. (An effect of this kind is probably operative in ordinary sparks in air. There are always a few ions present, and when the electric field becomes great enough, they acquire sufficient velocity to produce new ions by collision, the process being then rapidly cumulative and ending in the spark.)

It appears, therefore, that the magnitude of the current depends not only upon the applied P.D., but also upon the number of available ions present. Now, increasing the distance between the plates increases the volume of gas exposed to the ionising agency, which, therefore, increases also the number of ions produced per second, and consequently the saturation current.

The absence of the saturation state in the case of conduction in solids and electrolytes is an indication that relatively inexhaustible supplies of ions are present.

Measurement of Extremely Small Currents.—A suspended coil galvanometer of ordinary sensitiveness will easily measure currents of the order of 10^{-6} ampere, but it is often required to measure much smaller currents in experiments of the kind described above. For this purpose, a quadrant electrometer, or a gold-leaf electroscope of suitable design, is used—previously calibrated by means of a standard cell, so that the P.D. corresponding to any given deflection is known. The apparatus is then arranged as shown in Fig. 342. A battery, ZC, is connected *in series* with the plates

A and B and a quadrant electrometer, E, one terminal of the battery and one of the electrometer being earthed. Hence, there is a P.D. between A and B, but the plate B and both sets of quadrants are initially at zero potential, and there is no deflection. An ionising agency, such as an X-ray tube, is then made to act upon the gas between A and B, and as a result the potential of B gradually rises and a deflection is produced.

Let this deflection increase from d to d_1 divisions in time t seconds. Then $d_1 - d$ measures the increase of P.D. between the quadrants of the electrometer, and its value is known from the

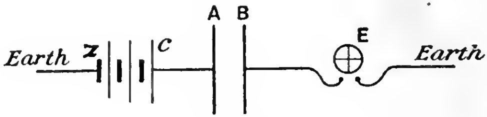


FIG. 342.

previous calibration. Let this value be e, and let K be the capacity of the electrometer (also previously determined by experiment).

Then, the quantity Q passing into the electrometer (and also between A and B) is given by

$$Q = eK.$$

Let i be the *average* current during the time t (or the true current if it happens to increase uniformly with time),

$$\begin{aligned} \text{then} \quad Q &= it \\ \textit{i.e.} \quad it &= eK \end{aligned} \qquad \text{or} \qquad i = \frac{eK}{t}$$

From this equation we see that the smaller the value of K, the smaller will be the value of i corresponding to a given change in deflection. This shows us why gold-leaf electroscopes are even superior to the electrometer for the purpose of measuring small currents. Suitably designed, an electroscope can be given a much smaller capacity than any electrometer, and will, therefore, indicate correspondingly smaller currents.

Nature of Discharge in Sparks and in Vacuum Tubes.—We are now in a position to indicate the general nature of such discharges, regarding them as dependent upon the presence of free ions. There is good reason to believe that gases at ordinary pressure contain a few ions, and in a spark-gap these tend to move along the electric field, acquiring velocity exactly as a falling body does in a gravitational field. If this velocity reaches a sufficiently high value, they may produce other ions by impact with neutral particles, and then, as already stated, the process is rapidly cumulative and ends in a brush or spark. The velocity acquired depends partly upon the P.D. per unit of length, and partly upon the length of path available, although, as this path is small at ordinary pressures, a very high voltage is necessary to initiate ionisation by shock. At low pressures, the mean free path of an ion is much longer, and it can acquire sufficient velocity under the action of smaller voltages. Again, the velocity acquired depends upon the mass of the particle, and hence, if electrons are present, they are much more effective in producing ionisation than positive ions, for, owing to their small mass, they can acquire the necessary velocity in moving through a smaller difference of potential. In a highly exhausted tube, the potential gradient is, as already stated, particularly steep between the cathode and the negative glow, and there the greatest ionic velocities are acquired. In all probability, it is the impact of positive ions which facilitates the escape of electrons from the cathode, and when there is an aperture in it, some of these positive ions may pass through to form the canal rays. The liberated electrons similarly acquire their enormous velocity in moving along the potential gradient away from the cathode, and become active in ionising the neutral particles in the cathode glow. When, however, they have lost some of their velocity by collisions, they also lose their ionising power, and it is necessary for them to travel a certain distance in order to again acquire sufficient velocity for that purpose. Hence, under certain conditions, stratifications are produced, the dark spaces between them representing convective motion, and the striæ themselves the places where ionisation is going on by collisions.

In the electric arc the action is somewhat different. In this case, the voltage is much too low to produce ions by impact, and they must be supplied by the electrodes themselves. Consequently, the formation of an arc depends very largely upon the nature of the electrodes (see p. 553).

Certain substances, chiefly metallic oxides (of which lime is a good example), when heated possess in a conspicuous degree the property of readily emitting electrons. If a metallic cathode in a highly exhausted tube is coated with lime, and arranged in the form of a loop so that it can be readily heated by an independent current, the discharge passes readily at quite low voltages. This discovery is due to Wehnelt.

Metallic Conduction.—Good conductors must be regarded as bodies containing enormous numbers of free electrons.

A piece of metal is a region in which they can move about freely, although they cannot readily pass through its surface into the surrounding space. If their number were not exceedingly great, the phenomenon of saturation would occur as it does in gases, and Ohm's law would fail for very large current densities.[1]

[1] This remark applies also to electrolytes. It has, however, been shown recently that certain very badly conducting solutions (*e.g.* lead oleate in hexane or light petroleum) *do* give a saturation current.

Experimental evidence supporting these views is to be found in the Hall effect (see p. 498).

Sources of Ionisation.—Gases may be ionised, *i.e.* brought into a conducting state, in many ways, of which the most important are—

(1) By raising the temperature to a very high value.

The effect of high temperatures on a gas is somewhat complicated, depending largely upon the proximity of the gas to heated surfaces. Hot metals, in particular, have a strong tendency to give off positively charged particles. In some gases the effect is small; when large, it can always be traced to chemical decomposition. Hence, gases drawn from the proximity of flames or arcs, or from contact with glowing metals, are found to conduct.

(2) By passing X-rays, radium rays, or Lenard rays through the gas.

The effect of radium and other similar bodies is referred to separately. Lenard rays are produced *outside* a vacuum tube when cathode rays are made to impinge upon a small aluminium window. Their influence is confined to the immediate neighbourhood of the window, and they are relatively unimportant as a source of ionisation.

(3) Air is ionised by passing it over phosphorus, or by bubbling it through water.

(4) By ultra-violet light.

Hertz found that the incidence of ultra-violet light on a spark-gap facilitated the passage of a spark. This observation formed the starting-point of a large amount of research. It was discovered that a clean zinc surface charged negatively, rapidly loses its charge when ultra-violet light falls upon it. If uncharged to begin with, it becomes positively charged, *i.e.* it emits electrons. If positively charged, there is no loss. Other metals also show this effect in varying degrees. The more electro-positive elements are the most efficient, the order of efficiency being identical with Volta's electro-chemical series. With copper, iron, lead, &c., the effect is negligible; with magnesium it is stronger than with zinc; sodium and potassium are sensitive to ordinary daylight; whilst rubidium, the most electro-positive of all metals, is sensitive to a glass rod just heated to redness.

(The student may find some difficulty in understanding why we have given the latter statements under the heading "ultra-violet light." The point to be brought out is that waves of short length, *i.e.* ultra-violet, are the most effective. The alkaline metals are, however, sensitive to longer waves, and rubidium is sensitive even to the extreme red rays.)

Becquerel Rays, Radio-active Bodies.—Immediately after Röntgen's discovery, Becquerel made experiments to ascertain whether ordinary phosphorescent bodies emitted penetrating radiations. Amongst other substances he tried uranium salts, which had been made to phosphoresce by exposure to daylight, and found that they gave out a radiation similar to X-rays; further investigation, however, showed that the exposure to sunlight was unnecessary, the action being just as intense when the salts were kept in the dark. Shortly afterwards, Schmidt discovered that thorium possessed similar properties. Crookes then discovered that uranium salts could be separated by ordinary chemical means into two portions—one of which was radio-active and the other was not. He called the active portion Uranium X. Becquerel found that, if these two portions were kept for some months, the non-active part gradually regained its activity, whilst the active part gradually lost it.

M. and Mme. Curie discovered that certain minerals containing uranium were more radio-active than uranium itself, and hence they inferred that other active bodies were present. After some years of laborious research, they isolated two new substances, *polonium* and *radium*, enormously more active than uranium. Debierne discovered yet another intensely active substance associated with thorium in extremely small quantity, which he called *actinium*.

These new substances possess, in a conspicuous degree, the power of ionising surrounding gases. They have been found to emit three distinct kinds of radiation [1]—

(1) Streams of positively charged particles—known as the α-rays—of atomic mass, moving with a velocity something like $\frac{1}{10}$th that of light. These have proved to be atoms of helium.

(2) Streams of negatively electrified particles—known as the β-rays—moving with a velocity approaching that of light itself. These seem to be identical with the electrons present in the cathode rays, although they move with a much greater velocity.

(3) An unelectrified radiation—the γ-rays—which closely resemble X-rays, and may be identical with them.

Apparently these radio-active elements are gradually breaking up into constituents of smaller atomic weight; the disintegration being accompanied with the liberation of relatively enormous amounts of energy. These newly formed bodies, after a life of very varied duration, may again break up; and the final result may be the ordinary stable elements as we know them.

The experiments of Rutherford have shown that the rate of disintegration follows a perfectly definite law, and is unaffected by any change of experimental conditions; it is always proportional to the amount of substance present.

This condition can be stated in a simple form. If M_o is the mass of active substance initially present, and M_t the mass after a time t, then

$$M_t = M_o e^{-at}$$

[1] This term usually implies wave motion, but it is now generally used to denote the emissions from radium and other radio-active bodies.

where e is the base of natural logarithms and a is a constant, whose value depends on the substance, and which can be determined by experiment. For example, the time required for half the substance to break up is given by

$$\tfrac{1}{2}M_o = M_o e^{-at}$$

$$\text{or} \quad e^{at} = 2$$

$$\therefore \quad t = \frac{\log_e 2}{a}$$

It will be noticed that the time is independent of the amount of material present, and hence it has become usual to employ it as a measure of the rate of change. It is often called the "period" of the body. Only products of long period are likely to accumulate naturally in any large quantity.

The following table, corrected to June 1911, by the kindness of Professor Rutherford, represents the state of our knowledge:—

Substance.	Period.	Emitted Radiation.
Uranium	5×10^9 years	(Two) α
Uranium X	23 days	β and γ
Ionium	?	α
Radium	2000 years	α
Radium Emanation	3·85 days	α
Radium A	3 minutes	α
Radium B	26 minutes	β and γ
Radium C	19 minutes	α, β, and γ
(Radio-lead) Radium D	16 years	Slow β
Radium E	5 days	β and γ
(Polonium) Radium F	140 days	α
Thorium	3×10^{10} years	α
Mesothorium (1)	5·5 years	None
Mesothorium (2)	6 hours	β and γ
Radio-thorium	2 years	α
Thorium X	3·6 days	α
Thorium Emanation	54 seconds	α
Thorium A	10·6 hours	Slow β
Thorium B	55 minutes	α
Thorium C	?	α
Thorium D	3·1 minutes	β and γ
Actinium	?	None
Radio-actinium	19·5 days	α
Actinium X	10·5 days	α
Actinium Emanation	3·9 seconds	α
Actinium A	36 minutes	None
Actinium B	2·1 minutes	α
Actinium C	4·7 minutes	β and γ

It will be noticed that the γ-rays are never produced without β-rays, and as they resemble the X-rays and cathode rays respectively, it seems probable that the γ-rays represent a disturbance due to the sudden starting of an electron, just as the X-rays are produced by its sudden stoppage. The various products are all solid except the "emanations." These are true gases, inactive chemically and closely allied to the argon group. They can be liquefied by extreme cold, and appear to possess characteristic spectra, but their life is so short that only the radium emanation has been studied at all carefully, and even that has been done in the face of very great experimental difficulties.

Faraday, Kerr, Hall, and Zeeman Effects.—Certain outlying phenomena of great theoretical importance may conveniently be grouped together. We may summarise the more important of these in historical order as follows:—

(1) **The Faraday Effect.**—In 1845 Faraday discovered that when certain substances were placed in a strong magnetic field, they acquired the power of rotating the plane of polarisation of a beam of plane polarised light, if that beam passed through them in the direction of the lines of magnetic force. If the two directions are not the same, the effect is weaker, becoming zero when the path of the light is at right angles to the field. It is now known that all substances exhibit this effect to some extent, but in varying degrees. It is especially marked in bodies having large refractive indices for light, like Faraday's "heavy glass," in which it was discovered, and carbon disulphide. Non-transparent bodies like the metals can be used only in extremely thin films, and it is noteworthy that an *iron* film, when very strongly magnetised, produces a relatively enormous effect.

The experiment is usually performed by placing the substance between the poles of a powerful electromagnet, the pole pieces being perforated in order to permit the passage of the light. As a rule the plane of polarisation of the beam is rotated in the same direction as that of the current, which produces the field, *e.g.* if we suppose that the field is produced by a solenoid, inside which the substance is placed, then the direction of rotation is usually in the same direction as the current in the solenoid, although there are a number of substances for which the rotation is in the opposite direction. The amount of rotation is directly proportional to the length of path in the field (l) and to the strength of the field (B), and is approximately inversely proportional to the square of the wave length of the light used; hence, if θ be the observed rotation, we may write

$$\theta \propto B \times l.$$

From p. 375 we know that, *in air*, $Bl = 4\pi in$, and therefore we see that the rotation for a given length is proportional to the *magnetic difference of potential* acting on that length. This is known as Verdet's law, and the rotation produced in any given substance by unit magnetic P.D. is known as **Verdet's constant** for that substance. If the light be reflected backwards again through the substance, the rotation is increased, just as though the length of the path had been doubled, *i.e.* the effect is independent of the *direction* in which the light passes along the field—a fact which sharply differentiates the phenomenon from that observed in the case of naturally optically active substances like quartz, sugar solutions, &c., for in these the direction of rotation *does* depend on the direction of the light in the substance, and the reflection of the light back through them would simply annul the rotation altogether.

(2) **Kerr Effects.**—Kerr discovered that when a beam of plane polarised light is reflected from the polished surface of the pole of a strongly excited electromagnet, it becomes elliptically polarised. Apparently, this is a special instance of the Faraday effect, the light penetrating very slightly, but sufficiently to cause its plane of vibration to be affected by the molecular rotations which must be present in magnetised iron.

Another effect discovered by Kerr is of a totally different nature. He found that a transparent dielectric placed in a strong *electrostatic* field becomes doubly refracting. This might be regarded as the natural result of the strain thereby set up in the dielectric, for it is a familiar fact that singly refracting substances, like glass, become doubly refracting when subjected to strains produced by purely mechanical means. But Kerr found that it occurred in *liquids* also, in which mechanical strains cannot exist, and hence it must be regarded as a molecular action of some kind.

(3) **The Hall Effect.**—Let AB, Fig. 343 (taken from Carey Foster and Porter's *Electricity and Magnetism*) represent a very thin plate through which a current is passed from A to B. Let a sensitive galvanometer be connected to the points *a* and *b* at the sides of the strip. If these points are symmetrically placed, they will be at the same potential, and there will be no deflection, a result easily obtained by slightly adjusting the position of the wires. Hall discovered that, if a very strong magnetic field is then produced at right angles to the plane of the strip (*i.e.* in the direction of the arrow), this state of balance is disturbed and the galvanometer is deflected, thus showing that a P.D. has been produced between the points *a* and *b*. The effect is inversely proportional to the thickness of the plate and to the field strength, and is much larger in bismuth than in any other substance. Its *direction* varies—in gold and nickel it is the same as in bismuth, but the deflection is reversed in iron, antimony, cobalt, and tellurium.

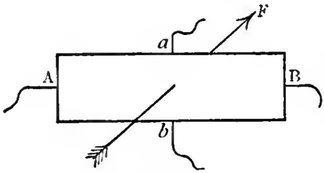


FIG. 343.

The Hall effect has not been observed in liquids, but it is well-marked in gases. Up to a certain point, its explanation is fairly easy if we regard the current in a metal as being due to a stream of negatively charged electrons in one direction and (more doubtfully) a stream of positively charged particles of some kind in the opposite direction. We know that, in the magnetic field, a mechanical force acts on the conductor, tending to drive it sideways (see p. 343), and we must now suppose that this force acts primarily on the electrons, which, whether positive or negative, will be urged in the *same* direction. (For, in the magnetic field, a positively charged particle moving in a certain direction will be urged in the same way as a negatively charged particle travelling in the *opposite* direction). If the carriers of opposite sign behave in exactly the same way, there will be no effect, but if there is any difference of velocity between them, so that there is an excess of one kind at the side (*a* or *b*) of the plate, then there will exist the transverse P.D. discovered by Hall. If the current consisted merely of a stream of electrons, the deflection would in all cases be the same as that observed in bismuth; but the fact that its sign varies in different metals is somewhat puzzling, and might be interpreted as indicating the existence of free positively charged bodies, which in these metals have greater mobility than the electrons. It is extremely improbable that this is the correct explanation, and the question must be regarded as open until the true nature of a positive charge is determined. The absence of the effect in electrolytes is probably due to the fact that the velocities of the oppositely charged ions do not differ sufficiently, for the negative charge is not carried by rapidly moving free electrons, but by relatively massive bodies to which electrons are attached. In gases, the Hall effect is very easily observed, because free electrons are again met with, which move much more rapidly than the positively charged atoms or atomic groups also present.

Various other phenomena, which may be regarded as dependent upon the Hall effect, have been observed and measured in bismuth. Perhaps the most important of these is an increase in the resistance of the metal, a fact that has been applied with considerable success to the measurement of strong magnetic fields. A flat spiral of bismuth, wound non-inductively, is mounted on a suitable holder, and, when placed in the field in question, its resistance is measured in the usual way, a calibration curve having previously been prepared by measurements in fields of known strength. The increase of resistance amounts to about 50 per cent. in a field for which B=10,000 lines per square centimetre.

Other secondary actions are (1) a slight decrease in the heat conductivity of the material, and (2) a transverse difference of temperature between its sides. These temperature effects can be deduced as necessary consequences of the electron hypothesis. In fact, a current of *heat* through the strip is found to act like an electric current. It is deflected by a magnetic field, and the result is the production of an *electrical* P.D. between the sides of the strip, which can be detected by a galvanometer.

(4) **Zeeman Effect.**—Zeeman discovered that if a source of light, giving line-spectra, be placed in a very strong magnetic field and then viewed through a spectroscope of great dispersive power, some of the lines widened out and became resolved into doublets, triplets, and sometimes into more complex systems, although all the lines, even of the same substance, were not affected alike. In the simplest case, when the source was viewed at right angles to the direction of the magnetic field, a line became a triplet, each constituent plane polarised, but the plane of polarisation of the middle one was at right angles to that of the two outer ones. When viewed along the direction of the field, the line became a doublet, with constituents circularly and oppositely polarised. This discovery strongly supports our ideas as to the existence of charged particles, for according to the electro-magnetic theory of light, the source of a light-wave must be an oscillating electric charge, and this is probably an "electron" rotating around some central body as a nucleus. Now, all possible motions of such a particle can be resolved into two opposite circular paths at right angles to the field and a rectilinear path along the field, the magnetic force accelerating one circular component and retarding the other, but leaving the rectilinear component unaffected. Thus the original line gives rise to three, the middle one being of the original frequency and in the original position, and the two outer ones of higher and lower frequency respectively; and evidently, when viewed along the direction of the field, two circularly polarised components should be produced. It is not difficult to calculate the alteration in frequency in terms of the strength of the field, assuming the original vibration to be simple harmonic, and thus by measuring the alteration in frequency by spectroscopic observations, another value of $\frac{e}{m}$ is obtained, which agrees very nearly with the results derived from quite different methods.

Until this discovery of Zeeman in 1896, the only known relation between magnetism and light was Faraday's rotation of the plane of polarisation in a magnetic field. In the latter, the effect depends upon the action of the field on light-vibrations previously produced, *i.e.* it produces a difference in the velocity of the two circularly polarised constituents of a plane polarised beam, but does not alter their frequencies. In the former, the magnetic field acts directly upon the vibrating particles which are the source of light-waves, and by influencing their velocities causes them to emit light of different frequencies.

Exercise XXIII

1. Describe the mode of production of Röntgen and Lénard rays, and discuss the possible origin of the phenomenon. (B. of E., Stage III., 1909.)

2. Describe the arrangements for generating Röntgen rays, and discuss briefly their character and the question of how they are produced.
(Lond. Univ. Inter., B.Sc., Honours, 1904.)

3. An electrified particle traverses an electric field, the intensity of the field being normal to the original direction of motion of the particle. Find an expression for the deflection of the particle. What other experiments must be made in order to determine the ratio of the mass of the particle to its electric charge? (Lond. Univ. B.Sc., 1906.)

4. What is meant by a saturation current? Explain the experimental details you would adopt to investigate the relation between the electromotive force and current for a gas ionised by Röntgen rays.
(Lond. Univ. B.Sc., Honours, Internal, 1909.)

5. Give a short account of the effects of transmitting light through a magnetised material, and of reflecting it from the surface of a magnetised body. (Lond. Univ. B.Sc., Honours, Internal, 1909.)

6. An electron of mass 10^{-25} grams rotates 5×10^{14} times a second round an atom supposed to be at rest; find the force with which the electron must be attracted to the atom in terms of the radius of its orbit. If the atom has a positive charge of 10^{-10} electrostatic units and the electron an equal negative charge, calculate the radius of the orbit, assuming that the attractive force is purely the force between the charges.
(Lond. Univ. B.Sc., Internal, 1903.)

CHAPTER XXX

TELEGRAPHY AND TELEPHONY

One of the most important applications of electro-magnetism is the electric telegraph, by means of which messages are transmitted from one place to another at a considerable distance apart. The essential parts of most telegraphic systems are :—

(1) Line wires connecting the two stations.
(2) Batteries for generating the current.
(3) Instruments for sending and receiving the signals.

Lines.—In the earlier days of the electric telegraph, two wires were employed, one to carry the current from the sending station to the receiving station, and the other to convey it back to the battery. If was soon discovered, however, that if each end of the first wire was connected to a large plate sunk in the ground, the earth itself would act as the return wire. The conductivity of the earth appears to be chiefly due to the moisture contained in it. Although its *specific* resistance is high, this is counterbalanced by the enormous cross-section available; indeed, the resistance between two earth plates rarely exceeds a few ohms, no matter how far they are apart.

Lines may be divided into three classes: (*a*) overhead or aerial; (*b*) underground; (*c*) submarine.

Overhead lines are constructed chiefly of iron, but the use of copper for long and important circuits is increasing. The poles are generally made of fir or larch, treated with creosote or with a solution of copper sulphate to prevent decay. As wood is to some extent a conductor—especially when wet with rain—the wires are supported on glass, glazed earthenware, or porcelain *insulators*, to stop, as far as possible, the current leaking to the earth. A modern form of insulator is shown in Fig. 344 (taken from Slingo and Brooker's *Electrical Engineering*). As most of the leakage takes place, not through the substance of the insulator, but along a film of moisture on its surface, it will be seen that the insulator is designed to expose as long a surface as possible between the wire and the

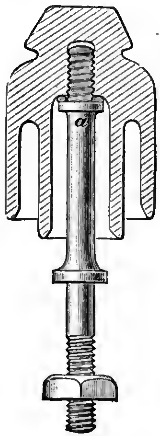


Fig. 344.

supporting bolt, *a*, and that, moreover, part of the surface is sheltered from the rain.

Underground wires are generally made in the form of a cable, containing a large number of copper wires, each usually about 40 mils[1] in diameter, and insulated from each other by means of a gutta-percha covering. The cable, generally placed in cast-iron pipes, is laid a foot or so below the surface of the ground. During recent years, "dry-core" cables have been much used. The separate wires of such cables are insulated from each other by wrappings of manilla paper, the whole being bound together with cotton and enclosed within a sheathing of lead—the external appearance being that of a leaden pipe. (All paper-insulated cables are sheathed in lead to exclude moisture, as paper is very hygroscopic.) These cables are also laid in cast-iron pipes or in earthenware conduits. The chief advantages are (1) a lower electrostatic capacity; (2) economy of space, owing to the fact that the wires are placed

Fig. 345.

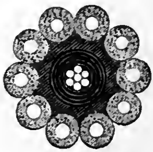


Fig. 346.

nearer together than in the guttapercha type; and (3) material decrease in cost.

Submarine cables, in which great strength and perfect insulation are necessary, vary considerably in detail, although the principle involved is the same in all. Fig. 346 represents a cross section of the longitudinal piece of an Atlantic cable shown in Fig. 345, which consists of a *core* of seven strands of copper wire, each about $\frac{1}{30}$ of an inch in diameter (forming one conductor), insulated by being covered with alternate layers of guttapercha, and of a mixture of tar, guttapercha, and resin. This is covered with hemp, and then wrapped round with a coiled sheath of steel wires to take the tensile strain and to protect the cable from injury.

Batteries.—The cells commonly used are the Daniell, the Leclanché, the chromic acid, and, for some special purposes, the so-called dry cells. In the larger offices, accumulators are generally used. The voltages required for different circuits vary from 5 to 180, and are obtained by joining up a sufficient number of cells in series.

[1] A "mil" is $\frac{1}{1000}$ inch.

Sending and Receiving Instruments.—The simplest forms of sending and receiving instruments are the *single current key* and the *sounder* respectively.

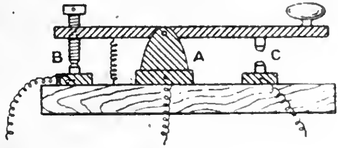


FIG. 347.

The key consists of a brass lever with its fulcrum at A (Fig. 347), and normally held on the back contact stop at B by means of a spiral spring. On depressing the ebonite knob, the front of the lever makes contact with the contact stop at C. The lever moves through a distance of $\frac{1}{30}$ inch only. Its general appearance is shown in Fig. 348.[1]

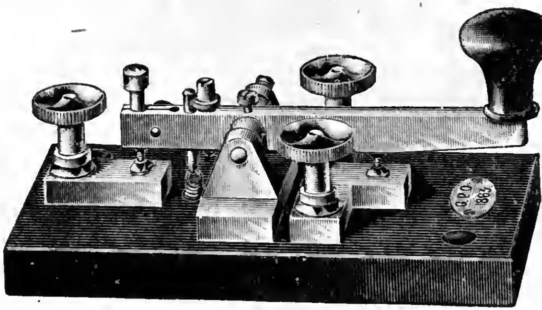


FIG. 348.

The Sounder (Fig. 349) consists essentially of a double core electromagnet, the upper poles of which lie immediately below a soft iron armature A, fixed to a brass bar. One end of the bar is supported on bearings at B, while the up-and-down motion of its free end is limited by the screws at C and D. Normally the bar is held against the upper screw C, by tension of a spring (shown in diagrammatic form at S); but when a current flows through the coils, the cores are magnetised, and therefore attract the armature, so that the screw D strikes the "anvil" N, and emits a sharp metallic click, which denotes the commencement of a signal. When the current ceases, the cores are demagnetised,

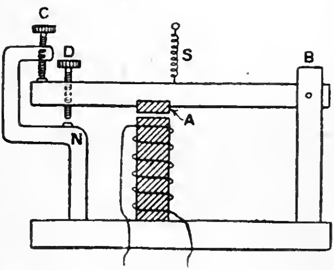


FIG. 349.

[1] Many figures in this chapter (*e.g.* 348, 350, 352, 356, 362, 368, 369, 371, 372, 374) are taken by permission from Preece and Sivewright's *Telegraphy*, published by Messrs. Longmans.

and the spring S lifts the bar against the screw C—another sharp click announcing the end of the signal. Fig. 349 illustrates the principle of the instrument; the actual form being shown in Fig. 350.

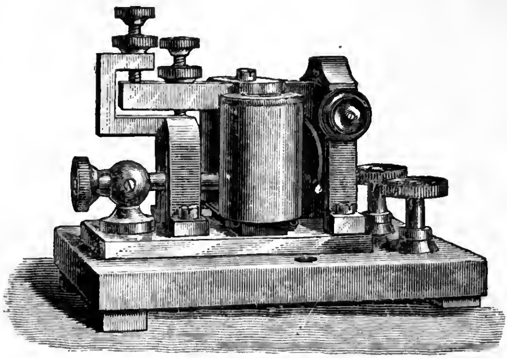


FIG. 350.

The Simple Telegraphic Circuit (Fig. 351) shows diagrammatically the scheme of connections. D and D′ are the keys; A and B the batteries; E, E, the earth plates; the galvanometers are supposed

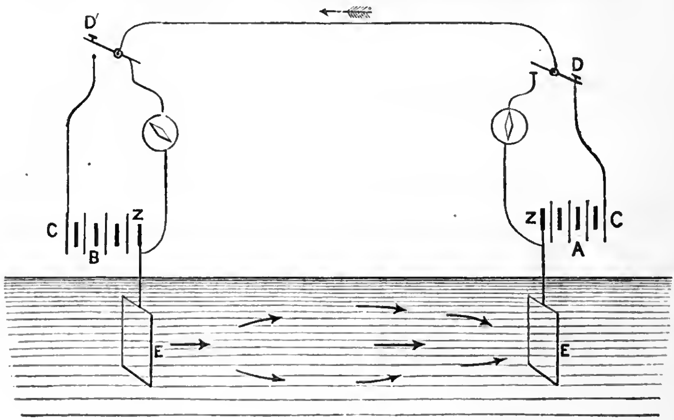


FIG. 351.

to represent any form of receiving instrument. We may mention that a special kind of galvanometer, with a vertical needle, was used before the advent of the sounder; and in fact is still used, on account of its simplicity, on railways. When a sounder takes the place of the galvanometer shown in the diagram, it is usual to insert a simple form of galvanometer at the ends of the line, so that the operator, when "sending," may know that his apparatus is working properly. (These galvanometers are shown at G and G′ in Fig. 352). On depressing the key, say D, at one station, the sounder there is disconnected, and the battery becomes connected to the line. The current flows along the line, through the key D′ (by means of its back contact), to the sounder, thence to earth, and so back to the battery at the sending station. As long as the key is depressed, the current flows through the sounder—thus, by depressing the key for short or for longer

	Printing.	Single Needle.		Printing.	Single Needle.
A	- —	\/	N	— -	/\
B	— - - -	/\\\	O	— — —	///
C	— - — -	/\/\	P	- — — -	\//\
D	— - -	/\\	Q	— — - —	//\/
E	-	\	R	- — -	\/\
F	- - — -	\\/\	S	- - -	\\\
G	— — -	//\	T	—	/
H	- - - -	\\\\	U	- - —	\\/
I	- -	\\	V	- - - —	\\\/
J	- — — —	\///	W	- — —	\//
K	— - —	/\/	X	— - - —	/\\/
L	- — - -	\/\\	Y	— - — —	/\//
M	— —	//	Z	— — - -	//\\

periods, the *dots* and *dashes* of the Morse code, shown above, are signalled. An expert operator can send 25 to 30 words per minute. In the table, the column for "single needle" refers to the galvanometer with a single vertical needle, mentioned above, and indicates the direction and duration of the deflection.

The direct-sounder circuit just described should be carefully studied, as it illustrates very clearly the main principles underlying the more complicated systems.

It can be used, however, only on circuits of a few miles in length. On longer circuits, difficulties arise from many causes—defective insulation being, perhaps, the chief. A portion of the current leaks down each pole to earth, especially in wet weather; and as a hundred-mile line requires some 2500 poles, the total leakage becomes so great, that a current does not reach the distant sounder of sufficient strength to produce loud, clear, and reliable signals. The higher resistance of the longer wire, also, of course, weakens the current. This would be remedied by increasing the number of cells, except for the fact that such increase of E.M.F. also increases the leakage. The method adopted in practice is to employ an additional sensitive instrument called a *relay*, which actuates the sounder.

The Relay is practically a very delicate form of sounder, with an electromagnet containing many turns of fine wire, and attracting a very light armature, which plays between contact points only $\frac{1}{100}$ inch apart. This armature closes and opens a circuit containing the sounder and a local battery of a few cells. The weak line

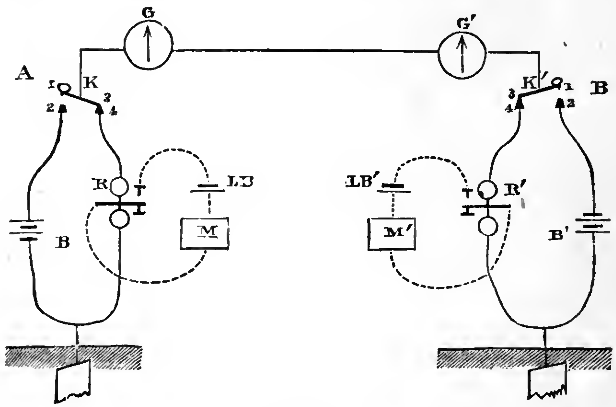


FIG. 352.

currents need only be of sufficient strength to move the relay armature, the energy required to produce the loud beat of the heavy sounder being furnished by the local battery. The connections are shown in Fig. 352, where RR′ are relays; LB, LB′, local batteries; MM′, the sounders (or any variety of receiving instrument); GG′, the line galvanometers already referred to. Notice that the relays take the place occupied (in Fig. 351) by the sounders.

There are many forms of relay, but the one almost universally used at the present time is the Post Office Standard Relay, which is described on p. 508.

Difficulties with Long Lines.—The introduction of the relay thus overcame the difficulty of dealing with weakened currents, but, when it became necessary to employ still longer lines, other difficulties, which we must now consider, were produced. After the working current has ceased, some means of bringing the relay armature back to its normal position must be provided. This could be done by means of a spiral spring (as in the case of the sounder) or by the attraction of a magnet. In either case, this controlling force must be adjustable to suit stronger or weaker line currents. If the relay is delicately adjusted to respond to very weak currents (by moving the armature nearer to the poles of the electromagnet, or by reducing the tension of the spring), the residual magnetism of the cores may cause the armature to stick during the interval between two signals, so that, say, two dots become one dash; if, on the other hand, this adjustment is reversed, the currents may be too weak to attract the armature, so that dots may be lost although dashes are recorded. Again, when the relay is in its most sensitive position, stray currents from other wires on the same poles may cause false signals to be received. The chief obstacle to rapid working on long lines is, however, due to their *electrostatic capacity*. Every telegraph circuit may be regarded as a condenser—the line wire forming one "plate," and the earth the other, the air between them being the dielectric. As we have shown in Chapter V., the capacity of a condenser is directly proportional to the area of the plates, and inversely proportional to the distance between them, so that, although the "plates" are usually some 15 to 20 feet apart, this is counterbalanced by their great length, which may be 100 miles or more. When a signal is being transmitted, the "plates" are joined together through the receiving apparatus at the distant end, while the poles of the battery are joined, one to each, at the sending station. The first portion of the signal is, therefore, absorbed as a static charge, the nearer parts of the line being first raised to the potential of the sending battery, and the distant parts later; the maximum current not flowing through the relay until this effect has been produced. The nett result is that the first signal is delayed. If now the key be raised, the battery is disconnected, both ends of the line becoming connected to earth. The static charge drains out at each end, thus prolonging the signal on the relay. If another signal is sent into the line before this discharge is completed, the returning discharge neutralises the first portion of this second signal, and the dash may become a dot, or a dot may be completely lost. In order that undistorted signals may be transmitted, sufficient time must be allowed to elapse between two successive signals, so that the accumulated charge may leak out. The longer the line, the greater its capacity and its consequent charge, which, of course, makes the time required for discharge longer, and, therefore, the speed of signalling slower.

The above statements must be regarded only as a very elementary explanation. The rate of working depends upon the capacity, resistance, inductance, and leakance[1] of the circuit, the four quantities being connected by very complicated formulæ.

All the difficulties mentioned above were overcome, or at any rate greatly reduced, by the introduction of double-current working, but before discussing this in detail, it is advisable to describe the Post Office Relay and the Double-Current Key.

The Post Office Standard Relay consists essentially of two chief parts—the permanent magnet combination (Fig. 353) and the electro-magnetic combination (Fig. 354).

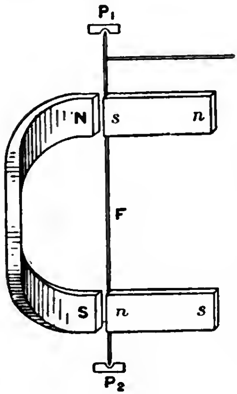


FIG. 353.

NS, Fig. 353, is a large, semicircular permanent magnet, near the poles of which are two soft-iron armatures of rectangular section, rigidly fixed to the spindle F, which is pivoted at P_1 and P_2. Above the armature is the tongue or contact maker, also rigidly fixed to the spindle. The free ends, *n* and *s*, of the armatures are seen between the pole-pieces of the electromagnet in Fig. 354. These armatures are kept magnetised with the polarity shown in the figure by induction from the permanent magnet. (It may be necessary to point out that, if permanent steel magnets were used in place of the armatures, they would gradually lose their strength owing to the vibration. As it is very important that the strength should remain constant, soft iron magnetised inductively by a fixed permanent magnet is used.) Each of the electromagnets shown in Fig. 354 consists of a straight core of well-annealed iron, about 3 inches long, upon which is wound a coil of fine silk-covered copper wire of many turns. At each end of the cores are soft-iron pole-pieces, $S_1N_1S_2N_2$. In the spaces between the pole-pieces play the free ends of the armatures *n* and *s* of Fig. 353. The two coils are joined in series, and a current in the proper direction will magnetise the pole-pieces with the polarity shown in the figure. As unlike poles attract and like poles repel, it will be seen that all four pole-pieces unite in moving the two armatures to the left—and the contact maker with them—in which position the circuit of the local battery is broken and the sounder is unaffected. If the direction of the current through the relay is

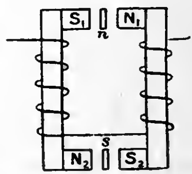


FIG. 354.

[1] This is a term much used in telegraphy. It may be defined as follows:—

$$\text{Leakance} = \frac{1}{\text{Insulation resistance in megohms}}$$

reversed, the polarity of all four pole-pieces is also reversed—the polarity of the armatures, of course, remaining unchanged. The contact maker is then urged to the right and closes the sounder circuit. This reversal of the current is performed by the double current or reversing key described later.

The metal plate carrying the contact maker and the contact screws is shown in Fig. 355. The screws, S_1 and S_2, are fixed so that the contact maker, T, does not move more than $\frac{1}{100}$ inch. In order to adjust for stronger or weaker signals, the plate may be slightly rotated about its centre by means of a thumb-screw, thus bringing the armature nearer to or farther from the left or right hand pair of pole pieces.

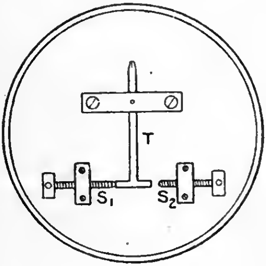


FIG. 355.

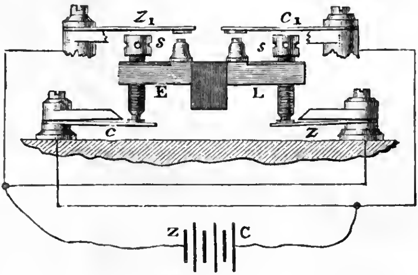


FIG. 356.

Double-current Key.—There are two forms of double-current key. With the first form, a single battery is used, the poles of which are alternately joined to the line and to earth; with the second, separate batteries are used for each direction of current. The principle of the first type is shown in Fig. 356. The brass bar, EL, is fixed at the end of the key lever, and moves up and down between four flat, steel contact springs, Z_1C_1ZC, which are connected to the battery, as shown in the figure, alternate pairs being also joined together. The two halves E and L of the bar are insulated from each other by an ebonite partition. In the position illustrated, the bar is resting on the two lower springs, the positive pole of the battery is connected to the line (which, although not shown in the figure, is permanently connected to E), and the negative pole to earth (which is permanently connected to L). This direction of current moves the distant relay tongue to the left, and the sounder there is not actuated. On depressing the key, the bar rises against the upper springs. The negative pole is now joined to the line, and the positive pole to earth. In this direction, the current moves the distant relay tongue to the right, closes the local circuit, and attracts the sounder armature. The long and short up-and-down movements of the key are thus reproduced on the sounder. When the operator has finished signalling his message, the line is connected to his own relay by turning a switch, which forms part of the key. Such a switch is indicated in Fig. 357, which illustrates the second method of double-current working. When the key is resting on the back contact, the positive pole of the battery A is joined to the line. On depressing the key, this is replaced by the negative pole of the battery B. It may be remarked here, that the direction of current which moves the relay tongue to the right and actuates the sounder is called by telegraphists a "marking" current; the oppositely directed current, which causes the sounder to rise, is called a "spacing" current. These terms arise from the fact that, in an obsolete form of instrument, which preceded the sounder, the dots and dashes of the Morse alphabet were printed on a moving ribbon of paper. While the current was flowing in the first direction, a line or "mark" appeared on the paper. On reversing it, a "space" was left.

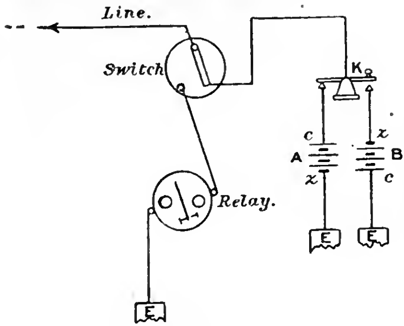


Fig. 357.

Advantages of Double-current Working.—The advantages of double-current working on long lines are as follows:—(1) The movement of the relay tongue to either side being effected by the line currents, no controlling or "antagonistic" force is required to pull it back after each signal. Thus, any cause which reduces the working currents affects both the "spacing" and the "marking" currents alike, and the relays can, therefore, be adjusted to possess great sensitiveness. (2) A current in one direction or the other is always flowing during transmission. Stray leakage currents are, therefore, shut out. (3) The reversal of current after each signal obliterates any residual magnetism in the relay cores. (4) Last, but not least, the application of a reversed E.M.F. greatly facilitates the electrostatic discharge.

With modern appliances, and especially with the greatly improved construction and insulation of the lines now attained, no difficulty is experienced in fair weather in working over lines 200 miles in length with double-current apparatus.

Repeaters or Translators.—When it is desired to work direct to greater distances, a *repeater* is used. This instrument is placed in some office situated somewhere near the middle of the line, and is a special form of relay in which the local battery currents, instead of working the sounder at the office where it is placed, are transmitted to the distant office relay and again actuate the sounder there. Thus, a message from London to Glasgow is sent over two lines, each connected to earth at Leeds, where relays repeat the London signals on to Glasgow.

Duplex Working.—This is a method by means of which it is possible to send messages simultaneously in both directions, *i.e.* station A may be signalling to B, while B is signalling to A.

The two chief methods of working duplex are (1) the Bridge system, mainly used in submarine cables, and (2) the Differential system, which we shall now describe.

The Differential Duplex.—The essential feature of this system depends upon the fact that the cores of the relay electromagnets are each wound with two separate and distinct coils of wire

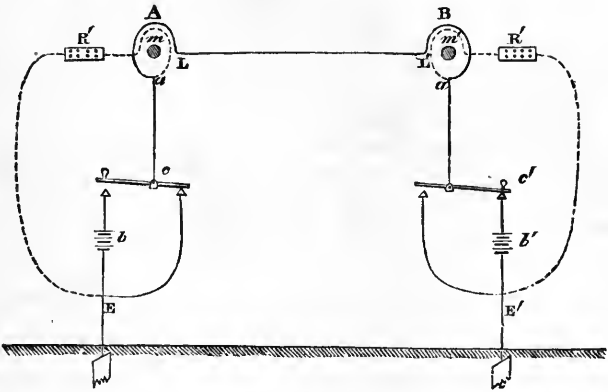


Fig. 358.

of equal resistance, and of an equal number of turns—the two wires, indeed, being laid on side by side.

If currents of *equal* strength are sent in *opposite* directions through the two coils, their magnetic fields cancel each other and the cores are not magnetised. If one current is stronger than the other, the polarity produced in the cores will be determined by the *difference* between the current strengths. Hence, the term *Differential.*

The arrangement of the apparatus is shown diagrammatically in Fig. 358, where *mm′* represent the cores of the double wound relay coils; *cc′*, the keys; and RR′, two variable resistance coils called *rheostats,* by means of which the currents in the inner relay coils may be made equal to those in the outer relay coils.

A little consideration will show that in order to work duplex, the following conditions must be fulfilled :—

(*a*) When both keys are at rest, both relays must be unaffected.
(*b*) When the key at station A is depressed, the relay there must be unaffected, but the relay at B must work.
(*c*) When the key at B is depressed, the relay there must remain neutral, but the relay at A must work.
(*d*) When both keys are depressed, both relays must work.

The last case is the only one which may strictly be called duplex working; the other three cases being those that occur in ordinary single working.

We have now to show how these conditions are fulfilled.

Case (*a*) is self-evident. The keys being at rest, no current can flow to actuate either of the relays.

In case (*b*), the current leaving the key at A has two paths of equal resistance open to it: (1) through the inner relay coil of A, through the rheostat to earth and back to the battery; (2) through the outer coil of A's relay, the line, the outer coil of B's relay, and then through the back contact of B's key to earth. Two currents of equal strength, therefore, flow through the coils of A's relay, and, being in opposite directions, that relay remains neutral. At B, however, the arriving current flows through the outer coil only, and therefore its armature is attracted, which closes the local circuit of the sounder (not shown in the figure).

Case (*c*) is exactly similar to case (*b*)—the signalling being merely in the reverse direction.

In case (*d*) both keys are depressed, and therefore equal and opposite electromotive forces are applied to the line, the points a and a' being raised to the same potential. Therefore, no current flows through the outer coils of the relays and the line. At each station, however, a current can flow from a and a' respectively through the inner coils and the rheostats, and it is these currents which actuate the relays and produce the signals.

The description given above refers to the single-current duplex. In practice, however, the double-current system is used, and although the principle remains the same, the explanation is somewhat more complicated, so that if the student wishes to study the subject more fully, he must consult some technical work on telegraphy, *e.g.* *Telegraphy*, by Preece and Sivewright.

The Central Battery System.—One of the most recent developments in telegraphy is the Central Battery system, chiefly used on the circuits which radiate from a large central office to surrounding smaller towns and villages or to branch offices. Instead of providing a battery at each end of the line, the whole of the current required is supplied from one large storage battery at the central office. The only apparatus used at the distant office are a key, a condenser, and a special form of receiving instrument called a **polarised sounder**. The principle of the polarised sounder will be understood from Fig 359. The lower ends of the electromagnet, instead of being joined, as in ordinary form, by a soft-iron yoke, rest upon the poles of a large compound horseshoe permanent magnet placed under the base of the instrument, so that the cores form its pole-pieces. The path of the lines of force of the magnet is, therefore, from the N pole up the left-hand core, across the armature, and down the right-hand core to the S pole. The tension of the spiral spring can be so adjusted that, if the armature is forced down to the cores, the attraction of the permanent magnet is sufficient to keep it down, but if the armature is raised, it remains up in consequence of the increased air-gap reducing the magnetic flux. The armature is fixed to the sounder lever as in the ordinary form of the instrument.

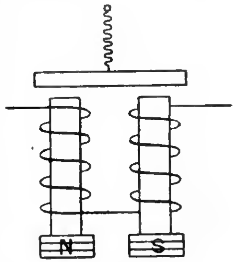


FIG. 359.

A momentary current, *e.g.* a condenser discharge, passing in the right direction to increase the magnetism of the cores, will suffice to pull down the armature, which will remain down until another momentary current in the opposite direction weakens the cores and allows the spring to pull it up again.

The arrangement of the apparatus is shown in Fig. 360, which

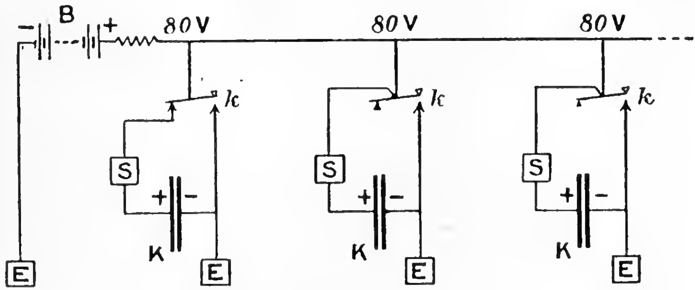


FIG. 360.

indicates the connections for three offices on one line. B is the central battery, S the sounders, *k* the keys, K the condensers, and E the various earth-connections. The battery, usually of 80 volts, has one pole connected to earth and the other to the line, the distant end of which is insulated, instead of being connected to earth. A resistance of about 3000 ohms is intercalated between the battery and the line, indicated by the zigzag line in the figure. This is to avoid short-circuiting the battery when a key is depressed.

It should be noted that, if the line is perfectly insulated, no current flows while the apparatus is at rest, but that the condensers are maintained in a charged condition—the P.D. between their plates being (say) nearly that between the poles of the battery, viz. 80 volts. In fact, every portion of the circuit between the positive pole of the battery and the positive plates of the condensers is 80 volts above the potential of the earth.

When a key at any station is depressed, the line is connected to earth. As all the positive plates of the condensers are connected through the sounders to the line, they also become earth-connected. The negative plates of the condensers are permanently joined to earth, so that, in effect, the plates are connected together and the condensers therefore discharge. This momentary discharge pulls down the armatures of the sounders, and they remain down after the current ceases. When the key is permitted to rise, the earth connections are broken, and the battery immediately re-establishes the original P.D. of 80 volts between the plates of the condensers. Then the transient current flowing through each sounder to recharge the condenser permits the armature to rise and denotes the completion of the signal. In the ordinary sounder, the dots and dashes are formed by long and short currents, while with the polarised sounder they are due to longer or shorter intervals between two currents of equal duration but opposite in direction.

It will be noticed that two alternative methods of connecting up the key are shown in the figure. When arranged as in the first office, depressing the key operates all the sounders on the line except the one at the sending station, and when arranged as in the second or third office, the sender's own sounder is operated also. As a rule, the latter plan is adopted, but sometimes the former is convenient.

Telephony.—Before we discuss the electrical instruments—known as **telephones**—which reproduce distant sounds, it may be pointed out that all sounding bodies are in a state of vibratory motion, which sets up waves in the surrounding air, and that it is these waves which, striking the ear, cause the sensation of sound. These sound-waves differ among themselves in three particulars, viz. (1) the rate of vibration, *i.e.* the number of vibrations per second; (2) amplitude, *i.e.* the amount of displacement; and (3) shape, due to the coexistence, with the principal, of other secondary vibrations. These physical conditions determine respectively (1) the pitch, (2) the loudness, and (3) the quality or timbre of a sound. Given, then, a sound which has these three characteristics in a fixed and definite way, the problem of practical telephony is to produce an exactly similar wave-motion at a distance.

The first person to construct an apparatus for this purpose was Reis of Friedrichsdorf. His apparatus, although fairly successful with musical notes, only imperfectly reproduced speech, mainly owing to its faulty receiver. The sending arrangement of the instrument depended upon the alteration of a loose contact under the influence of sonorous vibrations, and was identical in principle with the best modern transmitters.

The Bell Telephone.—This well-known instrument was invented by Professor Graham Bell in 1876. Its original form is shown in Fig. 361. A bobbin of fine silk-covered copper wire, B, is fixed

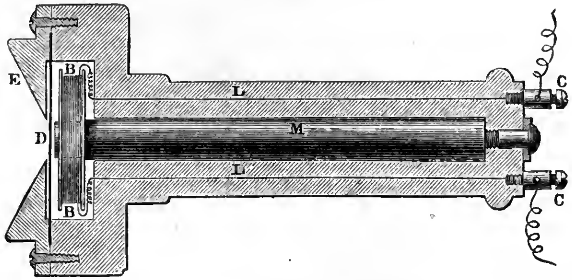


Fig. 361.

on a short soft-iron core, attached to one end of a steel permanent magnet, M, supported in a wooden case. The ends of the coil pass through holes in the case, and are attached to binding-screws, CC. In front of the soft-iron pole-piece, and as close to it as possible without actual contact, is placed a very thin disc, D, of soft iron. This disc, or diaphragm, is held in position by the mouthpiece, E.

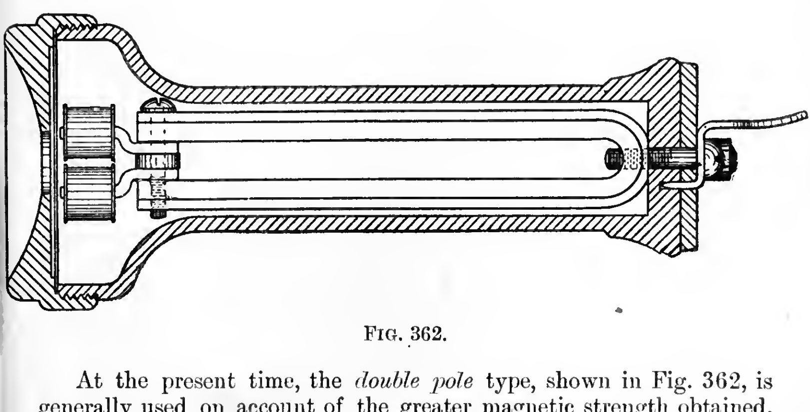


Fig. 362.

At the present time, the *double pole* type, shown in Fig. 362, is generally used on account of the greater magnetic strength obtained. The principle is unaltered, the only difference being that a horse-shoe magnet is employed instead of a bar magnet. The magnet is made from a long thin bar of steel bent double in the middle, thus bringing the poles within ½ inch of each other. The two coils of wire are joined in series, and the ends brought out by a flexible connection (shown at the end) consisting of two separate insulated conductors bound up together. In Fig. 363 two such instruments are shown, connected to a line wire, the circuit being completed through the earth, although in practice another wire is generally used. Their action is easily understood. Considering one of the instruments, we see that the magnetic circuit is partially completed through the diaphragm. The distribution of the lines of force is very unstable, and varies greatly with any alteration in the distance between the diaphragm and the pole-pieces. The smaller the distance, the greater the number of lines completing their circuit through the disc, and *vice versâ*, and any movement of the lines means that some of them must cut the turns of the coils. (This action is well illustrated by Experiment 213, p. 349.)

When sound-waves strike the disc, it vibrates in unison with them,

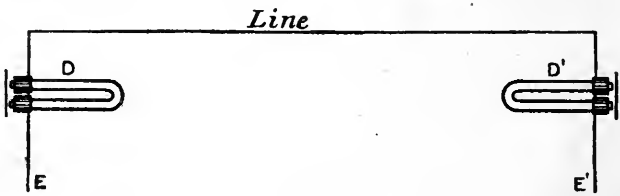


FIG. 363.

and the distribution of the lines through the coils on the bobbins is disturbed, the opening-out and closing-in of the lines setting up E.M.F.'s in opposite directions, which generate alternating currents, when the circuit is completed. The impulses of current thus set up are more or less wave-like in form, and reproduce with remarkable fidelity the chief characteristics of the sound-waves producing them. We may, therefore, regard the instrument as a very delicate alternating current generator, which transforms the energy of sound-waves into oscillatory currents of electricity. The currents from the transmitting telephone, as they pass through the coils of the receiving instrument, alternately increase and decrease the strength of the steel magnet, thus varying the flexure of the diaphragm, the disc moving inwards when the pull increases, and outwards (by virtue of its own elasticity) when the attraction decreases. The discs of both instruments, therefore, vibrate in harmony, with the important difference that the amplitude of the receiver is much less than that of the transmitter, and as a consequence the loudness of the resulting sound is much reduced.

The Watch Receiver.—This is the typical form of receiver used in the instruments of the National Telephone Company, in which the transmitter and receiver are fixed to a common support held in the hand. For this construction, the Bell type would be too heavy.

The watch receiver is very compact and light, the essential feature being the shape of the steel magnet, which in the earlier forms was a nearly semicircular plate of steel, B (Fig. 364). This is fitted with soft-iron pole-pieces, AA, on which the coils are wound, as shown in section in Fig. 365. In later patterns, the permanent magnet is a complete ring of steel, magnetised so that consequent poles, N and S respectively, are produced on opposite sides of a diameter, although Fig. 364 represents this type equally well.

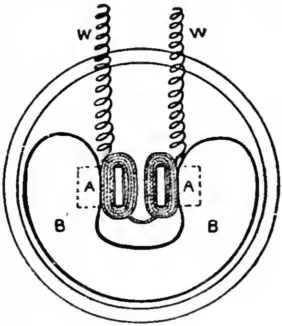


Fig. 364.

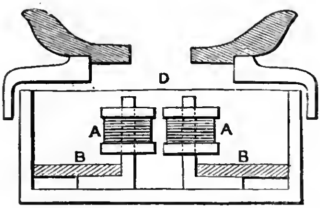


Fig. 365.

Transmitters and Receivers.—Any of these forms of Bell telephone will act either as transmitter or as receiver (without the use of a battery), if connected up as shown in Fig. 363. As a receiver, the telephone is unequalled, but as a transmitter it is not so satisfactory. It will transmit speech over 10 to 15 miles of well-insulated wire, but as the currents generated by the motion of the diaphragm are very small and are incapable of augmentation, the sound becomes more and more inaudible as the distance increases. In order to be able to converse over long distances, it became necessary to devise some new form of transmitter, which would permit of a stronger current being used. This was achieved by the introduction of a microphone in circuit with a battery.

Carbon Transmitters. Hughes' Microphone.—In 1878, Professor Hughes showed that any loose or imperfect contact interposed in a conducting circuit could be used as a telephonic transmitter, owing to its action in varying the resistance, and consequently the current. For example, two ordinary nails were made the terminals of a battery, in the circuit of which a telephone was included. The nails were connected by a third nail lying across them, thus forming an imperfect contact. When speaking near the nails, the vibrations of the voice were communicated to them, and as differences in pressure produced differences in resistance, similar variations were, of course, produced in the current through the telephone.

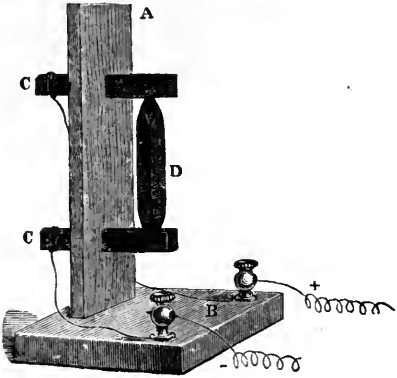


FIG. 366.

The most satisfactory results were, however, obtained when points of carbon, mounted on a sounding-board of thin flexible pine-wood, were used instead of the nails. In Fig. 366 is shown Hughes' original form of microphone, which merely consists of a carbon pencil, D, with pointed ends loosely fixed between two carbon blocks, CC.

If the microphone be connected in circuit with a battery and telephone (Fig. 367), and any sound be produced near it, the sound-waves, falling upon the carbon pencil, will cause it to vibrate slightly. The alterations in resistance thus produced result in a varying current through the telephone, which reproduces the sound in the usual manner. To transmit very feeble sounds or vibrations, such as the ticking of a watch, the carbon pencil must be delicately balanced; for spoken words or louder sounds, a coarser adjustment is necessary to give good results.

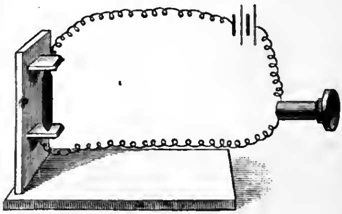


FIG. 367.

Further improvements were made by using a number of pencils joined in parallel.

Granular Transmitters.—All transmitters in use at the present time are of this type. The principle is indicated in Fig. 368. Two discs of carbon, placed very near each other, are connected respectively to the terminals, the space between being filled loosely with carbon granules. One of the carbon discs is fixed to the back of the case, and the other is a thin plate which serves as a diaphragm and is provided with a mouthpiece. When this diaphragm vibrates under the influence of the voice, the resistance of the granules varies with the compression. The granules may, in fact, be regarded as equivalent to an enormous number of pencils. The outside of the diaphragm is varnished to exclude moisture from the breath; and inside, both the carbon surfaces are made extremely smooth and are highly polished. The carbon granules are very carefully selected; they should be extremely hard and well polished.

The chief difficulty with granular transmitters is due to "packing," *i.e.* a tendency of the granules to cohere into a more or less solid mass. Various devices are employed to prevent this defect. In the Deckert type (largely used in the Post Office systems) the surface of the fixed carbon plate, instead of being smooth, is cut into a large number of four-sided pyramids (Fig. 369). The summits of these pyramids are very close to the carbon diaphragm, so that the granules can only move freely along the zigzag grooves between the pyramids. The apexes of about fifteen of the central pyramids are cut off and replaced by little tufts of fluffy material. The tufts

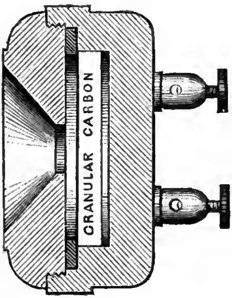


FIG. 368.

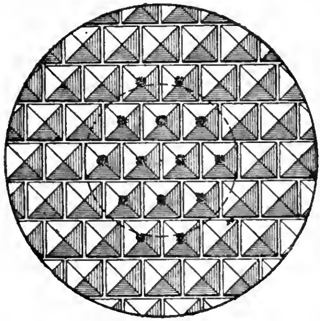


FIG. 369.

on the tops of the pyramids assist in delaying the packing of the granules, prevent short-circuiting between the two carbon electrodes, and damp the vibrations of the diaphragm, *i.e.* check any tendency of the vibrations to continue after the exciting sound-waves have ceased.

The Telephone Induction Coil.—It should be clearly understood that the efficiency of a carbon transmitter depends not only upon the number of ohms resistance by which it varies the total resistance in its circuit, but also upon the ratio which that number bears to the total resistance; consequently, the longer the circuit the less the efficiency of the transmitter. A microphone, whose resistance varies between some small value and 10 ohms, if placed in a circuit whose resistance is also 10 ohms, would alter the total resistance from 10 ohms to 20 ohms, and *vary* the current by 100 per cent. If it were placed in a circuit of 1000 ohms it would vary the current by only one per cent. A transmitter is, therefore, most efficient when it is practically the only resistance in the circuit. The longer the line, the less *percentage change in total resistance* (and therefore in the current). Hence, it is imperative that the total resistance should be kept low, but on long lines this is impossible. Edison ingeniously overcame this difficulty by using a small induction coil. This is a small and very simple affair, consisting of two coils on a core of iron wire. The general arrangement is shown in Fig. 370. A battery, B, and microphone, A, are joined up to the primary of the induction coil, C. By using a special type of low-resistance battery, the resistance of the microphone becomes practically the only resistance in the circuit, that of the battery and primary not exceeding 1 ohm. The secondary coil of many turns, having a resistance of 25 to 100 ohms, is connected to the line L and to the receiving telephone D, the circuit being usually completed by means of a return wire, and not by the earth connection shown in the figure. Although the resistance of the

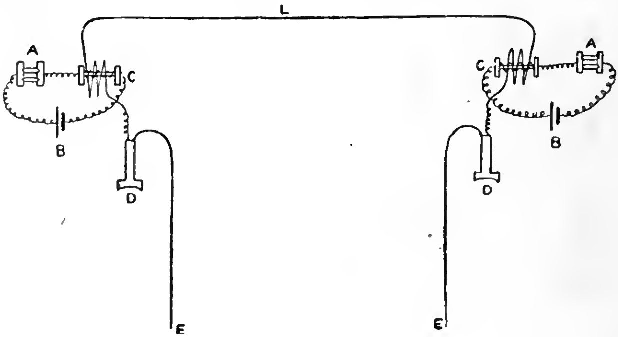


FIG. 370.

coils is fairly high, their insertion in the line whose resistance is already considerable, does not increase the latter by a large percentage.

The action is as follows: The variations of resistance, caused by the sound-waves striking the transmitter, produce corresponding changes in the strength of the current flowing through the primary of the induction coil, and these fluctuations are considerable because the microphone resistance forms so large a portion of the total resistance.

The variations of current in the primary induce similar currents in the secondary, but at a much higher E.M.F., which is an advantage on account of the high resistance of the line. These induced currents actuate the receiving telephone, D, which is of the usual Bell type.

The Magneto-Generator.—The magneto-generator is of the type shown in Fig. 374, p. 526, having an H pattern armature wound with a coil of fine wire of about 500 ohms resistance, and driven by a hand-wheel through gearing. No commutator is used, connection being maintained with the ends of the coil in a very simple manner, and an automatic device is arranged to cut it out of the line circuit except when the handle is being turned.

The Magneto-bell.—As the generator produces an alternating current, the bell must be designed to work efficiently with such currents. Its principle is indicated in Fig. 371. E is an electromagnet, provided with a soft-iron armature rocking on a pivot, A, and carrying a hammer which strikes the gongs as it moves to and fro. This armature is "polarised," being magnetised inductively by means of a steel magnet (not shown in the figure), which is bent so that one pole is just below A, and the other just above the middle of the base of the electromagnet, E. The lines of force, entering at A, divide to pass along the armature to both ends, and then return to the distant pole through the cores of the electromagnet. Hence, the armature is kept magnetised with "consecutive" poles, both ends being of the same polarity, and this magnetism is not affected by its vibrations.

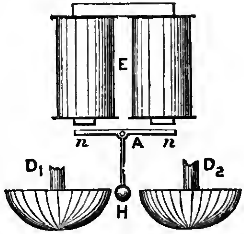


Fig. 371.

The alternating current from the line passes round the coils of the electromagnet, and produces an alternating polarity superposed upon the steady inductive effect due to the fixed magnet. If the polarity alternates, a little consideration will show that each end of the armature is in turn attracted and repelled, thus oscillating rapidly, and if the current is so small that the poles are merely strengthened and weakened respectively, the resulting inequality in pull will produce the same effect.

Return Wires. Effect of Induction.—Except on very short lines, it is not possible to use the earth as a return wire for telephone circuits. The modern telephone is such a remarkably sensitive detector of weak currents that it may almost be regarded as an electrical microscope. When two earth plates, situated some distance apart, are connected by a wire, a sensitive galvanometer placed in the circuit will show that there is a continual ebb and flow of current between the two plates, owing to the fact that their potentials are constantly changing. This effect is very pronounced in towns having an electric tramway system. These currents, although they are too weak to affect ordinary telegraph instruments, produce a very disagreeable buzzing in the telephone, which renders conversation very difficult, if not impossible.

Another trouble is due to the fact that telephone circuits are especially affected by induced currents from other wires—more particularly from high-speed telegraph circuits—running parallel to them. Owing to such induced currents on two telephone circuits, it is possible for a conversation taking place on one circuit to be overheard on the other.

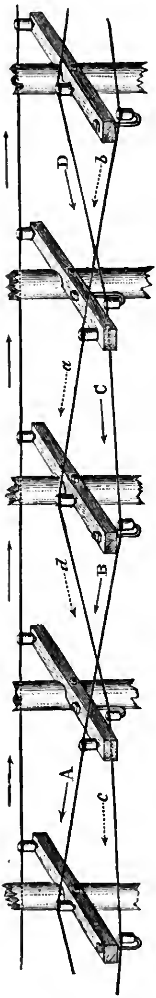


Fig. 372.

To overcome these difficulties, it is usual to provide two wires, thus forming a metallic loop and dispensing with the "earth return."

In Fig. 372, the upper wire is part of a telegraph circuit, and the two lower lines represent a telephone "line" and "return" wire, carried on the same poles. The "starting" of the current in the telegraph line would tend to induce currents in the opposite direction (as shown) in both wires of the telephone circuit. If the two latter wires were equidistant from the former, the two induced currents would be equal in strength and would neutralise each other, as they tend to flow in opposite directions round the circuit; but unless precautions were taken, the telephone wires would not be equidistant, and there would be a resultant induced current, which would introduce a disturbing influence. Hence, the wires are arranged as shown, being made to change places at every fourth or fifth pole. Equal portions of each wire thus become alternately nearer to, or farther from, the source of disturbance, so that no resultant induced current is produced in the circuit.

Attenuation and Distortion of Speech.—The currents in a long-distance telephone circuit may be regarded as belonging to a complex series of electro-magnetic waves propagated through the space surrounding the conducting wire. Such waves of current can be recorded by means of an oscillograph; they have approximately a sine-curve form, and are usually between ten to twenty miles long. The speed and mode of propagation are determined by the four electrical constants of the circuit, viz. resistance, leakance, inductance, and capacity.

The resistance and leakance, by abstracting energy from the circuit, reduce the amplitude of the waves at the distant end of the line to a mere ripple with scarcely enough energy to vibrate the diaphragm of the receiver. The sounds reproduced are faint but distinct. This effect is called *attenuation.*

The effects of inductance and capacity are more serious. Inductance tends to cause the wave to lag in phase behind the E.M.F. producing it, and as waves of higher frequency are the more retarded, the sounds of the voice—including, as they do, many different frequencies—become *distorted*. Capacity affects the waves in much the same way, with, however, the important difference that a *lead* instead of a *lag* is produced in the wave-form. This *distortion*, which is much more pronounced in cables than in aerial circuits (owing to the relatively enormous capacity of the former), may so modify the resultant wave-form, that speech becomes quite unintelligible, although the sound produced may be loud.

The fact that inductance and capacity displace the wave-form in opposite directions, indicates the possibility of using one to neutralise the other—a problem which has been attacked in various ways.

In all cables the capacity greatly exceeds the inductance, and as it is not possible to reduce the former, the only remedy is to increase the latter. This is done by placing specially constructed induction coils (in series) in the cable, their number and distance apart being determined by the "wave-length constant" of the cable.

In the year 1910, a specially designed cable, "loaded" (as it is termed) with such coils, was laid between Dover and Cape Grisnez in France, with, it is said, excellent results. It is hoped that, by means of this cable, London may hold telephonic communication with Berlin, Amsterdam, or any other centre in Northern Europe.

The investigations of Pupin in America, of Heaviside in England, and of other workers have provided data such that, knowing the electrical constants of any type of cable, we can calculate the distance in miles over which it is possible to conduct a satisfactory conversation. A unit or standard cable, of the paper-insulation type, has been constructed having the following dimensions per mile of loop: weight of conductors, 20 lbs.; resistance, 80 ohms; capacity, ·054 microfarad; insulation resistance, 200 megohms; inductance, ·001 henry; and speech limit, 43 miles. The properties of all other cables can be conveniently expressed in terms of this standard cable.

Exercise XXIV

1. Describe the construction and explain the use of a relay.

2. What is meant by electrostatic induction in cables?

3. Describe Bell's telephone, and explain how it transmits and reproduces the vibrations.

4. Describe the method of communicating telegraphically between two stations provided with Morse instruments, relays, and local batteries. (B. of E., 1904.)

5. Give a general explanation of the action of a telephone, and describe some form of transmitter. (B. of E., 1905.)

6. Describe the construction of some one form of telephonic receiver. Why would not soft iron do instead of steel for the electromagnet? (B. of E., 1907.)

7. Describe the ordinary form of Morse sender and receiver, and explain

how a line may be used for sending signals simultaneously in opposite directions. (B. of E., 1908.)

8. Describe and explain the reproduction of sound by a telephonic receiver of the Bell type. Why should the core be of magnetised steel and not of soft iron? (B. of E., 1909.)

9. Describe, giving sketches of the instruments and connections, a complete arrangement for telephonic communication between two stations. (B. of E., 1911.)

CHAPTER XXXI

DYNAMOS AND MOTORS

All practical dynamo-generators are applications of the principle established in Experiment 210, p. 348, *i.e.* they are devices for obtaining relative motion between a number of conductors connected in series, (forming what is termed the armature), and a magnetic field, the direction of the field being as nearly as possible at right angles to the conductors, and also to the direction of motion.

When the magnetic field is produced by steel permanent magnets, the arrangement is known as a **magneto-machine.** Such machines are used only in very small sizes for special purposes, *e.g.* for ringing bells, for exploding fuses, and the like.

When the field is produced by soft-iron electromagnets, which are excited by the whole or by a portion of the current generated, and which are known as field magnets, the machine is called a **dynamo**, and may be either (1) a direct current, or D.C. machine, or (2) an alternating current, or A.C. machine.

In D.C. machines, it is usual to make the armature to rotate in a stationary field; in A.C. machines, it is now customary to make the field magnets rotate with reference to a stationary armature, although in the past both arrangements have been used.

In our treatment of this subject, we shall first consider the simple case of a coil of wire rotating in a magnetic field. We have already shown (see p. 352) that an alternating current, making one complete cycle per revolution, is produced, and we have now to give the contrivance a practical form. For this purpose, it is evidently desirable to concentrate the lines of force upon the coil, which is conveniently accomplished by winding the latter upon a soft-iron cylinder rotating with small clearance in the polar gap. Also, as the end portions of the coil are inactive, it will be advisable to make its length great in comparison with its diameter. We are thus led to the original Siemens type of armature, which is still useful for small magneto-machines, such as are employed in telephony for ringing bells (see p. 520).

Siemens or H Pattern Armature

Fig. 373 shows one of the early armatures of this type. The coil is wound in a deep longitudinal groove in the iron core, and when the machine is intended to give alternating currents, the two free ends are merely joined up to insulating rings (or other forms of rubbing contact) so that they remain in connection with the stationary terminals of the machine as the armature revolves. Fig. 374 shows the iron core and steel field magnet of a magneto-machine. The

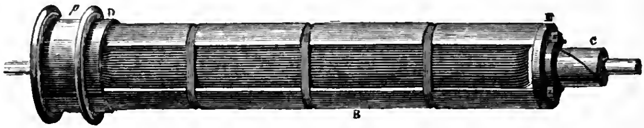


FIG. 373.

magnet, NS, is provided (1) with soft-iron pole-pieces, *ns*, which serve to concentrate the lines of force upon the armature; and (2) with distance-pieces, *bb*, of brass (or other non-magnetic material), which keeps them at the right distance apart.

When the permanent magnets are replaced by electromagnets excited by the machine itself, the current in the external circuit must be continuous in direction, and in order that this condition may be satisfied, some form of **commutator** must be used. The simplest is that shown in Fig. 373. It is merely a metal ring split into halves by two longitudinal and slightly oblique slits (the obliquity is not, however, essential, being a detail which tends to reduce sparking) insulated from each other and from the shaft, to which the two free ends of the coil are connected and upon which rest two stationary brushes. These brushes serve to maintain connection between the rotating coil and the remainder of the circuit. If their position is such that the contact changes from one half ring to the other at the instant the current reverses its direction, each brush will always have the same sign, and the current outside the armature will be continuous in direction, although pulsating from zero to some maximum value. Evidently the current may be led round the coils of the soft-iron field magnets in order to excite them, but as there are various ways of accomplishing this purpose, which do not in the least affect the consideration of armature details, it will be convenient to deal with such methods later, and at present to assume that field magnets of some kind exist.

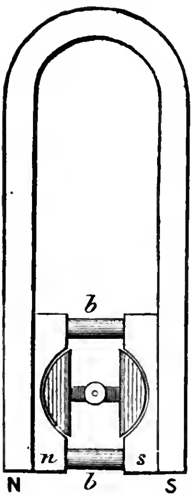


FIG. 374.

The E.M.F. of a machine, with a field of a given strength, depends upon the number of turns on the armature and upon the speed of rotation; the current then depends chiefly upon the resistance of the external circuit, and its safe value is determined by the heating of the armature conductors. At the same time, it must be remembered that, as the output increases, so does the mechanical power required to drive the machine.

The first armatures of this type were made (and are still made for very small machines) with solid iron cores, but then induced currents were not only produced in the winding, but also in the core itself. Such "eddy currents" involve a very serious waste of energy, and the iron core may become hot enough to injure the insulation of the winding. This difficulty is not a peculiarity of the type of armature just referred to. It is met with in all armatures, although it is very greatly reduced (but never completely eliminated) by *laminating* the armature core, *i.e.* by building it up of thin soft iron plates or stampings, whose planes are at right angles to the direction of the eddy currents. In large machines these stampings are usually insulated from each other by a thin coat of varnish; in small ones, the natural coating of rust or scale is sufficient. Such simple means are effective, because the E.M.F. producing eddy currents is very small.

Defects of Single-coil Armatures with Split-ring Commutator.—Of these defects, the two most important are: (1) the E.M.F., and therefore the current, falls to zero twice in each revolution—this makes the machine useless for charging accumulators; (2) the full P.D. the machine produces, exists between the two halves of the commutator, and as a consequence there is a serious danger of an accidental short-circuit. The latter defect prohibits the use of this winding for high voltages.

The Gramme Ring Armature.—A very great step in advance was made by the introduction of the Gramme armature. Although obsolete, it is worth attention, because it leads naturally to forms of winding now in common use. The study of its construction and action is best approached by considering a number of straight, insulated conductors, grouped symmetrically around a rotating cylindrical armature, as shown in section in Fig. 375, which indicates the direction of the E.M.F.'s on the two sides of the armature for the given polarity and direction of rotation. The problem to be solved is the method of connecting these conductors together and to the external circuit. Gramme solved it by making the armature hollow and providing inactive connecting wires inside. The arrangement is *equivalent* to

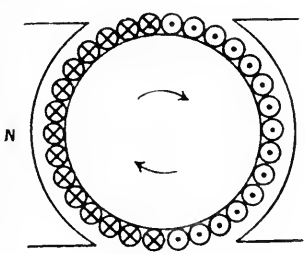


FIG. 375.

winding the iron ring with a uniform coil until the ends of the winding meet, and then joining the ends to form a closed winding. Of course, this is not the actual method of winding, but the result is, electrically speaking, the same. The principle is indicated diagrammatically in Fig. 376.[1] It will be seen that all the active conductors on each side of the armature are in series, but the E.M.F. induced on one side is opposed in direction to an equal E.M.F. on the other side, so that yet no current can flow. If, however, we suppose that the outside surface of the active conductors are free from insulation, and

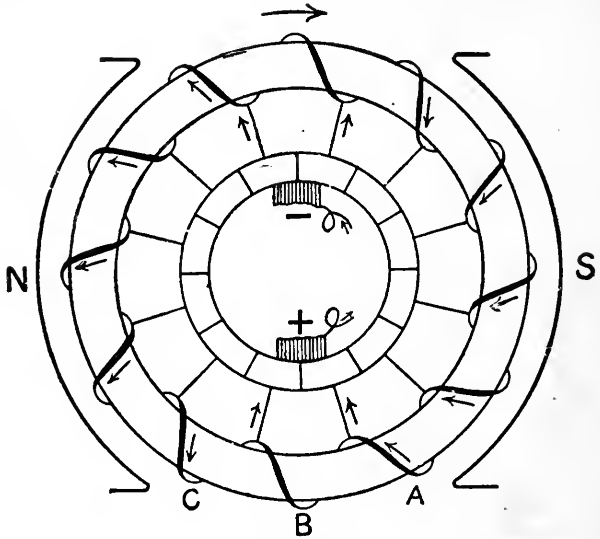


FIG. 376.

that two brushes, joined by a wire forming the external circuit, make contact with them at the top and bottom of the armature respectively, it is evident that the two halves of the winding are joined up *in parallel* to the external circuit, in which a current will flow of twice the strength of that in any one active conductor. In practice, it would be inconvenient to collect the current directly from the active conductors, and so they are connected to a number of insulated copper bars, forming the *commutator*, upon which the brushes (marked + and − in the figure) rest. It is not necessary to provide a bar for every active conductor, as, in most cases, such an arrangement would make

[1] This and several following figures (Figs. 376, 377, 378, 379, 380, 381, 383) are taken from Slingo & Brooker's *Electrical Engineering*, published by Messrs. Longmans.

the commutator impossibly large; but it is desirable to have as many bars as possible, because that helps to keep down sparking at the brushes. Consequently, the total winding may be regarded as being split up into a number of coils (known as "sections"), connected to the commutator bars as shown diagrammatically in Fig. 377.

Advantages and Disadvantages of Ring Armatures.—The advantages are: (1) the current in the external circuit is nearly uniform in strength, instead of falling to zero twice in each revolution; (2) the points, between which the P.D. is greatest, are at opposite ends of a diameter, both on the armature and on the commutator, and thus there is little difficulty as regards insulation.

On the other hand, there are two great drawbacks: (1) each section of the winding must be wound in its place by hand; (2) all

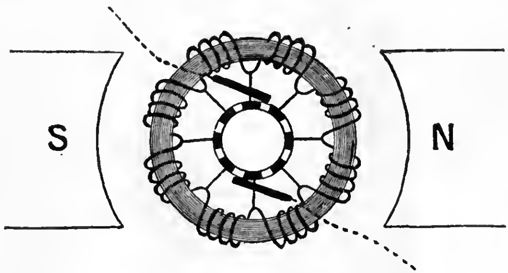


FIG. 377.

the lines of force of the field must pass through the iron of the ring, and it is difficult to give it a cross section sufficiently great to keep the magnetic reluctance within reasonable limits.

Drum Armatures.—Practically all modern armatures are of the drum type, in which these defects are avoided.

It has been stated that the conductors *inside* the Gramme ring are merely devices for joining together the external or active conductors in series, and we see that this can be accomplished equally well by connecting each of the external conductors shown in Fig. 375 to another diametrically opposite one, thus keeping all the winding on the outside of the armature, which need not then be hollow. For the actual details of the connections, the student must refer to special treatises; the result, however, is a closed winding, which behaves, from an elementary point of view, very much like the ring winding already described. In both, there are two parallel paths through the winding from brush to brush, and in each path half the external conductors are in series, the difference being mainly in the method of connection by which this result is obtained.

Early drum armatures were hand-wound, and the windings overlapped at each end, so that in case of injury, it was usually necessary to unwind and rewind the whole. This was a very great disadvantage, which is entirely avoided in modern practice—a damaged part being now easily replaced without seriously disturbing the remainder of the winding.

FIG. 378.

The iron core of both ring and drum armatures is, of course, built up of thin stampings, and the conductors are now invariably imbedded in slots. A stamping for a drum armature is shown in Fig. 378. The apertures around the shaft are partly to reduce weight where iron is not required, and partly for ventilation.

E.M.F. induced in Armature. Example.—An armature has 420 active conductors, and is to run at 900 revolutions per minute in a two-pole field. How many lines of force must pass through it in order to produce an E.M.F. of 110 volts?

Let Z lines of force be required. Then one active conductor cuts 2Z lines of force in one revolution, or $2Z \times \frac{900}{60}$ lines per second. By definition, this must be the average value of the E.M.F. per conductor in absolute units. Also $\frac{420}{2}$ active conductors are in series, therefore the total induced E.M.F. between the brushes is given by

$$E = 2Z \times \frac{900}{60} \times \frac{420}{2} \text{ absolute units.}$$

But E is to be 110 volts,

$$\therefore \quad 110 \times 10^8 = \frac{2Z \times 900 \times 420}{60 \times 2}$$

$$\text{or} \quad Z = \frac{110 \times 60 \times 2 \times 10^8}{2 \times 900 \times 420} = 1{\cdot}74 \times 10^6 \text{ lines of force.}$$

In this case, the result will be the same whether the armature is ring or drum wound.

It will be observed that the E.M.F. induced in any one active conductor is *alternating* in direction, whereas the method of collection ensures that the P.D. between the brushes shall be *continuous* in direction. Hence, this P.D. is the *average* value, as assumed in the above calculation.

It must not, however, be supposed that this is the reading actually obtained on a voltmeter connected across the terminals of the machine. Such may be the case under certain circumstances when no current is flowing, but the machine has a certain *internal* resistance, and to it applies all that has been said in Chapter XV. as regards the distinction between induced E.M.F. and P.D. at the terminals in current generators. For instance, if, in the worked example just given, the internal resistance is ·5 ohm, then, when the current is 20 amperes, 10 volts will be expended within the machine, and the P.D. at the terminals will be only 100 volts. But the case is not quite so simple as that of a cell or battery discussed in the chapter referred to, because there are other influences at work, which may alter the induced E.M.F., and, therefore, modify the P.D. at the terminals.

The Field-Magnet System. Self-exciting Principle.—If the field magnets possess initially a slight amount of magnetism, a definite but very small E.M.F. will be induced in the rotating armature. If the feeble current thereby produced be passed through the coils of the field magnets in the right direction, the existing magnetism will be strengthened, and consequently the induced E.M.F. and current will be increased. This current, in its turn, will increase the magnetism of the fields. The action is, therefore, cumulative, but as the iron tends to become saturated, further increases of current produce a smaller and smaller effect, and so the strength of the field soon reaches a limiting value for a particular speed of rotation.

In order to produce the required magnetic flux, the excitation must amount to a certain number of ampere-turns, which will depend upon the details of the machine, and which is calculated by the method outlined in Chapter XXV. The higher the permeability of the iron, the less will be the magnetic reluctance, and hence the best material for field magnets would be wrought iron, but as this cannot be cast, it is now usual to employ what is known as "mild cast steel," which is really a fairly pure form of iron, containing just enough carbon to make it sufficiently fusible.

Series Winding of Field Magnets.—In the oldest form of winding—known as series winding—the armature, the field coils, and the external circuit are in series with each other, and hence the fields must be wound with conductors thick enough to carry the whole output of current without excessive heating. However, as the exciting current is large, only a moderate number of turns is necessary to give the required number of ampere-turns.

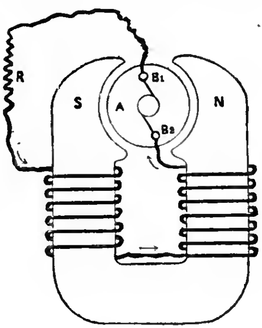


FIG. 379.

This winding is simple and inexpensive, and is also very satisfactory when the current output is always to be the same, but it is evident that, when the current alters in strength, the magnetic flux also alters, and with it the induced E.M.F. As, however, for most purposes, it is necessary that the current should be capable of variation without altering the E.M.F., the simple series winding is seldom used for generators. Such a winding is shown diagrammatically in Fig. 379. It will be noticed that no current can flow, and therefore the machine cannot excite itself, until the external circuit is closed.

Shunt Winding.—In a shunt winding, the required number of ampere-turns is produced by a small fraction of the total current passing round many turns of fine wire, the ends of the winding being connected directly across the brushes, and therefore in parallel with the external circuit. This form is shown in Fig. 380. Evidently the machine can excite itself even if the outer circuit is open; moreover, the exciting current is constant if the P.D. at the terminals is constant, and is, therefore, largely independent of changes in the current output. For this reason, the machine maintains a much closer approximation to a constant voltage at all loads, although it is easy to see that the voltage *must* drop to some extent as the load increases, for even if the induced E.M.F. remains constant, the P.D. at the terminals must fall on account of internal resistance.

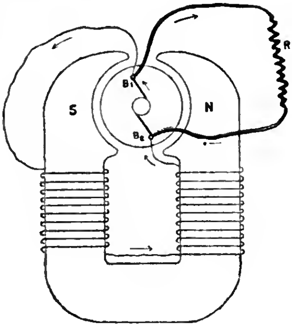


FIG. 380.

One method of keeping the pressure constant is to place an adjustable resistance in the shunt circuit; then when the voltage falls, this resistance can be reduced (thereby increasing the field current and the ampere-turns) until the voltage rises again to its original value. Such a method, of course, requires the presence of an attendant. The same result can, however, be obtained automatically by using a compound winding.

Compound-wound Machines.—These may be regarded as *shunt*-wound machines provided with an auxiliary *series* winding for regulating purposes, as shown in Fig. 381. The shunt coil produces the number of ampere-turns required to give the desired voltage at no load, *i.e.* when the external circuit is open and the series winding is inoperative. When the circuit is closed and a current flows, it also flows round the turns of the series coil, which, therefore, produces an extra number of ampere-

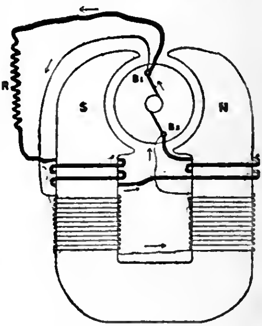


FIG. 381.

turns proportional to the load, and by adjusting the number of turns, it can be made to keep the P.D. at the terminals practically constant throughout the working range of current, or if necessary to make the voltage rise slightly as the current increases.

Armature Reaction.—It should be noticed that the symmetrical distribution of the field assumed to exist in Figs. 375 and 376, only holds good at no load. When a current flows through the armature, it must produce another magnetic field of its own, and the actual flux in the air-gap will be the resultant obtained by superposing this field upon the original one. For an adequate discussion of this important subject, the student must consult text-books on electrical engineering; but we may state that in the case of a generator, the final result is very much as if the original field had been slightly dragged round in the direction of rotation, and thereby distorted. As this displaces the positions of zero E.M.F., it necessitates a similar forward displacement, or "lead," of the brushes in order to bring them into the correct position. Again, as the strength of the armature field depends upon the strength of the current, it must vary with the output of the generator, and in consequence it was once necessary to alter the lead of the brushes, when the load altered, in order to avoid sparking. In modern practice, machines are made to work with fixed brushes at all loads, a result obtained partly by improved design, and partly by taking advantage of the properties of carbon brushes, or of reversing poles, as mentioned in the next paragraph.

Commutation.—In the simple case shown in Fig. 376 we see that the current in each armature conductor must be reversed in direction as the latter passes a brush; whilst Fig. 377 tells us that if, as is generally the case, the conductors are grouped in sections, the current in each section as a whole must be reversed as that section passes a brush. Hence, while the current is stopping, there will be produced a self-induced E.M.F. in the *same* direction, and therefore tending to retard the stoppage; and another in the *opposite* direction while the current is starting again, retarding its rise. These effects lead to very injurious sparking between the brushes and the commutator bars, which must be eliminated in some way. The difficulty may be minimised, to begin with, by making the number of turns in each section as small as possible, because the magnitude of the induced E.M.F. is proportional to the square of the number of turns. This means using numerous sections and many commutator bars. It is then customary to provide what is known as a "reversing field" at the brushes. According to Fig. 376, the correct position for a brush is exactly at the neutral point of the field, so that the section undergoing commutation is not at that moment cutting the lines of force. If, however, the brush is advanced slightly in the direction of rotation, the section will be moving in a magnetic field as it passes the brush, and the E.M.F. induced in it by this motion will be in the *opposite* direction to (and can be made to neutralise) the self-induced E.M.F. due to the stoppage of the current. Moreover, as each section passes the brush, there is a brief instant during which it is removed from the main circuit, and is short-circuited upon itself through the brush as the latter bridges across two consecutive commutator bars. Under these circumstances, the reversing field can induce an independent current in the section, and ideal commutation is obtained when this local current reaches the same strength as the working current at the instant the section becomes part of the winding beyond the brush.

These actions are so important that it is becoming an increasingly common practice to provide the field magnets with small auxiliary poles near the brushes, whose sole purpose is to provide the necessary reversing field, and thus to secure sparkless commutation at all loads.

Carbon brushes, wide enough to bridge over several commutator bars at once, are now invariably used. The extra width is beneficial in prolonging the time available for commutation, whilst carbon is remarkably effective in suppressing slight tendencies to spark, chiefly on account of (1) its relatively high specific resistance, and (2) the peculiar way in which the resistance of a carbon-metal contact varies with the pressure, the practical result being that energy, otherwise expended in a spark, is harmlessly dissipated as heat in the brush.

Multipolar Machines.—For convenience, we have thus far considered two-pole or "bipolar" generators, but at the present time multipolar types are preferred—four poles being used for small machines, and six, eight, or more poles for larger outputs. The poles are N and S alternately, and, with the usual form of armature winding, there are as many brushes as there are poles. For instance, an eight-pole machine will have four positive brushes all connected together to form the positive terminal, and four negative brushes similarly connected; and instead of there being two paths *in parallel* through the armature winding from brush to brush, there will be eight such paths, so that, when calculating the E.M.F. by the method given on p. 530, $\frac{1}{8}$ of the total conductors must be reckoned *in series.*

The parts of a small four-pole generator of modern type (made by the Lancashire Dynamo and Motor Co., Manchester) are shown in Fig. 382, which will serve to indicate the usual construction of such machines. A machine of this size gives out a current of 40 or 50 amperes at 500 volts pressure, but, of course, the winding may be adapted to any output of about the same number of watts.

The advantage of the multipolar type, apart from increased symmetry and better mechanical protection of the windings, is chiefly due to the fact that large current outputs may be obtained without using abnormally thick armature conductors. For instance, if the full load current is to be 400 amperes, each conductor in a bipolar type must carry 200 amperes, but with eight poles each conductor has only to carry 50 amperes, and can, therefore, be of proportionately

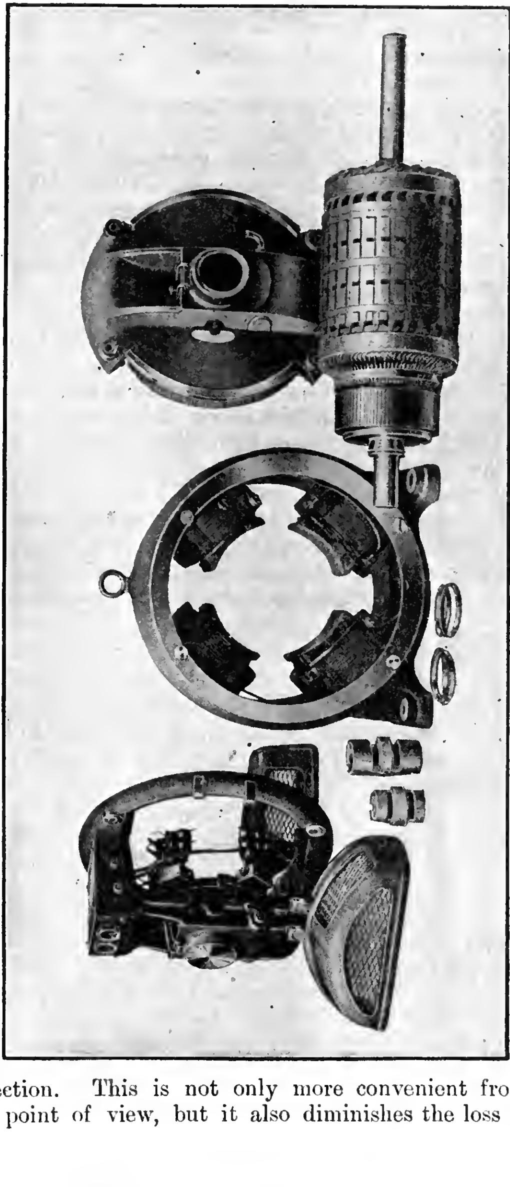


FIG. 382.

smaller section. This is not only more convenient from the constructive point of view, but it also diminishes the loss due to eddy currents in the copper itself, which always occurs in thick conductors; and, what is of much greater importance, it greatly facilitates the sparkless collection of current, owing to the fact that the self-induced E.M.F. which impedes commutation is proportional to the current in the section undergoing reversal, and therefore will be only $\frac{1}{4}$ as great as in the previous case.

Efficiency of Direct-current Generators.—Evidently, the mechanical power supplied to drive a current generator must be greater than the output of electrical power by an amount equal to the various sources of loss. These are (1) the C^2R loss in the armature and field windings, (2) the eddy current loss, due to induced currents in the framework, iron cores, &c., (3) the hysteresis loss in the armature core, and (4) frictional losses, due to bearings, brushes, air resistance, &c.

Theoretically, the first source of loss can be made as small as we please (see p. 252) by making the resistance of the armature and series coils sufficiently small, and that of the shunt coil sufficiently large. As this increases the size of the machine, and, therefore, the first cost, there is a practical limit which it does not pay to exceed. Obviously, these losses are (apart from that in the shunt coil) proportional to the load.

The eddy current loss can be kept down by careful attention to details of construction, but, as we have said, it can never be eliminated. It is proportional to the *square* of the speed, because the E.M.F. producing the eddy currents is *directly* proportional to the speed, and, in a circuit of constant resistance, the watts lost are proportional to the *square* of the E.M.F. $\left(\text{watts} = \frac{E^2}{r}\right)$.

The third loss depends upon the fact that the iron in the armature core is continually passing through cycles of magnetisation. As there is the same loss in each cycle, it is proportional to the speed, and can only be reduced by carefully selecting the iron used for the armature stamping.

The fourth or frictional loss is, of course, unavoidable, although in well-designed machines it is relatively small. Like the hysteresis loss, it is proportional to the speed.

Now, efficiency may be defined as the ratio $\frac{\text{Power given out}}{\text{Power supplied}}$, and if e be the P.D. between the terminals and C the current given out, this may be written—

$$\text{Efficiency} = \frac{eC}{eC + (C^2R \text{ loss in windings}) + (\text{eddy current loss}) + (\text{hysteresis and friction losses})}$$

The student must be referred to technical manuals for the methods adopted in measuring the various losses, and thereby determining the efficiency. Here, it is sufficient to say that in the case of large machines at full load it is well above 90 per cent.

Direct-current Motors.—These do not differ in any essential respect from current generators, although certain details may be modified to suit the working conditions. For instance, Fig. 382 represents a motor just as well as it represents a generator. In generation, the force resisting motion is the force on armature conductors carrying a current in a magnetic field. Evidently, if the driving-power were suddenly removed, and if, at the same time, the current were kept flowing in the same direction *as before* from some external source, this force would still act and would drive the machine backwards as a motor. A reversal *either* of the field *or* of the direction of the armature current would reverse the direction of the force, and therefore of the rotation; but if *both* be simultaneously reversed, the direction of the force is unaltered. This is exactly what is found to occur with a series-wound machine—it runs in one direction as a generator and in the opposite direction as a motor—but its direction of running as a motor is not altered by reversing the current through the machine as a whole.

A shunt-wound machine behaves somewhat differently. If a current from some external source be sent through it, the student will see, by drawing a simple diagram, that it can never flow in the same direction as before in *both* armature and field. In one of the two windings it must be reversed, and this reverses the direction of rotation, so that a shunt-wound machine runs in the *same* direction both as generator and as motor. Compound windings are very little used for motors, and need not be discussed here.

Contrasting the three windings, it will be sufficient to say that a series-wound motor slows down under a heavy load and races dangerously under small loads; a shunt-wound motor runs at nearly the same speed at all loads within its range, although it slows down slightly as the load increases; a compound-wound motor can be made to run at practically constant speed at all reasonable loads.

Back E.M.F. of a Motor.—In accordance with the principles stated on p. 371, an induced E.M.F. will be produced in the armature as soon as the motor begins to rotate. This E.M.F. will be exactly the same in value as it would be if the machine were running as a generator at the same speed and in a field of the same strength, with the important difference that it will be now in opposition to the impressed voltage.

It is, as we have already pointed out (see p. 372), this back E.M.F. which is the essential factor in the performance of external work. For instance, if E be the impressed voltage (assumed to be constant) and r the internal resistance of the motor, the current through it at rest will be $\frac{E}{r}$ amperes. As r is quite small, perhaps only a fraction of an ohm, this current is excessively large, and would burn out the armature in a very short time. Hence, the full working voltage is never applied to a motor at starting; an adjustable resistance being included in the circuit, which is gradually cut out as the speed rises. When running, $C = \frac{E - e_m}{r}$, where e_m is the back E.M.F. of the motor; the power supplied to the motor is EC watts, and the power developed is $e_m \times C_a$ watts (here C_a is the current through the armature. This is obviously the same as C in a *series* motor, but slightly less in a *shunt* motor). The difference $EC - e_m C_a$ is equal to $C^2 r$, and represents the power wasted as heat in the windings. However, not all the power represented by $e_m \times C_a$ is available for external work, for from it must be deducted the losses due to hysteresis, eddy currents, and friction.

Exp. 245. Connect up any small machine, that may be available, to a battery or to supply mains, including in the circuit an adjustable resistance and an ammeter. Watch the ammeter as the motor starts up from rest, and notice that the current tends to decrease as the speed increases. (If the motor is series wound, care must be taken to keep it from racing dangerously.) Slow it down by holding something against the pulley, and notice that the current increases, reaching a maximum when the motor is forcibly prevented from turning.

When a motor is run without a load, it speeds up, thereby reducing the current, until the power (EC) taken from the source is just sufficient to balance the total losses occurring at that speed. With a well-designed machine, the losses are small, and hence the speed must be great enough to make e_m nearly equal to E, and the current correspondingly small. Obviously, if a machine could be made in which no loss occurred, it would, at no load, speed up until e_m was actually equal to E (when the current would be zero), but, of course, it would not then be doing any work.

When the machine is shunt wound, its fields are always fully magnetised (because the impressed E.M.F., E, is always acting directly on the field coils), and in this case a very slight increase of speed, at no load, is sufficient to raise the back E.M.F. to the required value. When it is series wound, the field strength decreases as the current decreases, and this means that, at no load, a very much higher speed is necessary to raise the back E.M.F. to the required value.

When a load is applied, it tends to slow down the machine; and, if shunt wound, a very slight decrease in speed will lower the back E.M.F. sufficiently to permit the necessary increased current to flow; if series wound, the field strength rises as the current increases, and this raises the back E.M.F., so that a much greater slowing down must take place before the current can rise to the amount required.

The series-wound motor is much more useful than the series-wound generator. It is almost exclusively employed for traction purposes, because the tendency or effort to rotate (known as the torque or turning-moment) is greatest when the resistance to motion is greatest. This is because the turning-moment depends entirely upon the force on a conductor carrying a current in a magnetic field, and thereby depends, in a given armature, merely upon the strength of the current and the strength of the field. Now, in a series machine,

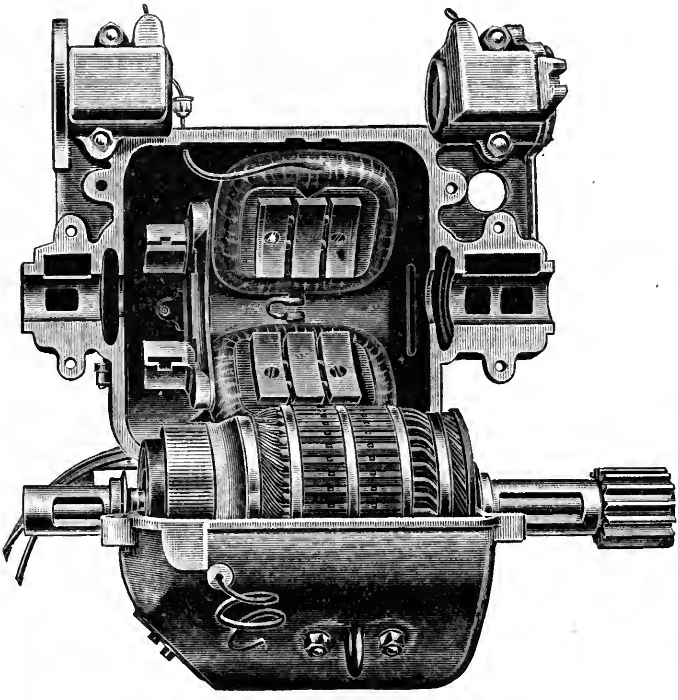


FIG. 383.

these two factors increase simultaneously, being greatest when the machine is at rest, *i.e.* forcibly restrained from turning. Thus, the motor makes its greatest effort when it is most needed.

Fig. 383 is a four-pole traction motor, made by Messrs. Dick, Kerr, & Co., and serves to show the nature of the modifications in design necessitated by the special conditions of service. As it has to be carried beneath a car, it must be dust-proof, take up little space, and be able to stand large temporary overloads without injury.

Alternating Current Generators.—These are simpler in principle than direct-current machines, because, as we have seen, the induced E.M.F. in a coil rotating in a magnetic field is naturally alternating. Hence, the commutator is unnecessary, and thus many possible sources of trouble are eliminated. On the other hand, a direct current is required to excite the fields—usually supplied by a separate D.C. machine, known as an "exciter."

On p. 352 we have considered a very simple type of alternator,

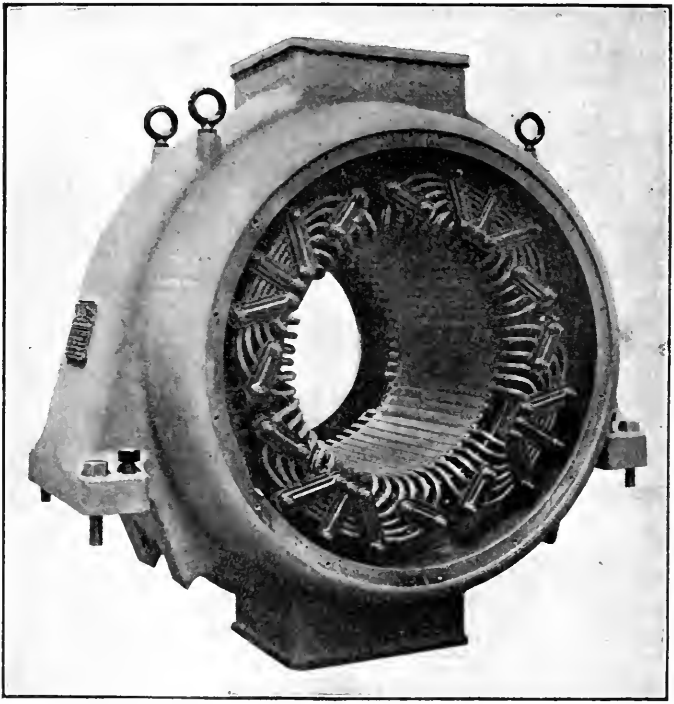


FIG. 384.

and have shown that in a two-pole field the frequency is equal to the number of revolutions per second. To obtain a frequency of 50 cycles per second (a very usual value) from a bipolar machine would, therefore, mean an excessive speed, and hence the construction is always multipolar, for a little consideration will show that if there are n pairs of poles there will be n complete cycles per revolution.

During recent years, however, turbine-driven alternators, running at very high speeds, have been largely used, and this has enabled the necessary frequency to be obtained with two or four poles (more usually the latter). It is the invariable practice to make the field-magnet system rotate inside a stationary armature, because the former is simpler in structure and it is easier to give it the necessary strength and rigidity to withstand centrifugal forces.[1] The stationary armature has also the advantage of enabling the current to be collected from fixed terminals, instead of through rubbing contacts—a matter of considerable importance in the case of large currents.

In Fig. 384 is shown the armature of an alternator, made by Messrs. Siemens Bros., to whom we are indebted for the illustration.

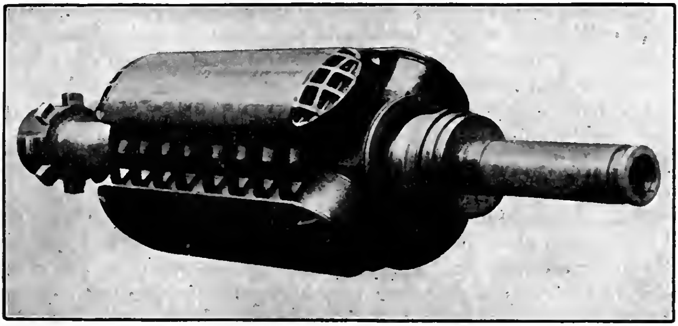


Fig. 385.

The active conductors are embedded in slots in the stampings, and the end connections are held in position by very strong clamps bolted to the framework. As these end connections are not in the magnetic field, there is, under ordinary conditions, no force acting upon them except the very small one due to the currents in the neighbouring conductors, and the necessity for much care in supporting them would not naturally be anticipated. However, the experience of all makers of alternators of this type has shown that, when an accidental short circuit or other mishap occurs when working, the end connections are liable to be torn out of place and broken, thus completely wrecking the armature, although the part of the winding embedded in the slots is uninjured. This happens because the current under such circumstances may reach an enormously high value for a brief instant, thus producing momentary forces of very great magnitude. Hence, all modern armatures are provided with clamps as shown in the figure.

A four-pole field magnet for a turbine-driven alternator is shown in Fig. 385. The four polar projections are each wound with an

[1] It is a convenient and usual practice to speak of the stationary and moving parts as the "stator" and "rotor" respectively.

exciting coil, and the whole structure is very firmly braced together. The exciting current is led into and out of the windings by means of two insulated rubbing contacts on the shaft, which, at the further end, is arranged for direct coupling to the engine. This field magnet was constructed by Messrs. Dick, Kerr, & Co., and is therefore not the one actually used with the armature shown in Fig. 384. There are, in fact, two distinct types of field magnets in use at the present time: (1) the salient pole type, as shown in Fig. 385; (2) the non-salient pole or cylindrical type, which resembles externally the ordinary drum armature of a direct-current machine, such as is shown in Fig. 382, and is built up of slotted stampings in much the same way. It is wound so that the necessary poles are produced on the cylindrical surface, although there are no pole-pieces. It is difficult to say which form will finally survive—probably the latter.

A cylindrical four-pole field magnet of this kind is used with the Siemens armature described above (Fig. 384), but as an illustration would merely show a plain cylinder supported on a shaft, it is not given here. The output of this particular machine is 1000 kilowatts at a pressure of 2200 volts, *i.e.* the full-load current is about 450 amperes. The frequency is 50, and as it is a four-pole machine, there will be two complete cycles per revolution, and hence the speed must be 1500 revolutions per minute. The surface speed at the circumference of the rotor is therefore very high, and the greatest care must be taken to secure perfect balance. The machine is totally enclosed (excepting for air inlets and outlets), to minimise noise, and it is usual to provide forced ventilation by means of fans fastened to the shaft.

The Winding of Alternator Armatures.—This is very simple in principle, as there is only one path through the winding from terminal to terminal. Two varieties are shown in Fig. 386.[1] The machine is supposed to be of the ordinary slow-speed type, having a large number of pole-pieces, of which only four are shown for convenience, and for the sake of clearness the armature conductors are projected on the polar faces. It is evident that as the poles move over the conductors, the induced E.M.F. will be in one direction under a North pole and in the opposite direction under a South pole, as shown, so that it is merely necessary to join the active conductors in series, leaving two free ends to be connected to the terminals. This condition is satisfied by either form of winding—electrically they are equivalent—but whereas the end connections in the upper figure are the shorter, thus reducing the resistance and the amount of inactive wire, the lower form—sometimes called a hemitropic winding—has the advantage of being more readily adapted to the varying requirements of different designs.

Polyphase Windings.—It will be noticed that only half the available space is filled up with active conductors. Although in practice rather more than half is utilised, this is an essential condition in "single-phase" alternator windings, because there must be an instant at which *all* the active

---

[1] This figure is taken from *Electric Light and Power* (Brooks and James), published by Messrs. Methuen.

conductors are so placed that there is no induced E.M.F. Evidently it would be possible to put on two independent windings, each occupying half the available space (thus increasing the output for a given size), and it is also obvious that one alternating E.M.F. will be a maximum at the instant the other is zero, *i.e.* they would differ in phase by $\frac{1}{4}$ period, or be "in quadrature." The result is a "two-phase" alternator.

When the space is filled with three independent and symmetrical windings, the result is a "three-phase" alternator. Such machines have most important properties, which, however, cannot be discussed here.

Alternating Motors.—In practice, it is more difficult to construct satisfactory motors for alternating than for direct currents. Although an

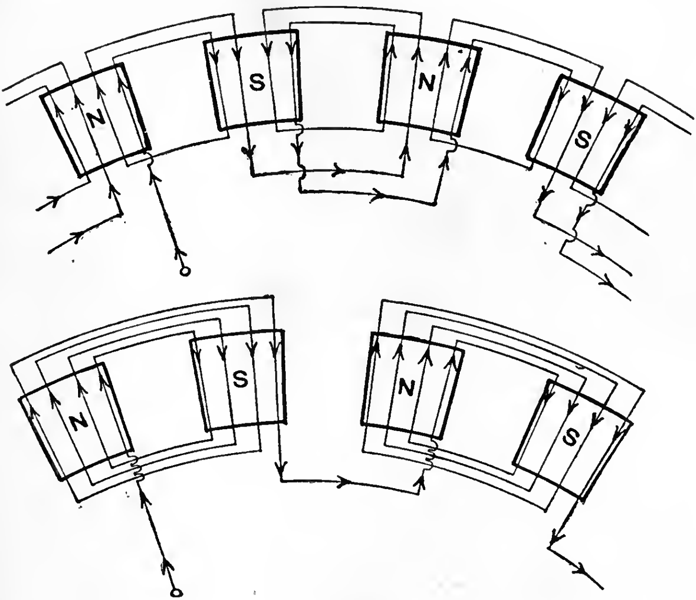


FIG. 386.

ordinary alternator *will* run as a motor, when its armature is supplied with an alternating current of suitable frequency, the property is of little value, because the machine will not start from rest, and it also requires an auxiliary direct current to excite its field. Moreover, the speed is determined by the frequency and the number of poles; it is, therefore, constant at all loads, and cannot be varied or controlled as in the case of direct-current motors. The first practical solution was based on the properties of the two and three-phase currents mentioned above, and such "polyphase" or "induction" motors are still largely used. These motors are quite distinct in principle from direct-current motors, being based on the properties of rotating magnetic fields. Single-phase motors took longer to develop, and several distinct varieties are in existence. One of these is similar to the polyphase motors, and another is really a modified series motor of the ordinary type with laminated field magnets—to avoid eddy currents. Motors of this kind can be made to run on D.C. or A.C. circuits indifferently, and are successfully used for electric railways.

Examples.—1. A motor running light takes 1 ampere at 100 volts. If its resistance is 20 ohms, how much power is spent in turning the armature, and how is it expended?

(B. of E., Day, 1907.)

Writing $C = \frac{E - e_m}{r}$, we have

$$1 = \frac{100 - e_m}{20}, \text{ or } e_m = 80 \text{ volts.}$$

The power supplied to the motor is $EC = 100 \times 1 = 100$ watts; the power expended in turning the armature is $e_m \times C = 80 \times 1 = 80$ watts, and the C^2r loss is $1^2 \times 20 = 20$ watts, which makes up the difference. But of the 80 watts expended in turning the armature, only a portion (of amount unknown) is spent in doing useful work—the balance being required to overcome resistances to motion due to eddy currents, hysteresis, and mechanical friction.

2. A motor whose resistance is 6 ohms is connected to a supply at 100 volts, and it is found that 3 amperes flow through the motor. Neglecting friction, and losses due to hysteresis and eddy currents in the iron, find the amount of mechanical work done by the motor.

(Lond. Univ. B.Sc., 1908.)

As before, we have $C = \frac{E - e_m}{r}$

$$\therefore \quad 3 = \frac{100 - e_m}{6}, \text{ or } e_m = 82 \text{ volts.}$$

Hence, with the above-mentioned limitations, the power expended in mechanical work is $e_m \times C = 82 \times 3 = 246$ watts. The "amount of mechanical *work*" cannot be stated unless the time for which the motor runs is given—the question in this respect being a little ambiguous, confusing "work" with "power."

3. A battery of 50 volts and negligible resistance supplies a current of 5 amperes to a motor at some little distance. The E.M.F. across the terminals of the motor is found to be 35 volts. Find the ratio of the heat lost in the leads to the energy supplied to the motor.

(B. of E., Day, 1908.)

The pressure required to send the current through the leads is $50 - 35 = 15$ volts, and the power expended in them will be $15 \times 5 = 75$ watts.

The power supplied to the motor is $EC = 35 \times 5 = 175$ watts.

Now, the heat produced in the leads per second is proportional to the power expended therein, and the energy supplied to the motor per second is measured by the power it takes in watts.

$\therefore$ we have $\dfrac{\text{Heat lost in leads (measured as work)}}{\text{Energy supplied to motor}} = \dfrac{75}{175} = \dfrac{3}{7}$.

The question is, however, worded ambiguously, for in asking for the ratio of "heat" to work, it should be stated clearly in what units the "heat" should be expressed.

4. Enumerate the principal sources of waste of power in an electric motor. Current is supplied to a series motor at 100 volts, the resistance of the circuit being 0·5 ohm. Determine the power expended in turning the armature when the current is 10 amperes. Determine also the current when the power thus expended is a maximum. Compare the values of the electric efficiency in the two cases.

(1) We have $C = \dfrac{E - e_m}{r}$

$$\therefore \quad 10 = \frac{100 - e_m}{\cdot 5}, \text{ or } e_m = 95 \text{ volts.}$$

Also, Power expended in turning armature $= e_m \times C = 950$ watts.

$$\text{Efficiency (electrical)} = \frac{\text{Power developed}}{\text{Power supplied}} = \frac{e_m \times C}{E \times C} = \frac{e_m}{E} = \frac{95}{100}, \text{ or 95 per cent.}$$

(2) Now C increases as e_m decreases, and there will be some value of C for which the product is a maximum. To determine this value, we may write

$$\text{Power developed} = e_m \times C = \frac{e_m(E - e_m)}{r}.$$

The numerator of this expression is the product of two quantities whose sum is a constant, and therefore the product will be greatest when the quantities are equal,

$$\textit{i.e.} \text{ when } \quad e_m = E - e_m, \text{ or } e_m = \frac{1}{2}E.$$

When this condition is satisfied, the motor is developing the maximum power. In the case given, it evidently corresponds to a current of 100 amperes. The efficiency, however, is only 50 per cent.

This theorem is often misunderstood. It is not of any practical importance, for a motor is never worked at such a rate, and it would probably be burnt out in a few seconds by the excessive current if an attempt were made to do so.

Exercise XXV

1. How do magneto-electric machines differ from dynamo-electric machines?

2. Describe the construction and explain the action of a magneto-electric machine for the conversion of mechanical work into current energy. (B. of E., 1899.)

3. Describe the two chief forms of armature in continuous current dynamos.

4. What is the meaning of "self-excited machines"? Describe the various methods of winding the field magnets.

5. A battery is employed to drive a magneto-electric engine. Does the rate of consumption of zinc increase or decrease (and why) if the speed of the engine is increased by lessening the work it has to do?

6. Describe the general principles of the construction of a simple form of dynamo. (B. of E., 1897.)

7. State the difference between a series, a shunt, and a compound dynamo. Why is it not advisable to use a series dynamo for charging storage cells? (B. of E., 1905.)

8. Describe a drum armature. What are the advantages of this form of armature over the Gramme armature. (B. of E.. 1906.)

9. Give a diagrammatic sketch of a shunt-wound motor. How will the speed of such a motor vary under a given load when the resistance in the field circuit is altered? (B. of E., 1907.)

10. Describe, illustrating your answer with a diagram of the winding of the armature, a motor of about 10 horse-power. If 90 per cent. of the energy supplied is turned into useful work, what current would the above motor take at 100 volts? (B. of E., 1908.)

11. Describe any simple form of electric motor to work on a direct-current system. (B. of E., 1910.)

12. In some forms of high-tension magneto machines (small machines with permanent magnets used to produce a spark to fire the charge in internal combustion machines) the armature has two sets of windings, one consisting of a few turns of thick wire, and the other of many turns of thin wire, and the spark occurs in the thin wire circuit when the thick wire circuit is interrupted. Explain the action of such an instrument. (B. of E., Stage III., 1910.)

13. A Gramme armature, intended for a 2-pole field, has 120 turns of conductor wound upon it. Give the total flux required through armature core if an electromotive force of 100 volts is to be induced at a speed of 1000 revolutions per minute. (C. and G., 1894.)

14. It is found that a motor (shunt-wound, separately-excited, or series-wound) that is supplied from mains at constant pressure runs faster if its field magnet is weakened. Explain (1) the reason for this fact; (2) what arrangements you would make to weaken the field magnet in the case of each of the three sorts of motors mentioned. (C. and G., 1896.)

CHAPTER XXXII

ELECTRIC LAMPS

Electric lamps may be divided into two classes—

(1) those in which the source of light is a solid body heated by the passage of a current;

(2) those in which the source of light is a gas or vapour rendered luminous by a current.

The distinction is important, because in the former class the efficiency of the lamp (*i.e.* the ratio of the candle-power produced to the watts supplied) is almost entirely dependent upon the temperature, although it is limited by the fact that, at all practicable working temperatures, an enormous amount of energy is wasted in the form of useless heat radiation. In the latter class, the emission of light is not necessarily a question of temperature, and hence it is possible to obtain a much higher efficiency.

In certain cases, *e.g.* in arc lamps, the emitted light is to some extent due to both sources—chiefly to the first in the ordinary "open" arc, and to the second in the more recent "flame" arc.

Incandescent or Glow Lamps.—Of all forms of lamp, these are the most extensively used. Evidently, they are to be included in the first category, inasmuch as they are a direct and simple application of the C^2R or Joulean heating effect. It will be convenient to summarise as follows the conditions which should be satisfied in such a lamp in order to obtain the maximum efficiency.

(1) The surface area of the heated substance should be great compared with its mass, in order that the radiating surface may be as large as possible. This leads naturally to the use of a thin filament.

(2) The material of the filament should permit of its being used at extremely high temperatures, without too rapid disintegration. The laws of temperature radiation are such that a small increase in working temperature produces a relatively enormous increase in the emitted light.

(3) As it is convenient to supply power to such lamps in the form of a small current at a high voltage, the filament should have a high resistance. Hence, it is an advantage if the material naturally possesses a high specific resistance.

(4) The filament should be placed in an exceedingly good vacuum

in order to keep it from being destroyed by the impact of the rapidly moving gas particles. An additional advantage is due to the fact that the loss of heat by convection is greatly reduced. (A badly exhausted lamp is much hotter to the touch than a well exhausted one.)

The first incandescent lamps, developed by Edison between 1878 and 1880, were made with platinum filaments. The chief disadvantage of this material arises from its comparatively low fusing point, which means that it has to be worked dangerously near its temperature of fusion to give out any useful amount of light.

About the same time, Swan introduced the use of carbon, which has the advantage of infusibility. At high temperatures, however, it softens and tends to volatilise, thus gradually blackening the bulb. But so conspicuous did this advantage of carbon appear, that all other substances were discarded, and until recently there seemed little probability of its being superseded.

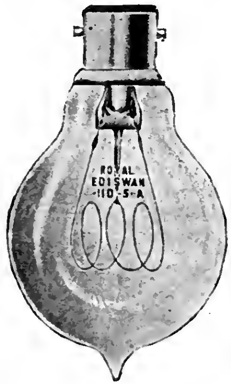


FIG. 387.

Fig. 387 shows a recent type of Edison and Swan carbon filament lamp. The raw material of the filament consists of cotton wool. This is usually dissolved in a strong solution of zinc chloride, and the viscous mass is then "squirted" through a fine nozzle into a hardening liquid (*e.g.* alcohol), which removes the zinc chloride. The threads thus formed are wound into shape, and gradually raised to a very high temperature in crucibles packed with carbon, to exclude the oxygen of the air, the result being the well-known thin carbon filament. During the operation of exhausting the bulb, both it and the filament are raised to a high temperature, so that any gases absorbed by the glass and carbon may be expelled.

Life and Efficiency of Incandescent Lamps.—The candle-power of a carbon filament lamp usually increases a little during the early stages of use; it then falls slowly but continuously, until at last it becomes desirable to replace the lamp by a new one. The "useful life" of the lamp is regarded as extending until the candle-power falls to 80 per cent. of its initial value.

As we have just mentioned, the efficiency of the lamp is really the value of the ratio $\frac{\text{Candle-power emitted}}{\text{Watts supplied}}$, *i.e.* it should be measured in candle-power per watt, although as a matter of fact it has become customary to express it in watts per candle-power.

It must be borne in mind that the normal working conditions are a matter of compromise between high efficiency and long life. For example, a certain carbon lamp, marked 100 volts, gave 16 candle-power, when worked at that voltage. It took ·56 ampere, and would have a useful life of about 800 hours, its efficiency being $\frac{100 \times \cdot 56}{16} = 3{\cdot}5$ watts per candle. When run at 77 volts, it took ·42 ampere and gave out 3 candle-power, its efficiency being $\frac{77 \times \cdot 42}{3} = 10{\cdot}8$ watts per candle. Under these conditions it would have a very much longer life than in the first case. On the other hand, at 112 volts it took ·62 ampere and gave out 29 candle-power, its efficiency being $\frac{112 \times \cdot 62}{29} = 2{\cdot}4$ watts per candle; but at this voltage its life would be considerably shortened.

Under ordinary circumstances, the average efficiency of a carbon lamp may be taken as about 4 watts per candle.

Resistance of the Filament.—As lamps are worked at a definite voltage, it follows that the smaller the candle-power, the higher must be the resistance of the filament, if the efficiency is to be maintained constant. For example, if a 16 candle-power lamp works at 100 volts and takes 64 watts, its resistance, when hot, must be given by $\frac{E^2}{r} = 64$, or $r = \frac{(100)^2}{64} = 156{\cdot}25$ ohms; whereas a 5 candle-power lamp of the same efficiency would take 20 watts, so that we have $\frac{E^2}{r} = 20$, or $r = \frac{(100)^2}{20} = 500$ ohms. This extra resistance can only be obtained either by reducing the section of the filament or by increasing its length, and hence it is always more difficult to make satisfactory lamps of small candle-power. This difficulty is still further increased when the voltage is raised, *e.g.* suppose that the 5 candle-power lamp is to work at 200 volts, then

$$\frac{(200)^2}{r} = 20, \text{ or } r = \frac{(200)^2}{10} = 2000 \text{ ohms.}$$

That is, if the voltage is doubled, the resistance must be quadrupled in order to obtain the same efficiency.

Metallic Filaments.—Within the last few years, lamps having metallic filaments have been introduced, which, apart from accidents, combine a very long useful life with the extremely high efficiency of from 1 to 2 watts per candle. The increase in efficiency is due to the fact that, *for a given useful life*, they can be run at a higher temperature than carbon. Evidently only metals of extremely high melting-point can be used.

The first lamp of the kind was introduced by Welsbach in 1902. Its filament was made of osmium, but owing to various practical difficulties it never passed into general use. In 1905, Siemens brought out the "tantalum" lamp, shown in Fig. 388, which really initiated a new era. Since then many other metals and substances have been tried, of which tungsten has been found to be most satisfactory, and, at the present time, lamps made of either tantalum or tungsten are almost universally used. As the specific resistance of metals is lower than that of carbon, the new filaments have to be made longer and finer than carbon filaments in order to obtain the required resistance, and at first they were very fragile. Although much improved in this respect, they are somewhat easily broken, and hence carbon lamps are still used in positions subjected to much vibration.[1] Again, metal filaments differ from carbon in having a positive temperature coefficient, *i.e.* their resistance increases with temperature, which on the whole is a distinct advantage. This fact is clearly brought out in Experiment 165, p. 258.

FIG. 388.

Use of Transformers with Metal Lamps.—On account of the smaller resistance necessary, it is easier to make thoroughly satisfactory and durable lamps for low voltages (*e.g.* 25 to 50 volts) than for the ordinary working pressure of supply mains, which is usually either 100 or 200 volts. Hence, where the pressure is alternating, small transformers are much used for house lighting, for the purpose of lowering the voltage to suit such lamps. The efficiency of the transformer is very high, and the small loss occurring in it is more than counterbalanced by the increased durability of the lamps.

Nernst Lamp.—In this lamp, advantage is taken of the fact that non-metallic substances have negative temperature coefficients, and hence may conduct fairly well at sufficiently high temperatures. The appearance of one of the many patterns is shown in Fig. 389,[2] and a key diagram in Fig. 390. The source of light—termed the "glower"—is a short and relatively thick rod, composed of certain metallic oxides (chiefly zirconia, with a little yttria). It is non-conducting at ordinary temperatures, but if connected to the supply mains, and then heated by some auxiliary means, it at length conducts well enough to

[1] Metallic filaments are more brittle when cold than when hot. Many breakages would be avoided if the lamps were always switched on before attempting to clean the bulbs or shades.

[2] Figs. 389, 391, and 393 are taken from Slingo and Brooker's *Electrical Engineering*.

maintain its high temperature by means of the C^2R heat developed in it. For the initial heating a spirit lamp may be employed, but usually a special heating coil (H) of fine platinum wire embedded in porcelain is provided, and also a small automatic switch, S, which cuts out this "heater" when the glower lights up. As, however, the resistance of the glower continues to fall (and thereby to increase the current), its condition is unstable, and without some regulating contrivance it would soon be destroyed by the excessive current. This is provided by intro-

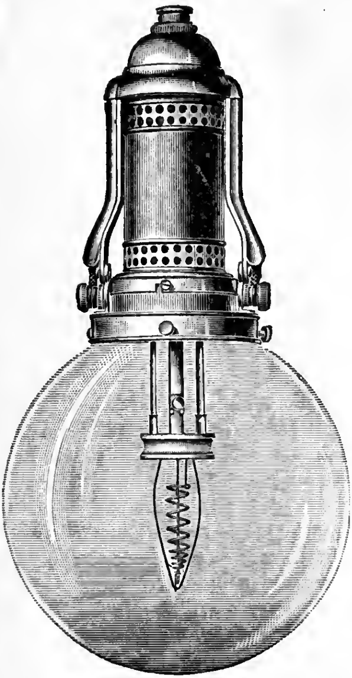


Fig. 389.

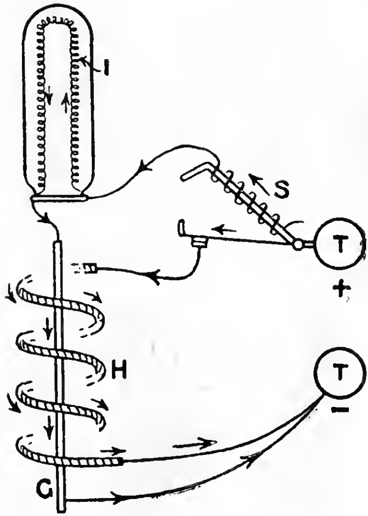


Fig. 390.

ducing, in series with the glower, a spiral of very fine iron wire, I, sealed up in a glass bulb containing hydrogen at low pressure. Now, the temperature coefficient of iron (which is positive, like metals in general) is remarkable for its sudden and abnormal increase at a temperature near redness, and hence, if the conditions are such that the iron wire is kept by the working current at a temperature just below redness, it will be in its most sensitive state, and as its behaviour is exactly opposite to that of the glower, the resistance of the lamp as a whole remains constant. In spite of the complicated details, and of the loss of energy in the iron wire, these lamps are fairly strong, and have the very high efficiency of about ½ watt per candle. Perhaps their greatest disadvantage is due to the fact that they do not light up instantly when the current is switched on. It will be noticed that they do not require an exhausted globe, and they also differ from incandescent lamps in working better, and in being more efficient at 200 volts than at lower pressures.

The Electric Arc.—If two carbon rods, connected to some source giving a steady pressure of about 500 volts, are brought into contact and then separated by a short distance—say $\frac{1}{10}$ inch—the current persists across the gap, and the carbon tips become intensely hot, emitting a very white and brilliant light. In order to avoid too great a rush of current, when the carbons are in contact, a resistance of from 1 to 2 ohms should be included in the circuit; in fact, such a resistance is necessary for other reasons in order to obtain steadiness of working. The P.D. between the carbons must not be less than about 39 volts, and it need not be increased much above that value unless a longer arc is required. The current strength is largely a matter of choice, depending on the intensity of the arc required—in ordinary lamps it is usually from 8 to 10 amperes. The low voltage and relatively large current are highly characteristic of the arc discharge, and distinguish it from the spark.

The processes occurring in the gap between the carbons are analogous to those discussed in Chapter XXIX., the chief difference being that the necessary supply of charged ions must be derived from the electrodes themselves. The essential factor is the negative electrode, which in all probability is the chief source of ions. This electrode *must* be hot, or the current cannot pass, whereas it does not matter whether the positive electrode is hot or not; in fact, in some recent lamps, it is made of copper massive enough to remain fairly cool, and which works for a long time without appreciable loss. At the same time, with carbon electrodes, the positive carbon actually becomes the hotter, and hence burns away more rapidly, most of the light emanating from a small hollow or "crater," which forms at its tip, whilst the negative carbon remains pointed and is less luminous (see Fig. 391). With alternating currents, both carbons naturally behave alike, the crater is not formed, and the efficiency as a source of light is much decreased. The arc is, in fact, not at its best with alternating currents; under such conditions, it dies out at each reversal, and has

FIG. 391.

to start again in the opposite direction before the bridge of vapour ceases to conduct.

Carbon electrodes are not essential to the formation of the arc discharge, although they are almost universally employed for lighting purposes. With direct currents, arcs may readily be formed between electrodes of iron or of other metals, but not so readily in the case of metals of lower fusing point and greater affinity for oxygen. With alternating currents, it is always difficult, and as a rule almost impossible, to run metallic arcs.

Theory of the Arc.—The various actions going on in the arc are somewhat complicated, and we can only briefly indicate certain general results as follows :—

1. When the current strength is kept constant, the relation between the P.D. across the arc (E) and its length (l) is given by $E=a+bl$, where a and b are constants for electrodes of a given material. Evidently a is the minimum voltage required to form the arc at all, and, as already stated, this is about 39 volts with carbon electrodes and a direct current.

2. When the length of the arc is constant, the relation between the P.D. and current is given by $E=m+\frac{n}{C}$, where m and n are constants under fixed conditions. This peculiar relation between voltage and current means that if E increases, C decreases, and *vice versâ*. Hence, the arc does not obey the simple form of Ohm's law, *i.e.* it does not act as if it possessed a definite resistance.

Very important methods of obtaining electrical oscillations of high frequency depend upon this property of the arc (see p. 615).

Arc Lamps.—The distinguishing feature of practical lamps lies in the mechanism employed to control the distance between the carbons and to "feed" them together as they wear away. For descriptions of the many forms in actual use, the student must consult special treatises. Here it is only possible to mention briefly certain distinct types of lamps.

Open and Enclosed Arcs.—So far, we have referred to arcs working with free access of air, and, until about 1895, all lamps were of this kind. Under these conditions, however, there is a rapid loss due to combustion, and, as a consequence, the carbons must be renewed frequently. To reduce this loss, they may be enclosed in a semi-airtight globe, in which case they become surrounded by the products of combustion. As the ready access of oxygen is thus prevented, a pair of carbons last a much longer time, and the lamps therefore require less attention. This construction also lends itself to the working of longer arcs with a larger P.D. between the carbons, and it was introduced mainly to make it possible to run a single arc from 100-volt supply mains. The great disadvantage of enclosed arcs is their relatively low efficiency.

Flame Arcs.—In the types previously mentioned, the light is almost entirely derived from the white-hot carbon tips, the arc itself giving merely a feeble bluish light. In the "flame" arc, the carbons are impregnated with a large proportion of metallic salts (chiefly of calcium), and the vapours thus produced become intensely luminous, the colour of the light depending on the composition used. The carbons themselves are usually of relatively small diameter, in order that they may become thoroughly and uniformly heated at the tips, and, in the majority of lamps of this type, they are placed side by side, like the letter V. As the chief source of light is a luminous gas and not a solid, the conditions which limit the efficiency of light sources of the latter type are avoided, and a greatly increased efficiency is obtained, although the carbons burn away very rapidly. In round numbers, the efficiency of an open direct-current arc is about ·5 watt per candle; of an enclosed arc, about 1 watt per candle; whilst that of a flame arc may be $\frac{1}{5}$ watt per candle. The general appearances of the various types of arc are shown in Fig. 392, adapted from *Electric Light and Power* (Brooks and James: Methuen), in which (1) is an open arc, having as usual the positive carbon uppermost and of greater diameter than the other. An open alternating arc is shown in (2); while (3) gives the characteristic appearance of any form of enclosed arc. It will be noticed that the carbons are farther apart than in (1) and (2), and as oxygen is excluded, *they do not round off at the ends.* In (4) the arrangement adopted with flame arcs is seen, the carbon tips passing just inside a small shallow vessel known as an "economiser," which steadies the arc by excluding draughts, and also serves as a radiator. If left to itself, the discharge would not remain at the bottom; it would at once ascend, and either go out as the gap widened or do mischief in the framework, and hence an important part of the lamp mechanism is a simple electromagnet, excited by the working current, which creates a magnetic field at right angles to the discharge in such a direction as to force the latter downwards, and of such strength as to keep it well extended without actually blowing it out.

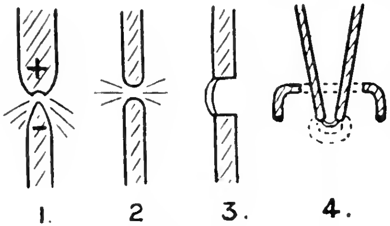


FIG. 392.

Magnetite and Titanium Arcs.—In America, magnetite arc lamps have been introduced with some success. In these, the electrodes are placed vertically, as in ordinary lamps, the lower one (negative) being a thin tube of soft iron packed with a mixture of magnetite (Fe_3O_4) and titanium oxide, and the upper (positive) being a massive copper rod. This rod is practically unaffected, the loss occurring entirely at the negative electrode. The light is derived from the vapour of iron and titanium, the idea being to obtain the advantages of the flame arc with more durable electrodes. These lamps are very efficient, and run a long time without attention. They are, however, inclined to flicker, and chiefly on this account they have not been much used in this country.

Mercury Lamps.—These are vacuum tubes with mercury cathodes, the discharge really being a very long arc in mercury vapour. One of the best-known forms is the Cooper-Hewitt lamp, shown in Fig. 393. The long glass tube is fixed to a support, which holds it in a slanting position, and is capable of being tilted. In the normal position the mercury—which forms the cathode—is in the bulb, B, making contact with a platinum wire sealed through the glass. The anode, A, is iron. To start the lamp, it is tilted until the mercury runs slowly from B to A, thus momentarily completing the circuit, and, when the lamp is allowed to return to its original position, the discharge persists through the vapour. As usual, when the conductor is gaseous, the current does not obey Ohm's law in its simple form, and hence it is necessary to include a steadying resistance in the circuit (not shown in the figure). The diameter of the tube is 1 inch and the length 40 inches for a working pressure of 100 volts and a current of $3\frac{1}{2}$ amperes. For running at 50 volts, the length is halved, the diameter and the current remaining unchanged.

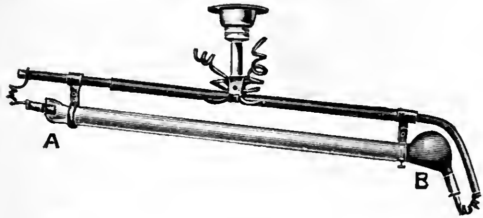


Fig. 393.

Originally the tubes were made of glass, but because glass is readily fusible, they are now frequently made of quartz. In the latter case, a strong smell of ozone is produced after the lamp has been working for a short time. This is due to the fact that the light is extremely rich in ultra-violet rays, and as quartz (unlike glass) is very transparent to such rays, they pass through it and produce ozone in the surrounding air. To avoid this action, as well as to obviate the injurious effect of ultra-violet light on the eyes, the quartz tube is surrounded by a larger one of flint glass.

Mercury lamps are, perhaps, the most efficient sources of light known, although the colour of the light is very unpleasant, owing to the almost complete absence of red rays. Hence, such lamps have at present a very restricted application.

The "Moore" Light.—This method of illumination has been used to some extent in America and elsewhere, although it seems very unlikely to meet with general acceptance at present. The "lamp" is really a long vacuum tube, built up in the room to be illuminated by welding together lengths of glass-tubing. In some cases, the total length is as much as 70 metres. This tube contains either nitrogen or carbon dioxide, according to the colour required (the former giving a reddish, and the latter a fairly white light). It takes about ·3 ampere at a pressure of 12,000 volts, which is supplied from alternating mains by a transformer of special form. A very complete system has been worked out, one ingenious device being a valve which admits a small quantity of gas, as it is required, to keep the pressure constant (for the vacuum in all tubes tends to improve with long running). The method may be regarded as an attempt to obtain light without a simultaneous production of useless heat, and is undoubtedly excellent in principle. The losses, however, which necessarily occur in the transformer, choking coils, &c., are sufficiently great to lower the efficiency to something like 1·5 watts per candle, while the complicated details required for high voltages and the difficulties of making repairs in the case of breakage are very serious objections.

It may be remarked that the nature of the gas used has an important influence on the efficiency. For the same consumption of energy, nitrogen gives out more light than carbon dioxide; and some recent researches appear to show that neon is superior in this respect to nitrogen.

Meaning of the term "Candle-Power."—We have used this term in the last few pages without comment, as being sufficiently definite for its purpose. It is, however, quite obvious that light sources in general do not radiate uniformly in all directions, so that their candle-power will depend upon the particular direction in which it is measured. For this reason, no exact meaning can be attached to the term when used in a general sense, and it has, therefore, become customary to express the power of a lamp in terms of either its "mean spherical candle-power," or its "mean hemispherical candle-power." The former may be defined as the candle-power of an ideal lamp, which would give out the same total amount of light emitted uniformly in *all* directions. In the latter, the light emitted above a horizontal plane is ignored, and it is, therefore, the candle-power of an ideal lamp in which the emission is uniform over a hemisphere.

Exercise XXVI

1. Write a short essay on the incandescent electric lamp, dealing particularly with any improvements which have been made within the last few years. (B. of E., 1910.)

2. State the conditions under which the electric arc can be formed and maintained between two carbon electrodes, and describe carefully the differences in shape, temperature, and rate of consumption of the two electrodes. (B. of E., 1903.)

3. A dynamo feeds 1000 16-candle-power lamps. What current must the dynamo supply, if the difference of potential at its terminals is 200 volts, and each lamp absorbs 3·6 watts per candle? (B. of E., 1903.)

4. Find the cost of running 20 16-candle-power lamps for 6 hours, if each lamp requires 3·6 watts per candle, and if each Board of Trade unit costs 4d.

5. Describe what arrangements you would make if you wanted to run a single 50-volt incandescent lamp off a 110-volt circuit. If the lamp takes ·5 ampere, how much power is taken from the mains, and how much power is absorbed by the lamp? (B. of E., 1911.)

CHAPTER XXXIII

MEASURING INSTRUMENTS

In the preceding chapters, various forms of measuring instruments have been repeatedly referred to and assumed to be available. We must now briefly outline the elementary principles on which their construction is based.

Voltmeters and Ammeters.—A voltmeter is an instrument with a scale graduated to read in volts the P.D. between its terminals, *i.e.* the P.D. between any two points to which it is connected by wires of negligible resistance. The ideal voltmeter should have an infinite resistance, for, if it takes an appreciable current, it may in certain cases alter the P.D. previously existing between the points to which it is connected, and, apart from this, many simple measurements depend upon the assumption that a voltmeter current is small enough to be negligible (see, for example, pp. 309 and 310).

An ammeter is an instrument graduated to read in amperes the strength of the current flowing through it. The ideal ammeter should have no appreciable resistance, otherwise its insertion in a circuit may alter the strength of the current previously flowing.

Both voltmeters and ammeters should be as dead-beat as possible.

Any property of a current (or charge), which can be made to produce motion of a pointer may be employed, the most important being—

(1) Electrostatic attraction between a movable and a fixed conductor.

(2) A coil moving in a magnetic field.

(3) Soft iron moving in a magnetic field.

(4) Expansion of a fine wire, by the heating effect of a current.

This list is by no means exhaustive, but it contains all the types we need consider here.

(1) **Electrostatic Instruments**, of which the electroscope and the quadrant electrometer may be regarded as types, are in practice limited to voltmeters. They are ideally perfect in having infinite resistance, and they also possess the very great advantage of working equally well on both direct and alternating circuits. But the force of attraction on which they depend is relatively feeble, and hence there is some difficulty in making instruments sufficiently sensitive to read low voltages, without being too delicate for ordinary commercial use. Lord Kelvin's multicellular voltmeters are perhaps

the best-known instruments of this class. One of the latest pattern, with part of the outer case removed, is shown in Fig. 394.[1] The moving system is a series of light metal vanes, V, mounted one above the other on a vertical rod, suspended by a fine wire, W. The bottom of the rod ends in a loop of wire (visible in the diagram), which dips into a small vessel containing a suitable liquid for damping the vibrations. To the top of the rod is attached a pointer, P, moving over a scale. In some patterns this scale is horizontal, but in the type illustrated it is vertical, in order to be readily seen from a

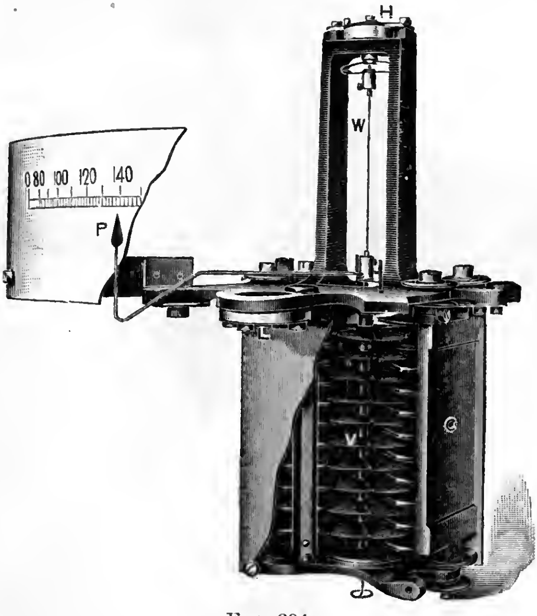


FIG. 394.

distance. When the pointer is at zero, the vanes are partly within a series of horizontal fixed plates or shelves carried by the metal plate, C. When the moving vanes and the fixed plates are connected respectively to two points between which a P.D. exists, lines of electric force pass from one to the other (*i.e.* they become charged, respectively positively and negatively). Hence, the attraction causes the movable vanes to approach the fixed ones, thereby setting up a rotation controlled by the torsion couple due to the suspension. It is usual to connect the moving part of the framework and to insulate the

[1] Figs. 394, 395, 396, and 398 are taken from Slingo and Brooker's *Electrical Engineering*.

fixed plates only. These instruments may be regarded as derived from the quadrant electrometer by multiplying the parts, and, like it, they are evidently condensers with one movable coating. Unfortunately they are not very dead-beat, which is sometimes an inconvenience.

(2) **Moving Coil Instruments.**—The principle of these instruments is similar to that embodied in the D'Arsonval galvanometer (p. 293). A steel horse-shoe magnet is provided with cylindrical soft-iron pole-pieces, and the polar space is very nearly filled up by a fixed cylindrical core of soft iron. A small coil, carrying a pointer, is pivoted to move freely in the narrow annular gap, its motions being controlled by two springs, similar to watch-springs. When

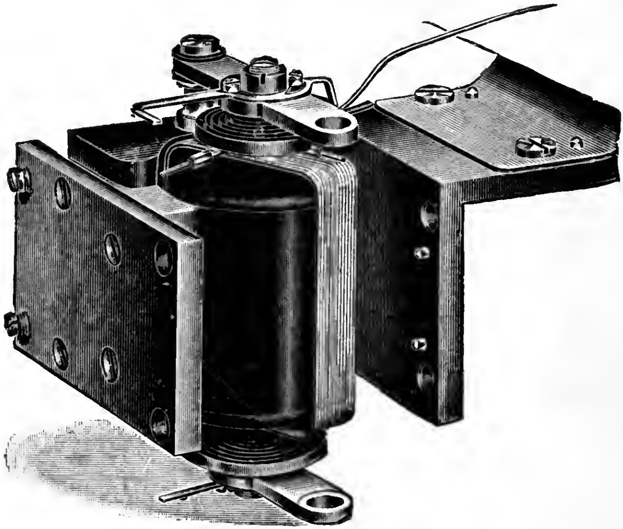


Fig. 395.

the instrument is used as a voltmeter, a large non-inductive resistance is placed in series with the coil; while, as an ammeter, the latter is shunted with a conductor of low resistance and of sufficient section to carry (without undue heating) the largest current to be measured. The general arrangement is shown in Fig. 395, in which the soft-iron pole-pieces have been removed in order to show the coil and the core more clearly.

Instruments of this type are convenient, very accurate, and the scale is remarkably uniform. They are unsurpassed for direct-current work, but as the direction of the deflection depends upon the direction of the current, they cannot be used on alternating circuits.

(3) **Soft-iron Instruments.**—A very small piece of thin soft

iron, attached to a spindle carrying a pointer, is pivoted inside a small coil. Motion may be obtained in several ways; perhaps the method most frequently employed is to place a fixed piece of iron in the coil, so that the two pieces become magnetised with the same polarity, and therefore repel each other. As in the previous case, a large non-inductive resistance is placed in series with the coil when it is to be used for a voltmeter. An ammeter is often wound with a coil of thick wire of low resistance.

Instruments of this type possess several advantages. They will stand much rough usage without injury, and can be made to work

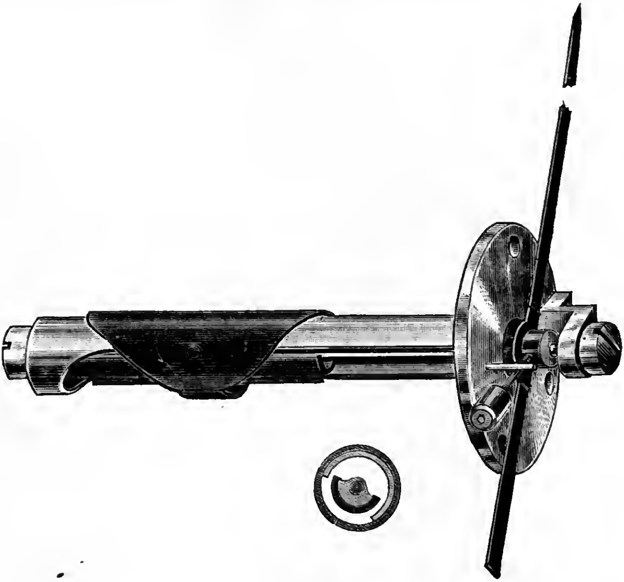


FIG. 396.

on either alternating or direct-current circuits. But they are naturally less accurate than the moving-coil type, and, in consequence of hysteresis, they are liable to read too low with gradually increasing currents, and too high with decreasing ones. Again, the scale is not uniform, for the force on the soft iron depends upon the square of the current, and therefore the deflection is also proportional to the square of the current (although this effect can be partially compensated by the details of construction). A well-known form is shown in Fig. 396, which represents the moving part of an instrument made by Messrs. Everett, Edgcumbe & Co. A spindle, pivoted in a brass frame, carries a pointer and also a half cylinder of thin soft iron (shown in the sectional view), and is balanced by a small adjustable weight, seen at the side of the pointer in the figure. Round the outside of the brass frame is fixed another strip of soft iron, bent round until the edges nearly—but not quite—meet, and cut away very much on that side. The whole arrangement is remarkably small and compact, being about 2 inches long and $\frac{1}{4}$ inch in diameter. This is surrounded by a coil of wire, which, when traversed by a current, produces an axial magnetic field passing longitudinally through both pieces of iron. If we regard them as becoming magnetised with like polarity, it is evident that the movable half cylinder will tend to set itself so that its ends are as far as possible from the ends of the fixed piece, *i.e.* it tends to take up the position shown in the figure.

(4) **Hot-wire Instruments.**—In this very important class of instruments, a fine wire (usually of platinum-silver alloy) under tension carries a current, and the slight increase in length due to the consequent rise in temperature (magnified by various devices) is made to move a pointer over a scale. As usual, voltmeters are provided with a large non-inductive resistance in series with the hot wire, and ammeters have the latter in parallel with a suitable shunt.

The great advantage of these instruments is due to the fact that they read as correctly on alternating as on direct-current circuits. They are also remarkably accurate. Their chief drawback is a tendency to "burn out," if by some mischance they are slightly overloaded. As the heating effect is proportional to the *square* of the current or of the voltage, the scale is not uniform.

Ammeters and voltmeters of all types are calibrated by sending known currents through them, or by applying a known P.D. to their terminals, the values being thus found by actual trial throughout the whole of the scale. Such calibrations are readily carried out by means of a potentiometer, but space does not permit us to describe the methods used in practice.

Dynamometers and Current-Balances.—In this category must be placed an extensive range of instruments depending upon what is known as the "dynamometer principle," *i.e.* the mutual forces between conductors carrying currents, when iron is not present. To some extent, this principle has been applied to the construction of direct-reading ammeters and voltmeters, but its great importance is due to the fact that it lends itself readily to the construction of wattmeters and current-balances, which may be used on either alternating or direct-current circuits.

Siemens' Dynamometer (shown in Fig. 397) was the first really practical instrument of the kind. It was devised for the measurement of alternating currents, and although now seldom used in its original form, it is worth studying as a good illustration of the principle in question.

The construction is shown diagrammatically in Fig. 398, where ABCD is a fixed coil composed of comparatively few turns of stout wire, the section depending upon the minimum current to be measured. At right angles to it is a movable coil, GEFH (which is usually one turn of thick copper wire), suspended by a silk thread from a specially made terminal forming the "torsion-head." A spiral spring, surrounding the thread, is connected below to the movable coil and above to the torsion-head, so that the thread carries the weight of the coil whilst the spring controls its position—the arrangement being such that the upper end of the spring can be rotated without

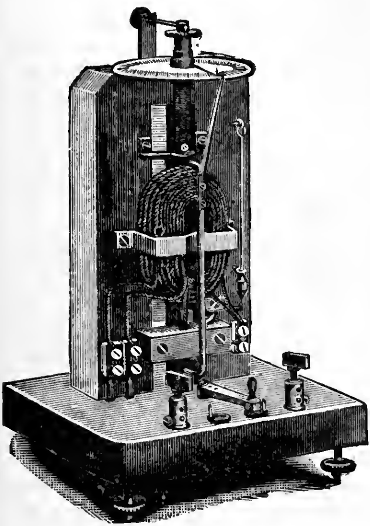


FIG. 397.

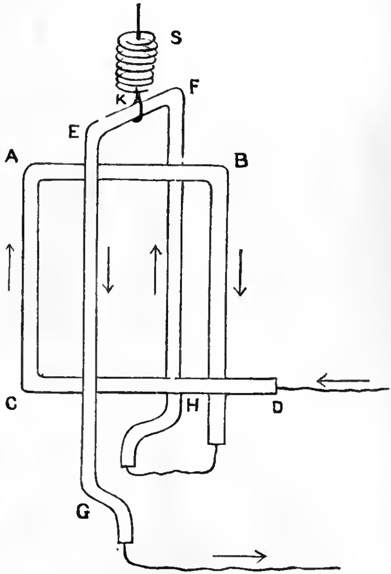


FIG. 398.

twisting the top of the suspension. The lower ends of the movable coil dip into mercury cups (not shown in the diagram), thus making good contact and yet allowing freedom of motion, and the two coils are connected in series. Remembering that currents in the same direction attract, and in opposite directions repel, it will be evident that the movable coil tends to move round until it is parallel to the fixed coil, and it will also be seen that the direction of motion is unaltered when the direction of the current is reversed. The torsion-head carries a pointer (shown in Fig. 397) which moves over a scale of degrees (or any convenient scale of equal parts). The movable coil also carries a pointer (also shown in Fig. 397) which passes upwards and bends just over the same scale, its motion (and, of course, that of the coil) being restrained by two stops.

Normally, both pointers are at zero, and when a current passes, the coil moves until its pointer comes in contact with one of the stops. The torsion-head is then turned in the opposite direction, thus putting a twist on the spiral and bringing the coil back to its exact zero position. The angle of rotation required for this purpose is then read off on the scale. Let this be d divisions. Then d is a measure of the force between the coils, and this can be shown to be directly proportional to the product of the currents in them. Hence, we have, if C is the current in each,

$$C^2 \propto d \qquad \therefore\ C \propto \sqrt{d}$$

which means that we can write $C = m\sqrt{d}$, where m is a constant for a given instrument. Evidently the value of m can be found by taking a single reading with a current of *known* strength (although it is, of course, better to confirm the result by taking a number of readings).

The importance of this instrument is due to the fact that when thus calibrated by means of *direct* currents, it reads correctly with *alternating* currents.[1] On the other hand, it is not direct reading and not very portable.

Application to Wattmeters.—Let the two coils of the dynamometer be fitted with separate terminals. The fixed coil need not be altered, but the movable coil should preferably consist of a few turns of fine wire, and should be connected in series with a large non-inductive resistance. (More turns are needed to give sufficient sensitiveness, because the current in this coil will now be very small; in any case, as few turns as possible are used.)

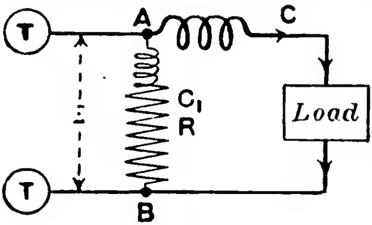


FIG. 399.

To measure the power expended in any part of the circuit, the thick wire coil is connected in series with the circuit (like an ammeter), and the movable coil, with

[1] It may be pointed out that any instrument which is to read with equal accuracy on either direct or alternating currents must satisfy two conditions: (1) The direction of motion must be independent of the direction of current, and (2) it must be practically non-inductive. Both conditions are satisfied in Siemens' dynamometer: (1) we have already mentioned, and (2) because it contains few turns and no iron. Hot-wire instruments also satisfy both conditions. The moving iron instruments satisfy condition (1), but as a rule only partially satisfy condition (2), and hence their readings depend upon the frequency, and they are correct only at the particular frequency for which they have been calibrated.

its resistance, is connected across that part as if it were a voltmeter. The arrangement is shown in Fig. 399, where the current is supposed to be derived from terminals TT, and the part of the circuit in which the power is to be measured is indicated by the word "load."

In the case of *direct currents*, the theory is very simple. If E be the P.D. across the mains, C the current through the load and the fixed coil, and C_1 the current through the movable coil and its resistance, we have

$$\text{Power} = \text{EC watts (expended in load and fixed coil).}$$

Also, as the reading of the torsion-head depends on the currents in each coil, we have

$$C \times C_1 \propto d$$

but $C_1 = \frac{E}{R}$, where R is the resistance between A and B (Fig. 399), *i.e.* practically the non-inductive resistance.

$$\therefore \quad \frac{CE}{R} \propto d$$

$$\text{or} \quad EC \propto Rd$$

$$\text{or} \quad EC = m.Rd.$$

Now, m is a constant for the given instrument, and as R is also constant, we may write $m.R = M$; therefore

$$\text{Power} = EC = M \times d$$

This includes the power expended in the fixed coil, which, however, is usually negligible. The value of M can be found by taking a reading with known values of E and C.

With direct currents, the same result could be obtained equally well by using a voltmeter and an ammeter, but with alternating currents we know that the product of volts and amperes (often called the "volt-amperes") does not give the true power.

Application to the Measurement of Power in Alternating Circuits.—Suppose that the load is inductive, then C lags behind E by some angle θ, and the true power is EC cos θ. Again, as the shunt-path between A and B (Fig. 399) is almost entirely made up of the non-inductive resistance R, the current C_1 must be practically in step with E. Therefore, C lags behind C_1 by the same angle θ. Now, it can be shown that, when the coils of a dynamometer are traversed by currents which are out of phase with each other, the force between them is proportional to the cosine of the angle of phase-difference (and therefore vanishes when that angle becomes 90°). Hence, in the present case, we have, if d is the scale reading,

$$CC_1 \cos \theta \propto d,$$

but $C_1 = \frac{E}{R}$, as R is non-inductive.

$$\therefore \quad \frac{\text{EC} \cos \theta}{\text{R}} \propto d$$

or $\text{EC} \cos \theta \propto \text{R}d$

or True power $= m \times d$, where m is some constant to be determined by experiment (using a known power).

In this simple proof, we have assumed that E and C follow a sine law, but fuller investigation would show that this is not an essential condition; and hence wattmeters may be used on alternating circuits without reference to the shape of the wave curve.

In practice, it is necessary to take care that no masses of metal are near the coils, otherwise eddy currents may be induced in them, and large errors thereby occasioned. Hence, metal covers should be avoided.

In any case, the argument given above is only approximately true, for the fine-wire coil must have *some* self-induction. The nature of the error thereby introduced can be readily investigated to a certain extent, for we know that C_1 really lags behind E by some very small angle α. Then the phase-difference between C and C_1 is $\theta - \alpha$.

$$\text{Hence.} \quad \text{CC}_1 \cos (\theta - \alpha) \propto d$$

$$\text{and} \quad \text{C}_1 = \frac{\text{E} \cos \alpha}{\text{R}}$$

$$\therefore \quad \text{EC} \cos \alpha \cos (\theta - \alpha) \propto \text{R}d = md.$$

The power deduced from the readings is therefore $\text{EC} \cos \alpha \cos (\theta - \alpha)$, whereas the true power is $\text{EC} \cos \theta$.

$$\therefore \frac{\text{Power given by reading}}{\text{True power}} = \frac{\cos \alpha \cos(\theta - \alpha)}{\cos \theta} = \frac{\cos \alpha(\cos \theta \cos \alpha + \sin \theta \sin \alpha)}{\cos \theta}$$

$$= \cos^2 \alpha + \cos \alpha \sin \alpha \tan \theta$$

If this expression is equal to unity, the instrument will read correctly.

Now, as α is very small, the first term is practically unity. Sin α is also small, but if θ be great, tan θ is very large, and the second term may not be negligible. Hence, the more inductive the load, the less accurate become the readings. (It may be pointed out in passing that the above expression becomes unity, not only when $\theta = 0$, but also when $\theta = \alpha$. This fact is, however, of no practical importance.)

Our argument has tacitly assumed that the resistance R is not only *non-inductive*, but is also destitute of *capacity*. If R be made non-inductive in the same way as coils in ordinary resistance boxes (*i.e.* by doubling the wire upon itself, and then winding it upon a bobbin), it may have sufficient capacity to introduce an error of considerable magnitude; for doubling the wire upon itself has no influence on the capacity, which depends mainly upon the contiguity of the layers in the winding. Resistances for use with wattmeters are never wound in that manner, but in such a way as to open up the winding as much as possible.

Measurement of Iron Losses.—Suppose that the load in Fig. 399 is a coil of wire, with an iron core, and that it is supplied with an alternating current from the terminals TT. The wattmeter reading gives at once the power supplied to the coil, which must be expended as (1) C^2R heat, (2) hysteresis, (3) eddy currents. The first source of loss is often negligible, and in any case it is easily estimated by a measurement of current and resistance. The actual "iron loss" is thus obtained in watts. The eddy-current loss can be made small by laminating the iron, but it is only possible to separate (2) and (3) by measuring the loss at different frequencies, and making use of the fact that (2) varies *directly* as the frequency, while (3) varies as the *square* of the frequency.[1]

What is required in practice, however, is not merely a knowledge of the iron loss, but the relation between that loss and the maximum value of B in the iron, *i.e.* we must make simultaneous measurements of the iron loss and B. Now, in discussing the theory of the transformer, it was shown that the value of B in the core can be deduced by measuring the P.D. at the secondary terminals on open circuit, when the area of section of the core and the number of secondary terms are known. Hence, one of the best methods of testing iron stampings for hysteresis loss is to make up a definite weight of them as the core of a simple form of transformer which is designed to be readily fitted with temporary cores, and to measure simultaneously the iron loss by a wattmeter and the P.D. at the terminals. In this way, the loss in watts per lb. of iron at a given induction is obtained.

Current-Balances.—Instruments of this type have been largely used for the absolute measurement of current; for instance, by Lord Rayleigh in his important research on the electro-chemical equivalent of silver. One of the latest instruments of the kind has recently been set up at the National Physical Laboratory. Two coils with their axes vertical are suspended from the arms of a delicate balance within vertical marble cylinders, on each of which two coils are wound. The general idea may be gathered from Fig. 400, which is merely a diagram, and in no way represents the actual details of construction. AA_1 and BB_1 are the two pairs of fixed coils, and CC_1 the two movable coils suspended from the arms of a balance. The current to be measured passes through all six coils in series in such directions that the forces act together to tilt the beam the same way, and the total force can be measured with great accuracy by adding weights until the balance is again in equilibrium. From what we have already said, it is evident that this force varies as the square of the current, and its value in terms of the current can be calculated from the dimensions of the apparatus without reference to any other electrical quantity.

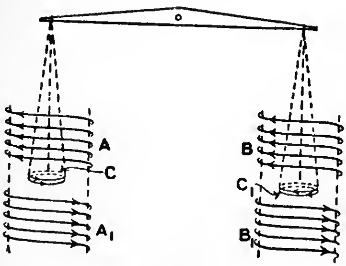


FIG. 400.

[1] The induced E.M.F. producing eddy currents varies *directly* as the frequency, and the power wasted as heat in a path of constant resistance varies as the *square* of E.M.F. (for watts $=\frac{E^2}{R}$). Hence, eddy current losses are proportional to (frequency)^2

As the reversal of the current in *all* the coils does not alter the direction of the force, such an instrument can be used in connection with alternating currents.

Kelvin Balances.—These are instruments capable of measuring currents with great accuracy, and yet not too delicate in construction for use as commercial standards. If in Fig. 400 we suppose that the coils CC_1 are carried by a horizontal lever suspended at the middle, and the tilt, due to the passage of a current, balanced by

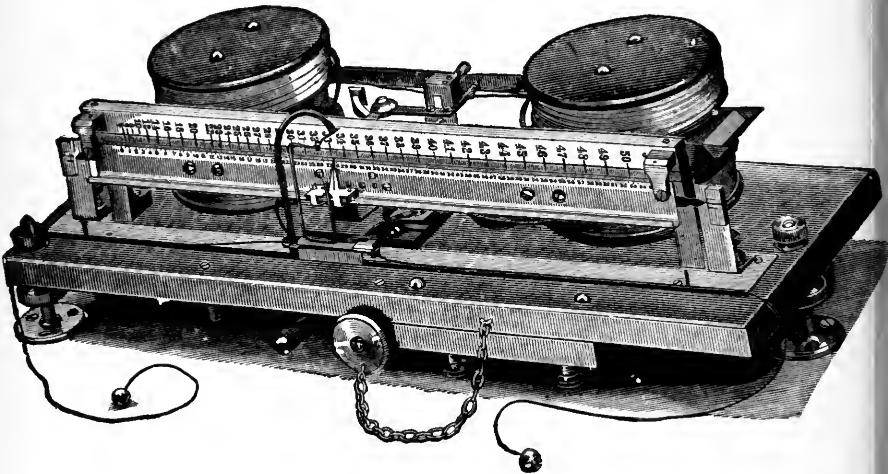


FIG. 401.

sliding a rider along the lever, the result will embody the principle of a Kelvin balance. These instruments are made in several forms, designed to measure currents from $\frac{1}{100}$ ampere up to 1000 amperes. Fig. 401 shows the appearance of the "centi-ampere" balance, whose effective range extends from $\frac{1}{100}$ ampere to 1 ampere, and also of the "deci-ampere" balance for currents between $\frac{1}{10}$ ampere and 10 amperes. Fig. 402 (from Aspinall Parr's *Practical Electric Testing*) indicates the arrangement diagrammatically. In the latter figure, *aa* is a light frame, which forms the lever carrying the coils BB^1, and a

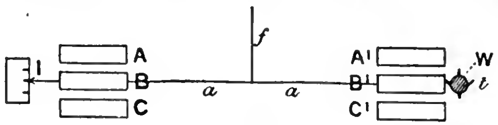


FIG. 402.

triangular scale-pan, *t*, in which a weight, W, can be placed. At each end is a pointer moving over a small fixed vertical scale—only one of which, however, is shown in the figure at I. AA^1

and CC^1 are the two pairs of fixed coils, and, as in the instrument just described on p. 567, the current passes in series through the six coils in such directions that the various forces act together to tilt the lever (the left-hand side moving downwards). In order to neutralise the effect of the earth's magnetic field, which would of itself produce a tilt, the current flows clockwise in one moving coil and anti-clockwise in the other. To the whole length of the lever is attached a light L-shaped bar of aluminium, the lower divisions forming a shelf and the upper edge having a scale of equal divisions engraved on it. On the shelf slides a carriage, drawn to and fro by means of threads. To this carriage is attached a pointer, by which its position on the scale can be read off. When the pointer is at zero and no current is flowing, the total weight of carriage and pointer balances the weight placed in the scale-pan at the other end, so that the arrangement is then equivalent to a loaded and balanced lever.

In Fig. 401, the scale-pan is visible just below the upper fixed coil on the right; the sliding carriage and pointer are seen in front of the graduated movable scale, *i.e.* the lower scale. After the sliding carriage has been placed exactly at zero, the beam is made horizontal (as indicated by the pointers at the ends) by means of a rider worked by the handle shown under the base of the instrument. At the back of the moving scale there is a fixed scale (*i.e.* the upper scale in the figure) *not* divided into equal parts. The numbers marked thereon are the doubled square roots of those coinciding with them on the lower scale (which are marked 1, 2, 3, &c. for 10, 20, 30, &c.); for instance, on the fixed scale 50 is visible near the right-hand end, which is the doubled square root of 625, coinciding with it on the movable scale. This *doubling* of the square roots is merely a matter of convenience, the essential point being that the readings on the fixed scale do not give the distance from the zero, but are proportional to the square root of that distance.

When the beam is tilted by the passage of a current, obviously equilibrium could be restored by diminishing the weight of the carriage, W, but it is more convenient merely to slide the carriage into some new position at distance d from its zero. Then it is evident that $W \times d$ balances the new couple due to the current, and, as already mentioned, this couple is proportional to C^2

Hence, we have $C^2 \propto Wd$

or $C \propto \sqrt{Wd}$

i.e. $C = m\sqrt{d}$, where m is some constant.

Thus the current is proportional to the square root of the distance through which the carriage is moved to restore equilibrium, *i.e.* to the reading on the fixed scale. This reading can be taken at a glance, and, when multiplied by a constant supplied by the maker, gives the current with sufficient accuracy. When the greatest accuracy is required, the reading on the lower scale is taken with the aid of a lens, and the corresponding number on the upper scale more exactly determined by reference to a table of doubled square roots provided for the purpose.

In order to extend the range of these instruments, it is usual to supply four pairs of weights, each pair consisting of one for the scale-pan and one for the carriage.[1]

These weights are in the ratio 1 : 4 : 16 : 64, and from the preceding argument, it follows that the currents corresponding in each case to one division of the fixed scale are in the ratio 1 : 2 : 4 : 8. For instance, in the size known as "the ampere-balance," there are 50 divisions on the fixed scale, and with the smallest pair of weights each division is equivalent to ·25 ampere. Hence, when using this pair of weights, currents up to 12·5 amperes can be measured. With the second pair, each division represents ·5 ampere, and the range is from 0 up to 25 amperes. Similarly, the third pair gives 1 ampere per division; and the fourth, 2 amperes per division—the total effective range being from about 1 ampere to 100 amperes.

The centi-ampere balance, with a range of from $\frac{1}{100}$ ampere to 1 ampere, serves equally well for the measurement of voltage. Its resistance is about 60 ohms, and if this is known exactly and multiplied by the current as measured, the result is the P.D. between the terminals of the instrument. In practice, special resistances are supplied for connecting in series with the balance in order to increase its range.

Special instruments are also made for use as wattmeters; in these, the fixed coils are of thick wire, and the movable coils of thin wire, the latter being provided with a separate non-inductive resistance. The connections and the theory are identical with those explained on pp. 564 and 565.

One of the most important details in these instruments is the ingenious way in which currents of any magnitude are taken in and out of the moving coils without interfering with their freedom of motion. This is the great difficulty with all dynamometers, mercury cups being a clumsy and troublesome expedient which is fatal to high accuracy. In a Kelvin balance, the frame of the lever is fixed to two metal bars, each suspended by a large number of very fine copper wires from two other fixed bars. This provides a soldered contact which can be made to carry large currents and is yet extremely flexible throughout the very small range of motion required. It was, in fact, the invention of this suspension which made these instruments possible.

[1] The carriage itself balances the smallest weight. Hence, there are four weights made to fit the scale-pan, and three in the form of riders to attach to the carriage.

Oscillographs.—An oscillograph is a variety of galvanometer, in which the deflection at any instant is proportional to the current strength at that instant, no matter how rapidly the current may be varying or alternating. It is used to obtain the actual shape of alternating current curves; to trace the rate of rise or fall of a current under various conditions; to record the number and amplitude of the oscillations set up by a spark discharge; and, in fact, to obtain exact data regarding the manner in which an E.M.F. or a current changes in strength under all possible conditions.

In order that the moving part may faithfully and instantaneously follow such extremely rapid changes, it must satisfy certain conditions: (1) its own natural time of vibration must be exceedingly small compared with the periodic time of the current to be investigated (say $\frac{1}{50}$ of the latter); (2) its motions must be very exactly dead-beat. The first condition implies that the moment of inertia of the moving system is small and the controlling couple relatively great; the second can be satisfied by a suitable method of damping.

The first instrument of the kind was devised by Blondel in 1892. More recently, Duddell has applied the principle of the suspended-coil galvanometer with great success, and it is chiefly to him that we owe the development of the oscillograph as a really practical instrument. In his form, the permanent magnet shown in Fig. 229 is replaced by a powerful electromagnet, and the moving coil is reduced to a single loop kept under tension by a spring. This is shown diagrammatically in Fig. 403,[1] where M represents a very small mirror attached to the loop between the pole-pieces, NS. The necessary damping is obtained by immersing the whole loop and mirror in oil.

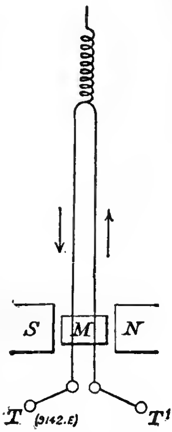


FIG. 403.

In order to obtain the wave-form, two methods may be employed. In one, the spot of light, reflected from the mirror, is received upon a photographic film or plate moving vertically with uniform velocity, thus giving a permanent record. In the other, the light from the mirror, M, falls first upon another mirror, which is rocked by a small motor in exact unison with the motion of M, and in a plane at right angles to it. Thence, the light passes to a screen, on which it is seen—on account of the persistence of vision—as a luminous curve.

Many other forms of oscillograph have been introduced—one of the most recent, due to Irwin, being of the hot-wire type. Braun has devised a special kind of highly exhausted vacuum tube, in which the wave-form is traced out on a phosphorescent screen by means

[1] From J. A. Fleming's *Principles of Electric Wave Telegraphy.*

of a beam of cathode rays deflected by a magnetic field due to the current under investigation. Gehrcke has also applied, for the same purpose, certain phenomena, which occur in much less highly exhausted vacuum tubes; but on the whole, the Duddell type just described appears to be the most generally useful.

Exercise XXVII

1. Describe carefully, stating the precautions necessary, how you would test the accuracy of an ammeter reading to about 1·5 amperes.

(B. of E., 1906.)

2. Explain why an electro-magnetic voltmeter should have as high a resistance as is practicable. Given a voltmeter of this kind reading from 0 to 5 volts with a resistance of 500 ohms, how may the same instrument be adapted to read from 0 to 50 volts with the same scale?

(B. of E., 1904.)

3. Describe the construction of an electrostatic voltmeter. An electrostatic voltmeter gives deflections of 15, 18, and 21 scale divisions for constant potentials of 50, 60, and 70 volts respectively; what deflections will be produced by an alternating electromotive force E sin pt, (a) when the amplitude E is 70 volts, and (b) when E is 90 volts?

(Lond. Univ. B.Sc., 1908.)

CHAPTER XXXIV

UNITS AND DIMENSIONS

EVERY physical quantity consists of the product of two factors—(1) a measure, and (2) a unit. Thus, if we speak of 10 metres, this is the product of the measure 10, and the unit 1 metre; and any one, who knows what a metre is, can form some idea of the length of 10 metres. If, however, we have no knowledge of the value of the unit, we can form no idea of the value of any quantity containing that unit; *e.g.* if a person, ignorant of electrical quantities, hears of a current of 10 amperes, no idea of the magnitude of that current is conveyed to his mind, and, for anything he knows to the contrary, it may be either an exceedingly small or an exceedingly large current.

In the case of most physical quantities, *e.g.* work, power, velocity, &c., the magnitude of the unit depends upon, and is determined by, the values we agree to take for the units of length, mass, and time. Hence, the latter units are known as **fundamental units**, and the others as **secondary** or **derived units.** There are, however, several physical quantities (μ and K, for instance), whose real nature is unknown, and which cannot be expressed in terms of the fundamental units.

The Centimetre-Gram-Second System.—In scientific work, the centimetre, the gram, and the second have been adopted, by common agreement, as the fundamental units.

Derived Units.—The unit of velocity is, therefore, a velocity of one centimetre per second, and is a *derived* unit. Other derived units are—

Acceleration.—By the term *acceleration* is meant the increase or decrease in velocity per second.

A body moves with unit acceleration when its velocity increases or decreases by 1 centimetre per second in every second.

The acceleration (g) produced by gravity on falling bodies is roughly 981 centimetres per second per second. This value, however, continually changes as we change our latitude, being greatest at the poles (983·1), and smallest at the equator (978·1).

Force.—The unit of force is called the *dyne*, and it is that force which, acting on a mass of 1 gram for 1 second, gives to it a velocity of 1 centimetre per second.

A force of 1 dyne is nearly equal to the weight of $\frac{1}{981}$ gram, *i.e.* of 1·02 milligrams.

Weight.—The student must be careful not to confuse the terms *mass* and *weight.* The weight of any mass is the force with which the earth attracts it, and as the value of this force varies at different places on the earth's surface, the weight of any mass also varies. In fact, the weight of a body is equal to the product of the mass of the body and the earth's acceleration, *i.e.* $W = mg$, in C.G.S. units, this means that a mass of 1 gram has a weight of 981 dynes.

Work.—Work is measured by the product of the magnitude of a force and the distance through which the point of application moves in the direction of the force.

The unit of work is called the *erg,* and is the amount of work done through a distance of 1 centimetre against a force of 1 dyne.

Energy.—As the energy of a body is its power of doing work, the erg is also the unit of energy.

Heat.—The unit of heat, a *calorie,* is the amount of heat required to raise the temperature of 1 gram of water from 0° C. to 1° C., and its dynamical equivalent is $4·18 \times 10^6$ ergs.

Dimensions of Units.—It is usual to employ capital letters in square brackets to represent the abstract ideas of length, mass, &c., apart from any numerical values. For instance, if we have $v = 6$, and wish to emphasise the fact that v is a velocity, we may write

$$v = 6\ [V],$$

or, because a velocity is measured by the ratio $\frac{\text{length}}{\text{time}}$,

$$v = 6\ [LT^{-1}]$$

where $[LT^{-1}]$ must be understood to mean the *unit* of velocity in the particular system we are using; in the C.G.S. system, for example, we must read

$$v = 6 \times \text{(one centimetre per second)}.$$

Definition.—The powers to which the fundamental units are raised in order to express the magnitude of any derived unit are called the dimensions of that derived unit. For instance, the dimensions of velocity are zero with respect to mass, + 1 with respect to length, and – 1 with respect to time, which is expressed by writing

$$[V] = [LT^{-1}]$$

This is known as a *dimensional* equation.

It is worthy of mention that, in any physical equation, the terms connected by + or – signs *must necessarily be of the same kind,* for it would be absurd to add or subtract quantities of different kinds. From this, it follows that all such terms must have the same dimensions. On the other hand, it is possible to *multiply* numbers expressing quantities of different kinds, the result being a third quantity different in nature from either, *e.g.* if we multiply a force by a length, the product represents work.

Dimensions of Various Derived Units.—

Area	$=$ length $\times$ length	$[L^2]$
Volume	$=$ Area $\times$ length	$[L^3]$
Velocity	$=\dfrac{\text{length}}{\text{time}}$	$[LT^{-1}]$
Acceleration	$=\dfrac{\text{velocity}}{(\text{time})^2}$	$[LT^{-2}]$
Density	$=\dfrac{\text{mass}}{\text{volume}}$	$[ML^{-3}]$
Force	$=$ mass $\times$ acceleration	$[MLT^{-2}]$
Kinetic energy	$=\frac{1}{2}$ mass $\times$ (velocity)^2	$[ML^2T^{-2}]$
Work	$=$ force $\times$ distance	$[ML^2T^{-2}]$
Power	$=$ rate of doing work $=\dfrac{\text{work}}{\text{time}}$	$[ML^2T^{-3}$
Angle	$=\dfrac{\text{arc}}{\text{radius}}=\dfrac{\text{length}}{\text{length}}$	$[L^0]$
Elasticity	$=\dfrac{\text{force per unit area}}{\text{a ratio}}=\dfrac{\text{force}}{\text{area}}$	$\left[\dfrac{MLT^{-2}}{L^2}\right]=[ML^{-1}T^{-2}]$

With regard to the meaning of these statements, it may be remarked that volume, for instance, is always the product of three lengths (there may be a numerical coefficient), and we may therefore alter the units of mass and time without, in any way, affecting the unit of volume; but, if we alter the unit of length, say by doubling it, the new unit of volume is then eight times as great as before, the unit of area would be four times as great, and the unit of velocity would be only twice as great.

The result for angle shows that a physical quantity may have zero dimensions, owing to the fact that it is expressed as the ratio of two similar quantities.

Applications of the Preceding Results.—There are two special uses to which a knowledge of dimensions may be applied.

The first of these soon becomes almost instinctive. It is to test the accuracy of any physical equation, which may have been obtained in some mathematical investigation, or which may, perhaps, have been written down from memory. This depends upon the fact, already mentioned, that, in any physical equation, the terms connected by $+$ or $-$ signs must necessarily be of the same kind, from which it follows that they must have the same dimensions. For instance, in abstract mathematics, we may accept the expression $a+b+c=d$ as one merely stating some relation between numbers; but in a physical equation, if a is a length and b is a force, such an expression becomes absurd. Let us, for example, consider the familiar equation of accelerated motion

$$s = ut + \tfrac{1}{2}ft^2.$$

As s is a length, the other terms must also be lengths.

$$\text{Now, } ut = \text{velocity} \times \text{time or } \left[\frac{\text{L}}{\text{T}} \times \text{T}\right] = [\text{L}]$$

In the last term, the numerical coefficient may be ignored, as it does not affect the dimensions, and we have

$$ft^2 = \text{acceleration} \times (\text{time})^2 \text{ or } \left[\frac{\text{L}}{\text{T}^2} \times \text{T}^2\right] = [\text{L}]$$

The student should test various equations, with which he is acquainted, in the same way; for instance,

$$t = 2\pi\sqrt{\frac{l}{g}}; \quad \text{V} = \sqrt{\frac{\text{E}}{\text{D}}}$$ (see p. 583).

The second useful application is to find the change that takes place in the numerical value of a certain quantity, when the fundamental units are altered. This depends upon the fact that the number expressing that quantity is inversely as the size of the unit, *e.g.* a certain sum of money is measured by 1, if the unit is a sovereign, and by 960, if the unit is a farthing. The dimensions of the quantity are, of course, unaltered by changing the units, and hence, if n_1 is the number expressing it when the fundamental units are M_1, L_1, T_1; and n_2 the number when these are changed to M_2, L_2, T_2, we have

$$n_1[\text{M}_1{}^x\, \text{L}_1{}^y\, \text{T}_1{}^z] = n_2[\text{M}_2{}^x\, \text{L}_2{}^y\, \text{T}_2{}^z]$$

This will be better understood if we consider an example.

Example.—Find the value of 42×10^6 ergs, when the units are the pound, the foot, and the minute.

As the dimensions of work are ML^2T^{-2}, we have

$$n_1[\text{M}_1\text{L}_1{}^2\text{T}_1{}^{-2}] = n_2[\text{M}_2\text{L}_2{}^2\text{T}_2{}^{-2}]$$

$$\text{or} \quad 42 \times 10^6\left[\frac{(\text{gram})\ (\text{centimetre})^2}{(\text{sec.})^2}\right] = n_2\left[\frac{(\text{lb.})\ (\text{foot})^2}{(\text{min.})^2}\right]$$

$$\therefore \quad n_2 = 42 \times 10^6\left\{\left(\frac{\text{gram}}{\text{lb.}}\right) \times \left(\frac{\text{centimetre}}{\text{foot}}\right)^2 \times \left(\frac{\text{min.}}{\text{sec.}}\right)^2\right\}$$

$$= 42 \times 10^6\left\{\left(\frac{1}{453{\cdot}6}\right) \times \left(\frac{1}{30{\cdot}48}\right)^2 \times \left(\frac{60}{1}\right)^2\right\}$$

$$= 358{,}790.$$

Had we not altered the unit of time, this result would have been in foot-poundals.[1] But as the dimensions of work are -2 with respect to time, increasing the time unit 60-fold has *decreased* the work unit 3600-fold, and so the result obtained above is in terms of a work unit, which is $\frac{1}{3600}$ of a foot-poundal.

Example.—What number would represent 33,000 foot-pounds per minute in the C.G.S. system?

The first step must be to express the quantity in terms of the fundamental units in the old system, for in ft.-lb.-second units, the unit of work is the foot-poundal—*not* the foot-pound.

Now, 33,000 foot-pounds per minute $= \frac{33{,}000 \times 32 \cdot 2}{60}$ foot-poundals per second $= 550 \times 32 \cdot 2$ foot-poundals per second.

This quantity is the *rate of working*, i.e. *power* or $\frac{\text{work}}{\text{time}}$, and its dimensions are $[ML^2T^{-3}]$

$$\therefore\ 550 \times 32 \cdot 2\{(\text{lb.})\,(\text{foot})^2\,(\text{sec.})^{-3}\} = n\{(\text{gram})\,(\text{centimetre})^2\,(\text{sec.})^{-3}\}$$

$$\begin{aligned} \text{or, } n &= 550 \times 32 \cdot 2\left\{\left(\frac{\text{lb.}}{\text{gram}}\right)\left(\frac{\text{foot}}{\text{centimetre}}\right)^2\right\} \\ &= 550 \times 32 \cdot 2 \times 453 \cdot 6 \times (30 \cdot 48)^2 \\ &= 746 \times 10^7 \end{aligned}$$

$\therefore$ 33,000 foot-pounds per minute, *i.e.* 1 H.P. $= 746 \times 10^7$ ergs per second.

This example indicates the origin of the number 746 in expressing the relation between watts and H.P. For it has been shown on p. 231 that 1 watt $= 10^7$ ergs per second, and is, therefore, $\frac{1}{746}$ H.P.

Dimensions of Electrical Units.—It will be convenient first to show how some of these dimensions may be obtained, and then to discuss certain peculiarities exhibited by them.

Static Units.—

Quantity or Charge—By Coulomb's equation,

$$\text{Force} = \frac{Q \times Q_1}{Kd^2}, \qquad \text{or} \qquad QQ_1 = K \times \text{force} \times d^2$$

$$\therefore\ [Q_s] = [(K \times MLT^{-2} \times L^2)^{\frac{1}{2}}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}]$$

Electric Force[2] *(at a point in an electric field)*—

$$\text{Mechanical force} = Q \times \text{electric force},$$

$$\text{or} \quad [\text{Electric force}] = \left[\frac{MLT^{-2}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}}\right] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}]$$

[1] A poundal is the unit of force in the ft.-lb.-sec. system, and equals $\frac{1}{32 \cdot 2}$ lb. weight.

[2] Denoted by U in this book.

Electric Field = K × electric force (or F = KU)

∴ [Field strength] = $[M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}K^{\frac{1}{2}}]$

Static Potential—

$$Q_sV_s = \text{work.}$$

$$\therefore \quad [V_s] = \left[\frac{ML^2T^{-2}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}}\right] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}]$$

Capacity $= \dfrac{Q_s}{V_s}$

$$\therefore \quad [\text{Capacity}] = \left[\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}}{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}}\right] = [LK]$$

[Hence, an electrostatic capacity is correctly measured in terms of a length, *e.g.* for a sphere, in terms of the *length* of its radius in centimetres.]

Electromagnetic Units.—

Pole Strength—By Coulomb's equation,

$$\text{Force} = \frac{m \times m_1}{\mu \times d^2}$$

$$\therefore \quad \begin{bmatrix}\text{Pole strength}\\(m)\end{bmatrix} = [(\mu \times MLT^{-2} \times L^2)^{\frac{1}{2}}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}\mu^{\frac{1}{2}}]$$

Magnetic Moment = pole strength × length,

$$\therefore \quad \begin{bmatrix}\text{Moment}\\(m \times l)\end{bmatrix} = [M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}\mu^{\frac{1}{2}}]$$

Magnetising or Magnetic Force—

Mechanical force = pole strength × magnetic force,

$$\therefore \quad \begin{bmatrix}\text{Magnetic force}\\(H)\end{bmatrix} = \left[\frac{MLT^{-2}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}\mu^{\frac{1}{2}}}\right] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}]$$

Field Strength, or Magnetic Induction (B)—

$$B = \mu H$$

$$\therefore \quad \begin{bmatrix}\text{Field strength}\\(B)\end{bmatrix} = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}]$$

Intensity of Magnetisation $= \dfrac{\text{magnetic moment}}{\text{volume}}$,

$$\therefore \quad [I] = \left[\frac{M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}\mu^{\frac{1}{2}}}{L^3}\right] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}]$$

Current— Force = B × *i* × *l*.

$$\therefore \quad [\text{Current}] = \left[\frac{MLT^{-2}}{M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}} \times L}\right] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}]$$

Quantity— $Q_m = it,$

$$\therefore \quad [Q_m] = [M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

Electromotive Force (*or Potential*)—

$$Q_m E_m = \text{work},$$

$$\therefore \quad \begin{bmatrix}\text{E.M.F.} \\ \text{or } V_m\end{bmatrix} = \left[\frac{ML^2T^{-2}}{M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}}\right] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}]$$

$$\textit{Capacity} = \frac{Q}{\text{E.M.F.}}$$

$$\therefore \quad [\text{Capacity}] = \left[\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}}\right] = [L^{-1}T^2\mu^{-1}]$$

$$\textit{Resistance} = \frac{\text{E.M.F.}}{\text{current}}$$

$$\therefore \quad [\text{Resistance}] = \left[\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}}{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}}\right] = LT^{-1}\mu]$$

(Hence, in magnetic units, a resistance can be expressed as a *velocity*, and it will be found that *all* absolute methods depend on the measurement of a velocity. (See also the dimensions of resistance in the static system, p. 580.)

Inductance.—For a solenoid, Inductance $= \dfrac{4\pi n^2 A\mu}{l}$

$$\text{or } [\text{Inductance}] = \left[\frac{L^2\mu}{L}\right] = [L\mu].$$

From which it appears that in magnetic units a coefficient of self-induction may be expressed in centimetres of length.

Contrasting these results with those previously obtained for mechanical quantities, we notice at once the prevalence of fractional indices, and the occurrence of K and μ in the static and magnetic systems respectively.

If we regard K and μ merely as numerical ratios, in terms of air taken as unity, they disappear from the dimensional equations, but then we meet with two initial difficulties: (1) that of understanding the physical nature of quantities whose dimensions are fractional, and (2) that caused by the dimensions of magnetic pole strength and of static charge becoming identical, which would lead to the absurd conclusion that they are really the same thing.

If, however, we regard K and μ as real physical quantities of unknown dimensions, both difficulties disappear.

It may be pointed out that the static system of units was obtained by ignoring current phenomena (*i.e.* the phenomena of magnetism), and the electro-magnetic system by similarly ignoring static phenomena; the two underlying conventions—that K and μ are to be taken as unity for air—being probably incompatible with one another. Thus both systems are entirely arbitrary, but it must be clearly understood that the terms potential, capacity, &c., mean the *same* physical quantity in either. Hence, the dimensions assigned to them in the two systems respectively must be really identical. Let us see what occurs, when we equate the two forms obtained for *quantity* or *charge*.

$$[M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}] = M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

$$\therefore \quad [LT^{-1}] = \left[\frac{1}{\sqrt{K\mu}}\right]$$

which indicates that $\frac{1}{\sqrt{K\mu}}$ has the dimensions of a velocity.

Repeating the process in the case of *capacity*, we have

$$[LK] = [L^{-1}T^2\mu^{-1}]$$

$$\text{or } [L^2T^{-2}] = \left[\frac{1}{K\mu}\right]$$

which is the previous result squared.

Similar results may be obtained with all the units represented in both systems. One obvious consequence is that we may change from one system to the other by making use of the relation $K^{\frac{1}{2}}\mu^{\frac{1}{2}} = \frac{T}{L}$, and substituting for either K or μ. For instance, if we know that in the static system

$$[Q_s] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}]$$

we can find the dimensions in the magnetic system by putting $K^{\frac{1}{2}} = \frac{T}{L\mu^{\frac{1}{2}}}$, which gives

$$[Q_m] = [M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

Example.—Find the dimensions of resistance in the static system.

$$[R_m] = [LT^{-1}\mu] \text{ and } [K\mu] = [T^2L^{-2}], \; i.e. \; [\mu] = \left[\frac{T^2L^{-2}}{K}\right]$$

$$\text{or } [R_s] = L^{-1}TK^{-1}]$$

From which we see that R_s comes out as the *reciprocal* of a velocity, *i.e.* as a *slowness*, and, in fact, the idea of *static resistance* may be interpreted as signifying the slowness with which a static charge leaks away through the substance in question.

Ratio of Units in the Two Systems.—It follows that if we substitute numerical values in the expression $\frac{1}{\sqrt{K\mu}}$, the result is a velocity; and our convention, that K and μ are unity for air, really amounts to making its value for air the *unit* of velocity. We can, however, estimate its actual value in C.G.S. units by applying the method already used for mechanical units.

Let Q_s, Q_m be the *numbers* representing the *same* quantity in the two systems. Then we must have

$$Q_s[M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}] = Q_m[M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

$$\therefore \frac{Q_s}{Q_m}[LT^{-1}] = \frac{1}{\sqrt{K\mu}}$$

In this equation, the ratio $\frac{Q_s}{Q_m}$ is a pure number (for instance, if Q_m is one absolute unit (*i.e.* 10 coulombs), Q_s will be the number expressing this quantity in static units), and as already stated $[LT^{-1}]$ stands for the unit of velocity, *i.e.* 1 centimetre per second in the C.G.S. system. Hence, the expression $\frac{1}{\sqrt{K\mu}}$ stands for a velocity, which, when $K=1$, $\mu=1$, is measured in centimetres per second by the numerical value of the ratio $\frac{Q_s}{Q_m}$.

It is usual to denote this velocity by v,

$$\therefore \frac{Q_s}{Q_m} = v.$$

Here, Q_s is a large *number* compared with Q_m, which, of course, indicates that the magnetic *unit* of quantity is v times as large as the corresponding static unit.

The value of v must be determined experimentally by actually measuring the same quantity in both systems of units. For instance, the capacity of a condenser in static units may be found by calculation from its dimensions (or by comparison with that of a sphere). It is then charged, and its potential is measured absolutely in static units by an electrometer, thus determining Q_s in terms of potential and capacity.

The condenser is then discharged through a ballistic galvanometer and the throw noted, Q_m being calculated from the theory of the instrument.

In this way, it has been found that $v = 3 \times 10^{10}$ centimetres per second.

Ratio of Units of Potential.—Obviously, the numerical value of v may be obtained by comparing any two units. In the case of potential, a steady current i may be allowed to flow through a resistance R, and the value of the P.D. across it, in magnetic units, found by measuring the current and the resistance, for $V_m = iR$. The P.D. may be directly measured at the same time, in static units, by means of an electrometer.

To ascertain the real meaning of the ratio thus obtained, we have only to notice that, if a current i flows for time t seconds, a quantity passes measured by Q_s in static units and Q_m in magnetic units, and that in one system the work done is V_sQ_s ergs, and in the other V_mQ_m ergs.

$$\therefore V_sQ_s = V_mQ_m$$

$$\text{or} \quad \frac{V_m}{V_s} = \frac{Q_s}{Q_m}$$

$$\textit{i.e.} \quad \frac{V_m}{V_s} = v = 3 \times 10^{10} \text{ (centimetres per second)},$$

from which we learn that the static *unit* of potential is v times as large as the magnetic unit.

Example.—How many volts correspond to 1 electrostatic unit of potential?

Now, 1 static unit $= 3 \times 10^{10}$ absolute magnetic units of potential, and 10^8 absolute magnetic units $= 1$ volt.

$$\therefore \text{ 1 static unit} = \frac{3 \times 10^{10}}{10^8} = 300 \text{ volts.}$$

Ratio of Units of Capacity.—The units of capacity may be compared by determining C_m as on p. 390, and calculating C_s from the dimensions of the condenser. To ascertain what this means, we have only to notice that, for a definite charge and potential, we have

$$C_m = \frac{Q_m}{V_m} \text{ and } C_s = \frac{Q_s}{V_s}$$

$$\therefore \frac{C_s}{C_m} = \frac{Q_s}{Q_m} \times \frac{V_m}{V_s} = v^2.$$

Magnetic Units of Capacity.—It has already been stated that the equation—

Quantity of charge in condenser = P.D. between its coatings × Capacity,

holds good generally, whatever system of units we may adopt in actual measurement, and we have written it in the form

$$Q = VC$$

to indicate that static units are employed, and

$$Q = EK$$

when using magnetic units.

From this it follows that *a condenser will have* 1 *absolute unit of capacity when a charge of* 1 *absolute unit of quantity produces a P.D. between its terminals of* 1 *absolute unit of E.M.F.*

Now, the absolute unit of quantity is 10 coulombs, which is fairly large, and the absolute unit of E.M.F. is $\frac{1}{10^8}$ volt, which is relatively very small.

Hence, a condenser must be of enormous size to have 1 absolute unit of capacity. It is very much as if we defined a vessel as having unit capacity for holding water, when a charge of 10 gallons would fill it to the depth of

$\frac{1}{10^8}$ inch. Evidently its area of section would be exceedingly great, or otherwise the 10 gallons of water would fill it to a greater depth than $\frac{1}{10^8}$ inch.

The result obtained above tells us that 1 absolute unit of capacity in the magnetic system is equal to 9×10^{20} static units, *i.e.* equal to the capacity of a sphere 11×10^{15} miles in diameter.

Again, as mentioned on p. 207, *a condenser will have a capacity of* 1 *practical unit (known as a "farad") when a charge of* 1 *coulomb produces a P.D. of* 1 *volt between its coatings.*

Obviously, this is a *smaller* capacity, and we can find its value in absolute units by writing

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{\frac{1}{10} \text{ absolute unit of quantity}}{10^8 \text{ absolute units of E.M.F.}}$$

$$\text{or } 1 \text{ farad} = \frac{1}{10^9} \text{ absolute units of capacity.}$$

This is still too large for ordinary purposes, and the "microfarad" is the most convenient unit for actual practice, where

$$1 \text{ microfarad} = \frac{1}{10^6} \text{ farad} = \frac{1}{10^{15}} \text{ absolute unit of capacity,}$$

which is equal to the capacity of a sphere about 11 miles in diameter.

Electro-magnetic Theory of Light.—Many careful experiments of the kind just given have produced results which have a remarkable agreement to one another, and which give a mean value of almost exactly 3×10^{10} centimetres per second.

Within the limits of experimental error, this is also the velocity of light in space, and Clerk-Maxwell, who gave to the mathematical theory of electro-magnetism its present form, suggested that light itself was an electro-magnetic phenomenon. In his time, no other experimental evidence existed to support this suggestion, but it has since been abundantly confirmed by the labours of Hertz and his successors (see p. 605).

According to Maxwell's theory, the numerical value of $\frac{1}{\sqrt{K\mu}}$ determines the velocity of an electro-magnetic wave (p. 607) in any given medium. If, for example, for a certain dielectric, $K = 4$, $\mu = 1$, then $\frac{1}{\sqrt{K\mu}} = \frac{1}{2}$, which means that the velocity in that substance is half its value in space, *i.e.* $\frac{1}{2} \times 3 \times 10^{10}$ centimetres per second.

The student should refer to more advanced treatises for a fuller treatment of the subject. We may, however, deduce the previous result by means of a simple analogy, as follows:—

It was shown by Sir Isaac Newton that the velocity of wave motion in any given medium may be expressed in the form $v = \sqrt{\frac{\text{Elasticity}}{\text{Density}}}$, where elasticity is defined as the ratio: $\frac{\text{stress applied}}{\text{strain produced}}$, and we may, therefore, try to evaluate this equation in the case of electrical quantities.

Let a small body, charged with $+Q$ units, be placed at the centre of a hollow conducting sphere of radius r. Then $+Q$ units appear on the outside of the sphere, which we may regard as being due to the introduced charge *displacing* an equal amount of charge, very much in the same way as a body lowered into water displaces its own volume of water. Now, $4\pi Q$ lines of force pass through the sphere, and the field strength at its surface is, therefore, $\frac{4\pi Q}{4\pi r^2}$.

Also, $F = KU$, $\therefore$ $U =$ electric stress $= \frac{Q}{Kr^2}$.

This stress has produced a displacement of charge per unit area of $\frac{Q}{4\pi r^2}$, which may be regarded as the measure of the strain produced by the applied stress. Therefore, we have, by analogy,

$$\frac{\textit{elasticity of dielectric}}{\text{(electric)}} = \frac{\text{stress}}{\text{strain}} = \frac{\frac{Q}{Kr^2}}{\frac{Q}{4\pi r^2}} = \frac{4\pi}{K}.^{1} \qquad (1)$$

We must next find an expression for density. We see from p. 377 that the kinetic energy of a current is $\frac{1}{2}Li^2$. Let us now apply this to the case of a ring-shaped coil, *e.g.* an endless solenoid, which is convenient because the whole of the field is contained within the turns of the coil. For such a coil we know (p. 376) that $L = \frac{4\pi n^2 A\mu}{l}$ where n is the number of turns, A the area of cross section, and l the mean length of the axis of the circular coil.

$$\therefore \text{ Energy of the magnetic field inside coil} = \tfrac{1}{2} \times \frac{4\pi n^2 A\mu}{l} \times i^2$$

$$= \tfrac{1}{2} \times 4\pi\mu \times \frac{n^2 A i^2}{l}$$

This energy is contained in a volume, $l \times A$ cubic centimetres,

$$\therefore \text{ Energy of field per unit volume} = \frac{\tfrac{1}{2} \times 4\pi\mu \times \frac{n^2 A i^2}{l}}{lA}$$

$$= \tfrac{1}{2} \times 4\pi\mu \times \left(\frac{ni}{l}\right)^2 \quad (2)$$

Now, the kinetic energy of a moving body is $\frac{1}{2}mv^2$, where m is

[1] In this and the following expression, the 4π is a coefficient depending on our initial choice of units, and has no physical meaning.

its mass, and v its velocity, which is strikingly similar to the expression (2) for the energy of a current.

If the analogy has any real basis, $4\pi\mu$ is the electrical mass per unit volume of the dielectric, *i e.* its density.

$$\text{Whence, we have } v = \sqrt{\frac{\text{Elasticity}}{\text{Density}}} = \sqrt{\frac{\frac{4\pi}{K}}{4\pi\mu}} = \frac{1}{\sqrt{K\mu}}$$

Examples.—1. Find the dimensions of specific inductive capacity in magnetic units, and of permeability in static units.

$$\text{We have } \left[\frac{1}{\sqrt{K\mu}}\right] = [LT^{-1}]$$

$$\therefore \quad [K] = [T^2L^{-2}\mu^{-1}]$$

$$\text{and} \quad [\mu] = [T^2L^{-2}K^{-1}]$$

It follows that the permeability of air (or space) in static units, and its specific inductive capacity in magnetic units, are each

$$\frac{1}{v^2} = \frac{1}{(3 \times 10^{10})^2}$$

2. Find the value of a quantity of electricity, of which the electrostatic measure is 250 (cent.$^{\frac{3}{2}}$ gram.$^{\frac{1}{2}}$ sec.$^{-1}$); (a) in magnetic units, (b) in coulombs, (c) in magnetic milligram-metre-second units.

(a)
$$[Q_s] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}]$$

$$\text{and} \quad [Q_m] = [M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

and these are identical,

$\therefore$ if n represents the value in magnetic units, we have

$$n[M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}] = 250[M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}]$$

$$\therefore \quad n = 250[LT^{-1}K^{\frac{1}{2}}\mu^{\frac{1}{2}}]$$

Now the numerical value of $K^{\frac{1}{2}}\mu^{\frac{1}{2}} = \frac{1}{v}$, and L and T are each unity

$$\text{whence } n = \frac{250}{v} = \frac{250}{3 \times 10^{10}} = 83{\cdot}3 \times 10^{-10} \text{ magnetic units.}$$

(b) In coulombs, the number will be ten times larger,

or $83{\cdot}3 \times 10^{-9}$ coulombs.

(c) We have to express $83{\cdot}3 \times 10^{-10}$ magnetic units (C.G.S.) in another system where the fundamental units are the milligram, the metre, and the second.

$$83{\cdot}3 \times 10^{-10}\left[\underset{\text{(gram)}}{M^{\frac{1}{2}}}\ \underset{\text{(cm.)}}{L^{\frac{1}{2}}}\ \mu^{-\frac{1}{2}}\right] = n\left[\underset{\text{(mg.)}}{M^{\frac{1}{2}}}\ \underset{\text{(metre)}}{L^{\frac{1}{2}}}\ \mu^{-\frac{1}{2}}\right]$$

$$\therefore \quad n = 83{\cdot}3 \times 10^{-10}\left\{\left(\frac{\text{gram}}{\text{milligram}}\right)^{\frac{1}{2}}\left(\frac{\text{centimetre}}{\text{metre}}\right)^{\frac{1}{2}}\right\}$$

$$= 83{\cdot}3 \times 10^{-10} \times \frac{1000^{\frac{1}{2}}}{100^{\frac{1}{2}}}$$

$$= 83{\cdot}3 \times 10^{-10} \times 10^{\frac{1}{2}}$$

$$= 83{\cdot}3 \times 10^{-9{\cdot}5}.$$

Clerk-Maxwell's relation between Dielectric Constant and Refractive Index for Light.—According to the theory of light, the refractive index of a substance is the ratio of the velocity of light in space to its velocity in that substance. Hence, if n be the refractive index of some transparent dielectric, we should have

$$\frac{1}{n} = \frac{1}{\sqrt{K\mu}},$$

but for all such substances, μ does not differ appreciably from unity, and therefore

$$n = \sqrt{K}, \text{ or } K = n^2.$$

When we examine the experimental values of K and n for various transparent dielectrics, we find that, whilst the above relation is satisfied in certain cases, it certainly does not hold good generally, and considerable discussion has taken place as to the cause of the discrepancies. If, however, we remember that neither n nor K are simple definite numbers for a given substance—the value of the former depending upon the wave-length for which it is measured, and that of the latter (as stated on p. 69) upon the temperature and the frequency—it will be evident that some difficulty must exist in selecting the values to be used in the comparison. As K is usually measured by processes equivalent to the use of very long waves, it may be supposed that we require the refractive index for waves of infinite length. Our ordinary tabulated values have been determined for waves of very short length, which lie within the range of vision, and if the refractive index varied with the wave-length according to some uniform law, it would be possible to calculate the corresponding values for waves of infinite length. But the phenomenon termed "anomalous dispersion," which is now known to be of general occurrence, makes the relation between wave-length and refractive index extremely complicated, the latter quantity varying between wide limits for one and the same substance. For instance, in the case of water, K = 80 (under ordinary conditions), and n for light waves is about 1·33. When, however, n is measured for electric waves having lengths of 5 centimetres and upwards, its value is found to be nearly 9, in accordance with Maxwell's law. Similar results have been obtained in the case of alcohol.

Relation between Opacity and Conductivity.—Maxwell also pointed out that a perfect conductor should be perfectly opaque, *i.e.* it should reflect completely all light falling upon it, and Drude has expressed the relation between reflecting power and conductivity in a convenient mathematical form. In Maxwell's time there were certain outstanding difficulties, *e.g.* gold-leaf is more transparent than it apparently ought to be. The discovery of the "electron" has enabled the theory to be extended to meet such cases, and it has been verified to a remarkable extent by the experimental work of Rubens on infra-red light waves.

The Ether.—In this book, we have implicitly assumed the existence of a medium, which is the seat of the phenomena denoted by the terms electric and magnetic lines of force. It may, however, be mentioned that at the present moment, the various questions associated with the ether give rise to problems of great complexity and difficulty. The experimental knowledge acquired during the last twenty years, taken in conjunction with recently acquired knowledge regarding the "electron" and the constitution of matter, leads to apparently irreconcilable results, and the real nature of the ether —if it exists at all in the old sense of the word—must be regarded as absolutely unknown. For instance, if the ether is incompressible, as it is usually assumed to be, we are driven, by one line of argument, to the conclusion that it is 2000 million times denser than lead and possesses enormous energy of internal motion. On the other hand, if it is compressible, it may be much rarer than the rarest gas. There is no intrinsic difficulty in either view, but at present no method is known by which we may hope to discriminate between them. The whole subject of the ether is in that state of uncertainty and apparent confusion, which in other branches of science has usually preceded some great advance in knowledge.

Practical Standards of Resistance.—The theoretical definitions of certain units have already been given. For instance, the absolute unit of E.M.F. has been defined as the E.M.F. produced when one conductor cuts one line of magnetic force in each second, and the volt has been *arbitrarily* taken as 10^8 absolute units. With regard to resistance, it will be seen (p. 579) that, in magnetic units, it has the dimensions of a velocity multiplied by μ, and hence, in a system of units in which μ is taken as unity for air, a resistance can be expressed as a velocity, and the absolute unit of resistance will, therefore, represent a velocity of 1 centimetre per second. The practical unit, or ohm, has been arbitrarily defined, perhaps unfortunately, as 10^9 absolute units, and it is, therefore, equivalent to a velocity of 10^9 centimetres per second.

These statements do not imply that a resistance is really the same physical quantity as a velocity, for obviously that cannot be the case if μ is regarded as a quantity of unknown dimensions, but they do imply that any method of measuring directly the resistance of a given conductor must resolve itself into a measurement of a velocity (or, of a length and a time).

These definitions, taken in connection with Ohm's law, determine the magnitude of the absolute unit of current, and incidentally it follows that an ampere is $\frac{1}{10}$ of that absolute unit.

It is, however, not sufficient merely to define these units in a consistent and logical manner; it is necessary to give them some concrete form, which may be used as standards for commercial purposes, and this is a problem of quite a different kind.

Evaluation of the Ohm.—The first step was to evaluate the ohm, *i.e.* to find the resistance of some given conductor in absolute measure. This task was undertaken in 1863 by a Committee of the British Association, who used a method suggested by Lord Kelvin. A circular coil—something like a tangent galvanometer coil—was rotated about a vertical axis in the field of the earth, and although we know that an alternating current was thereby induced in the coil, a small compass-needle suspended at the centre experienced a *steady* deflection. The theory of the method is rather too lengthy to be given here, but the resistance of the coil can be expressed in terms of the number of revolutions per second and the observed deflection of the needle (*i.e.* in terms of a velocity). The self-induction of the coil must also be taken into account. This method is subject to various sources of error, difficult to eliminate, and is not now used.

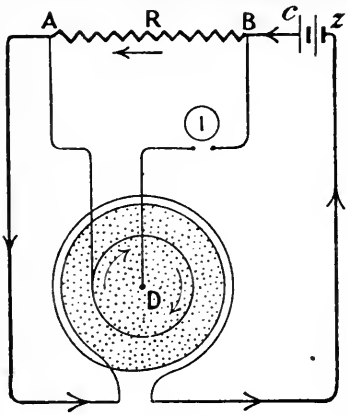


FIG. 404.

Of the numerous other methods proposed or carried out, perhaps the most important is that of Lorenz, which seems likely to become generally adopted. The principle of the method is indicated in Fig. 404.

A metal disc, D, is rotated inside a coil carrying a current, so that it moves at right angles to the field produced by the coil (indicated by dots). The conductor, R, whose resistance is to be measured, is placed in series with the coil, and the P.D. between its ends balanced against the induced P.D. between the centre and the circumference of the disc. The P.D. induced in the disc D depends upon its speed, and hence if it is connected up as shown there will be some speed at which the galvanometer will not be deflected (of course the two P.D.'s must be in opposition).

Let i = steady current in coil and in R,
R = resistance to be measured,
M = the mutual inductance between coil and disc,
n = number of revolutions of the disc per second,
e = P.D. between centre and circumference of disc,

then:— lines passing through disc $= M.i$,

also $$e = \frac{\text{lines cut in 1 revolution} \times \text{number of conductors}}{\text{time of 1 revolution}}$$

$$= \frac{Mi \times 1}{\frac{1}{n}} = M.i.n$$

Also, the P.D. between A and B $= i$R, and, therefore, if these P D.'s are equal and opposite,

$$iR = M.i.n, \text{ or } R = M.n.$$

Now M can be calculated from the dimensions of the apparatus as a purely geometrical problem, so that again R is measured as a velocity.

The chief drawback to this method is the fact that only comparatively small resistances can be measured by it.

The Ohm in Terms of a Column of Mercury.—In order that such absolute determinations may be practically useful, it is desirable to express them in terms of a substance readily obtainable in a state of purity, such as mercury, and the result of numerous determinations by different methods has been to show that the ohm is the resistance of a column of mercury 1 square millimetre in section and about 106·3 centimetres in length. In 1890, this was adopted as the legal definition of the ohm in this country, but as the only method of determining the cross section is to obtain the weight of mercury filling a given length of the tube containing it, the definition was afterwards amended, and at the present time it runs as follows: "The international ohm (as recommended by the International Conference on Electrical Units, held in London in 1908), is the resistance offered to an unvarying current by a column of mercury at the temperature of melting ice, 14·4521 grams in mass, of constant cross section, and of length 106·3 centimetres."

To construct a standard resistance embodying this definition, it is not necessary to use exactly the above dimensions. It would be exceedingly difficult to do so, and in practice a glass tube of uniform section, conveniently about 1 millimetre in diameter, is fitted into larger end vessels (to contain terminals), and the whole filled with mercury. The report of the Conference specifies that these end vessels shall be spherical, of approximately 4 centimetres in diameter, connection being made by means of platinum wires sealed through the glass.

The actual resistance of this standard is then calculated as follows: Let L=length of tube, r=radius, A=area of section, s=specific resistance, D=density of mercury, M=mass of mercury filling the tube, and R=its resistance in ohms. Then (neglecting at first the resistance of the mercury in the spherical ends) we have:—

$$R=\frac{L}{A}\times s$$

$$=\frac{L}{\pi r^2}\times s$$

$$\text{But } M=A\times L\times D=\pi r^2\times L\times D$$

$$\therefore \quad \pi r^2=\frac{M}{LD}$$

$$\text{and } R=\frac{L^2.D.s}{M}$$

Substituting in this expression the values given in the definition, we have

$$1=\frac{(106\cdot3)^2\times D.s}{14\cdot4521}$$

$$\text{or} \quad D.s=\frac{14\cdot4521}{(106\cdot3)^2}$$

$$\therefore \quad R=\frac{L^2}{M}\times\frac{14\cdot4521}{(106\cdot3)^2}$$

$$=\cdot001278982\times\frac{L^2}{M}\text{ ohms.}$$

Hence, we have only to determine experimentally the weight of the mercury filling the tube, and its length, in order to obtain a concrete standard of known resistance.

The report of the Conference gives the following formula for evaluating the slight extra resistance due to the spherical ends, when made in accordance with their instructions:

$$\text{Additional resistance}=\frac{\cdot80}{106\cdot3\pi}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\text{ ohms,}$$

where r_1 and r_2 are the radii in *millimetres* of the end sections of the bore of the tube.

Practical Standards of Current and E.M.F.—It is necessary to define one other practical standard (the third then follows from Ohm's law), but, whereas the ohm has been unanimously adopted by all countries, there has been some difference of opinion and of practice as to whether current or E.M.F. shall be selected for the second standard. If the former be chosen, the ampere must be defined in terms of chemical action, and, of course, cannot be represented by an actual standard. If the latter, the volt must be defined in terms of the E.M.F. of some standard cell, and there is a strong inducement to adopt this course, because, whatever be taken as the legal standard, practical men will always use a standard cell and a known resistance as the basis of their measurements.

The International Conference strongly recommended the adoption of the ampere, as a current can be measured quite independently of the ohm; and in carrying out practical measurements by means of a silver voltameter it is only necessary to use one salt (silver nitrate), which is easily procurable in a pure state. On the other hand, in a standard cell several salts are required, one of them (mercurous sulphate) is difficult to purify; and further, the absolute determination of its E.M.F. depends on the measurement of a current and a resistance. They, however, recommended the use of the Weston cadmium cell as a convenient sub-standard.

In order to define the ampere in terms of chemical action, it is necessary to measure a current absolutely, and to determine the amount of some substance liberated by it in a given time. A silver nitrate voltameter has proved to be the most suitable, and the current has usually been measured by some form of current-balance (see p. 567), because its absolute strength can be calculated in terms of the dimensions of the instrument. The final result shows that the ampere is the current which liberates ·001118 gram of silver per second.

The numerical value of the E.M.F. of a standard cell is based upon these two definitions. For the details of the modified potentiometer method most generally used for measuring the E.M.F. of such cells, the student should consult treatises on electrical measurements, but the principle is easily understood, and may be outlined as follows. Assume that a standard resistance of accurately known value—say 1 ohm—is available, and that it is able to carry one or two amperes without sensible heating. In any case it would be arranged so that its temperature could be measured and kept constant. Let this be connected to a suitable battery; an adjustable resistance and a silver voltameter being included in the circuit. The cell whose E.M.F. is to be determined is placed in series with a sensitive galvanometer, and the two connected across the terminals of the resistance R like the cell E_1 in Fig. 234, p. 306, in such a way that its E.M.F. is in opposition to the P.D. across R. The current is then adjusted until the galvanometer shows no deflection. If C is this current, then CR is the P.D. across R, and is equal and opposite to E. Hence, it is only necessary to measure C, but this must be done with the greatest possible accuracy. For this purpose the silver voltameter may be used, precautions being taken to ensure that C does not vary during the time required to deposit a convenient weight of silver.

For a Weston cadmium cell, set up in accordance with the specification published by the Conference, it may be taken as 1·0183 volts at 20° C. In the report of that body, the following expression is given for the value at other temperatures:—

If E_t = E.M.F. at t° C.; E_{20} = E.M.F. at 20°, then:—

$$E_t = E_{20} - \cdot0000406\,(t-20) - \cdot00000095\,(t-20)^2 + \cdot00000001\,(t-20)^3$$

Table of Electrical Units.—For convenience of reference we append the following table, indicating the various relations between the more important units and also the pages on which they are mentioned in this book.

It may be remarked that the ideas of *potential*, *quantity*, and *capacity* are met with in considering current phenomena, and also in electrostatics, and hence each of these possesses two well-known units.

On the other hand, the ideas of *current*, *resistance*, and *inductance* more naturally belong to the former section, and hence are represented by only one unit.

It is, of course, quite easy to express these in the static system (an instance is given on p. 580 in the case of the unit of resistance), but it would be misleading to insert such numbers in the following table.

	Name of Practical Unit.	Numerical Value of 1 Practical Unit in Absolute (*magnetic*) Units.	Numerical Value of 1 Practical Unit in *Static* Units.	Numerical Value of 1 Absolute Magnetic Unit in Static Units. ($v=3\times10^{10}$).	Page.
Resistance	Ohm	10^9	...	...	206, 587
Current	Ampere	10^{-1}	...	...	207, 590
Electromotive Force or Potential	Volt	10^8	3×10^{-2}	$\frac{1}{v}$	206, 351
Quantity or Charge .	Coulomb (or Ampere-second)	10^{-1}	3×10^9	v	207, 230
Capacity	Farad	10^{-9}	9×10^{11}	v^2	207
Inductance	Henry	10^9	...	...	377
Power	Watt	$=10^7$ ergs per sec. $=\frac{1}{746}$ H.P.			231
Work or Energy . .	Joule (or Watt-second)	$=10^7$ ergs			234

Multiples and Sub-multiples:—

Megohm $=10^6$ ohms.
Microhm, microvolt, microfarad, micro-ampere. Each of these is 10^{-6} of the corresponding practical unit.
Milli-ampere, millivolt. Each is 10^{-3} of the corresponding unit.
Kilowatt $=1000$ watts.
Kilowatt-hour, or "Board of Trade Unit" $=1000$ watt-hours $=1000\times10^7\times60\times60$ ergs.
Ampere-hour $=3600$ coulombs.

Exercise XXVIII

1. Define unit magnetic pole and unit electrical current in the electromagnetic system, and state the relation of the ampere to the latter. (B. of E., 1895.)

2. Draw up a table giving the practical units in terms of the C.G.S. electro-magnetic units for current, quantity, electromotive force, resistance, and capacity, detailing the fundamental relations on which the latter system is based. (B. of E., 1907.)

3. Describe the electrostatic and electro-magnetic systems of units. Find in each of the systems the dimensions of (1) potential difference, (2) capacity of a condenser, (3) specific resistance. (B. of E., Hon., 1903.)

4. Define the terms magnetomotive force, magnetic flux, and reluctance of a magnetic circuit. Find the dimensions of these quantities.

(Lond. Univ., B.Sc. Internal, 1909.)

5. According to the usual definitions, the dimensions of capacity in the electro-magnetic system are those of the reciprocal of an acceleration, while in the electrostatic system they are simply a length. Show how these results are obtained, and explain the apparent discrepancy.

(Lond. Univ., B.Sc. Internal, 1908.)

6. A disc 16 centimetres in diameter is rotated at a speed of 3700 revolutions per minute inside a long spiral coil of wire of 13 turns in each centimetre length of coil, the axis of revolution being parallel to the axis of the coil and perpendicular to the plane of the disc. When 1·5 amperes are passed through the coil, what difference of potential exists between the axis and the circumference of the disc. (Lond. Univ., B.Sc. Internal, 1903.)

7. Find the number of watts in 1 horse-power, given 1 foot=30·48 centimetres; 1 lb.=453·6 grams; $g=981$ centimetre/(seconds)^2.

Electrical energy is sold at the rate of 4d. per kilowatt-hour. The mechanical equivalent of the heat given by the burning of coal worth 4d. is 10^8 foot-lbs. Compare the prices of the two forms of energy. Why is electrical energy so much dearer than coal energy?

(Lond. Univ., B.Sc., 1902.)

APPENDIX

CHAPTER XXXV

EFFECT OF INDUCTANCE AND CAPACITY AT STARTING AND STOPPING A CURRENT—ELECTRIC OSCILLATIONS—RADIATION—WIRELESS TELEGRAPHY AND TELEPHONY.

WE have already shown (see p. 368) that, if Z lines of magnetic force cut N turns in t seconds, the *average* value of the induced E.M.F. is given by

$$\underset{\text{(average)}}{e} = \frac{ZN}{t} \text{ absolute units.}$$

If the rate of cutting is not uniform, the induced E.M.F. at any instant depends upon the rate of cutting at that instant, and we may write

$$\underset{\text{(instantaneous)}}{e} = \frac{dZ}{dt} \cdot N \qquad (1)$$

This expression holds good for *any* induced E.M.F. We have now to apply it more particularly to the phenomena attending the rise and fall of a current in an inductive circuit.

Let a steady current i flow in a coil of N turns, producing a flux of Z lines. Then equation (1) gives the value of the E.M.F. of self-induction at any instant whilst the current is starting or stopping (or more generally *varying* in strength). In this case, it is customary to write the equation with a negative sign in order to indicate that, when Z is increasing, e is a *back* E.M.F. Now, by definition (see p. 376)

$$ZN = Li$$

$$\therefore \quad \frac{dZ}{dt} \cdot N = L\frac{di}{dt} \qquad (2)$$

$$\text{and} \qquad e = -L\frac{di}{dt}$$

We have frequently pointed out, that this relation holds good only when the conditions are such that L may be regarded as constant, whereas equation (1) is always true.

Hence, in order that a current i may flow in a circuit (or part of a circuit) of resistance R and inductance L, we must have, at any instant, an impressed E.M.F., E, which, while the current is increasing, is numerically equal to $iR+L\frac{di}{dt}$, and while the current is decreasing, is equal to $iR-L\frac{di}{dt}$; or algebraically

$$\underset{\text{(instantaneous)}}{E} = iR + L\frac{di}{dt} \qquad (3)$$

where the sign of the second term depends upon the sign of $\frac{di}{dt}$; *i.e.* positive for an *increasing*, and negative for a *decreasing*, current.

From this equation, we may deduce the law of rise of current in a circuit, when a *steady* E.M.F. is suddenly applied to it, *i.e.* the law for direct-current circuits.

Helmholtz's Equation.—Writing expression (3) in the form

$$E - iR = L\frac{di}{dt}$$

by transposition we obtain

$$\frac{di}{\frac{E}{R}-i} = \frac{R}{L}dt$$

and integrating both sides of this equation,

$$\log_e\left(\frac{E}{R}-i\right) = -\frac{Rt}{L} + \text{constant.}$$

Now, $i=0$ when $t=0$, and, therefore, the constant is $\log_e\frac{E}{R}$

$$\therefore \quad \log_e\left(\frac{E}{R}-i\right) = \log_e\frac{E}{R} - \frac{Rt}{L}$$

$$\text{or} \quad \log_e\frac{\frac{E}{R}-i}{\frac{E}{R}} = -\frac{Rt}{L}$$

$$\text{or} \quad \log_e\left(1-\frac{iR}{E}\right) = -\frac{Rt}{L}$$

$$\therefore \quad 1-\frac{iR}{E} = e^{-\frac{Rt}{L}}$$

$$\therefore \quad \frac{iR}{E} = 1 - e^{-\frac{Rt}{L}}$$

$$i.e. \quad i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \qquad (4)$$

(where e is the base of natural logs.)

From this equation we can calculate the value of i at any time t after the start. We see that the second term gradually decreases as t increases, and becomes zero when t is infinite, which means that the current takes an infinite time to rise to its steady value. When $L=0$, the expression reduces to the ordinary form of Ohm's law, but as no circuit is absolutely non-inductive, equation (4) may be taken as holding good generally.

Time Constant of an Inductive Circuit.—Putting $t=\frac{L}{R}$ in the above equation, it becomes

$$i=\frac{E}{R}\left(1-\frac{1}{e}\right)$$

hence, $\frac{L}{R}$ is the time required by the current to reach a certain fraction $\left(1-\frac{1}{e}, \text{ or } 1-\frac{1}{2\cdot7182}, \text{ or } \cdot632\right)$, of its final value.

As the dimensions of self-inductance are [length $\times \mu$] and of resistance $[LT^{-1}\mu]$, it follows that the ratio $\frac{L}{R}$ has the dimensions of *time*, and is known as the *time constant* of the circuit.

This ratio has been already met with, *e.g.* on p. 454 it was shown that, if a steady current i be suddenly interrupted, a quantity, q, rushes round the circuit, such that $q=\frac{L}{R}\times i$. Hence, an inductive circuit behaves as if a quantity were stored up in it equal to the quantity conveyed by the full current flowing for a time equal to the time constant of that circuit.

In a *non-inductive* circuit, the current would rise *instantly* to its full strength, and, during t seconds after the start, a quantity, $i.t$, would have passed round the circuit. In an *inductive* circuit, a smaller quantity would have passed during this time, and it can be shown that the difference is equal to $\frac{Li}{R}$, or the circuit behaves as if the missing quantity were stored up in it. Of course, it is not "quantity," but "energy" that is really stored up, in the form of a magnetic field. An expression for this energy has already been found by simple reasoning (see p. 377); it may also be evaluated as follows:—

The current rises from 0 to i in time t against a back E.M.F., whose value at any instant is $L\frac{di}{dt}$.

If the current and the back E.M.F. were steady, the work done against the latter would be $i\times L\frac{di}{dt}\times t$, and this would be the energy stored up in the field. But, as both current and back E.M.F. vary, we must evaluate the above expression over the whole time, by writing,

$$\text{Work}=\int_0^i i\times L\frac{di}{dt}\times dt$$

$$=\quad \tfrac{1}{2}Li^2$$

Dying Away of a Current in an Inductive Circuit.—Equation (4) suggests the converse problem—given a current i_0 flowing steadily in the circuit, what occurs when the applied E.M.F. is suddenly removed without opening the circuit? To answer this question, we must put $e=0$ in equation (3), then we have

$$iR+L\frac{di}{dt}=0$$

from which, by rearrangement,

$$\frac{di}{i}=-\frac{R}{L}dt$$

and integrating we obtain

$$\log_e i=-\frac{R}{L}.t+\text{a constant.}$$

Now, when $t=0$, the current is i_o, therefore the constant is $\log i_o$.

$$\therefore \quad \log_e \frac{i}{i_o}=-\frac{R}{L}t$$

$$i.e. \quad \frac{i}{i_o}=e^{-\frac{R}{L}t}$$

$$\text{or} \quad i=i_o\times e^{-\frac{Rt}{L}} \qquad (5)$$

From which, we may calculate the current strength at any instant as it dies away.

Rise of Current in a Circuit containing Capacity and Resistance.—Let a steady E.M.F., E_o, be suddenly applied to a circuit containing a condenser of capacity K in series with a resistance R, and let E be the P.D. between the condenser terminals at any instant. At first E will be zero and will gradually rise to E_o in a direction opposed to E_o, while the current will gradually decrease and become zero at the instant $E=E_o$.

Let Q be the quantity in the condenser at any instant, then

$$Q=EK$$

$$\therefore \quad \frac{dQ}{dt}=K\frac{dE}{dt}$$

but $i=\frac{dQ}{dt}$. where i is the current at that instant,

$$\text{also} \quad i=\frac{E_o-E}{R}$$

$$\therefore \quad K.\frac{dE}{dt}=\frac{E_o-E}{R}$$

$$\therefore \quad \frac{dE}{E_o-E}=\frac{dt}{KR}$$

Integrating, $\log_e(E_o-E)=-\frac{t}{KR}+\text{a constant.}$

Now, when $t=0$, $E=0$, $\therefore$ constant is $\log E_o$

$$\therefore \quad \log_e\frac{E_o-E}{E_o}=-\frac{t}{KR}$$

$$\text{or} \quad 1-\frac{E}{E_o}=e^{-\frac{t}{KR}}$$

$$\text{or} \quad E=E_o\,(1-e)^{-\frac{t}{KR}}$$

$$\text{but as} \quad i=\frac{E_o - E}{R}$$

$$\therefore \quad i=\frac{E_o}{R}\times e^{-\frac{t}{KR}} \qquad (6)$$

The product KR has the dimensions of time. It is called the *time constant*[1] of the circuit. Evidently it is the time the P.D. across the condenser takes to reach $1-\frac{1}{e}$ or ·632 of its steady value.

Dying Away of a Current in a Circuit containing Capacity and Resistance.—Let the condenser be charged to a P.D. equal to E_o, and then be suddenly short-circuited through a resistance R. Then, if Q and E be the respective values at any instant during discharge, $Q=EK$.

Also $i=-\frac{dQ}{dt}$, the negative sign being due to the fact that, in this case, Q is decreasing as t is increasing.

As before, $\frac{dQ}{dt}=K\frac{dE}{dt}$, and as both Q and E are decreasing with the time, both sides of this equation are negative—and that sign cancels out.

Also, at any instant, $i=\frac{E}{R}$,

$$\therefore \quad i=-\frac{dQ}{dt}=-K\frac{dE}{dt}=\frac{E}{R}$$

$$\therefore \quad \frac{dE}{E}=-\frac{dt}{KR}$$

Integrating, $\log_e \frac{E}{E_o}=-\frac{t}{KR}$

from which, by the methods already given, we obtain

$$E=E_o\times e^{-\frac{t}{KR}}$$

$$\text{and} \quad i=\frac{E}{R}=\frac{E_o}{R}\times e^{-\frac{t}{KR}} \qquad (7)$$

This case is important, as it may be applied to the measurement of very high resistances.

For this purpose, the expression $\log \frac{E}{E_o}=-\frac{t}{KR}$ may be written

$$R=-\frac{t}{K\log_e \frac{E}{E_o}}=\frac{t}{K\log_e \frac{E_o}{E}} \qquad (8)$$

Hence, if a well-insulated condenser of known capacity is charged to any convenient potential, and then short-circuited by a resistance R so great that the charge leaks through it comparatively slowly, the value of R can be calculated by observing the P.D. at the beginning and end of any measured time, t seconds. As $\frac{E_o}{E}$ is a ratio, their values can be expressed

---

[1] This implies that $L=0$, just as a previous statement (that time constant$=\frac{L}{R}$) implies that $K=0$.

in any arbitrary units. If K is expressed in farads, R will be in ohms; if K is expressed in static units of capacity, R will be in static units of resistance. Now, a reference to p. 580 will show that, in static units, the dimensions of resistance are those of the reciprocals of a velocity (putting spec. ind. cap.$=1$), and the above result illustrates the meaning of this fact, for we see that the greater the value of R, the slower will the charge leak away through it.

As an interesting example of the use of this formula, the following well-known method may be mentioned. It was required to find the "insulation resistance" of a certain specimen of electric light cable, 110 yards in length, wound in the form of a coil. For such purposes the coil is immersed in a tank of water, with both ends protruding, and the resistance to be measured is that between the copper wire and a plate placed in the water. The best method for the purpose would be that given on p. 275, but, in this case, the resistance was much too great to be measured in that way with the apparatus available. The leakage method was, therefore, employed, and as such a coil has a capacity and acts like a Leyden jar (the wire forming one coating and the water the other coating), it was unnecessary to use a separate condenser.

The first step was to measure the capacity of the coil by comparing it with a condenser of known capacity, using the method described on p. 313. This was found to be $\frac{1}{32}$ microfarad. (In order to obtain a *small* known capacity for comparison, several condensers were connected in series, and use was made of the formula given on p. 65.)

Then the throw was taken, (1) when the coil, after being charged by a suitable battery, was *immediately* discharged through a galvanometer; (2) when three minutes were allowed to elapse between charge and discharge, during which period the charge was leaking slowly through the insulation. These throws were 54 and 42 scale divisions respectively.

Therefore, we have $$\frac{E_o}{E}=\frac{54}{42}$$

where $$R=\frac{t}{K\log_e\dfrac{E_o}{E}}$$

Hence, $$R=\frac{3\times 60}{\dfrac{1}{32\times 10^6}\times\dfrac{\log_{10}\dfrac{54}{42}}{\cdot 4343}}\text{ ohms,}$$

which, in round numbers, is $23{,}000\times 10^6$ ohms, or 23,000 megohms. It may be mentioned that the value for 1 mile of such cable would be $\frac{23{,}000}{16}=1440.$ This result would be expressed by saying that the insulation resistance was about 1400 megohms per mile.

In such experiments, all the apparatus must be carefully insulated in order to avoid large errors due to leakage other than that through the insulation, and in any case no great accuracy is likely to be obtained. It should be regarded as a method of obtaining approximate values for enormously great resistances, useful when no other plan is available.

Equation (8) may also be applied to compare the capacities of two condensers by means of an arrangement similar to Wheatstone's bridge. For this purpose, the condensers whose capacities are to be compared (C and C^1, Fig. 405[1]) are placed in the two arms of a bridge, the other two arms being

[1] From Carey Foster and Porter's *Electricity and Magnetism.*

occupied by resistance boxes a and a'. The circuit may be earthed at points shown in the figure, or these points may be connected by a wire. When the key is depressed, the condensers are charged by the battery, and when it is allowed to fly back they are discharged again. The resistances are adjusted until no throw is produced on working the key. The resistances should be fairly large, and it is preferable to rely upon balance during "charge," for, if the condensers have different dielectrics, absorptive effects may interfere with balance during discharge.

Evidently the condition for balance is that the potential at B should, at any instant, be equal to that at B^1. This implies that the P.D. across each

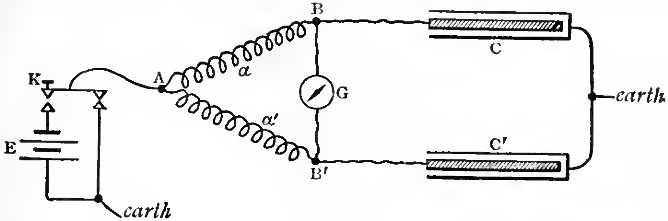


FIG. 405.

condenser rises or falls at exactly the same rate. When this is the case, the ratio $\frac{E_o}{E}$ in equation (8) is the same for each condenser. Hence, if R and R_1 are the resistances unplugged in the boxes a and a^1, and K and K_1 are the capacities of C and C^1 respectively, we have at any very short time t, after the commencement of charge or discharge,

$$R=\frac{t}{K\log\frac{E_o}{E}},\quad R_1=\frac{t}{K_1\log\frac{E_o}{E}},\quad \text{or}\quad \frac{R}{R_1}=\frac{K_1}{K}.$$

i.e. $KR=K_1R_1$, which means that the "time constants" of the two paths are adjusted to equality.

Discharge of a Condenser when the Circuit contains Self-inductance but not Resistance.—In this case, there are two E.M.F.'s in existence during discharge: (1) the condenser E.M.F.; (2) the E.M.F. of self-induction, which we know is in opposition to the rising current, and, therefore, also in opposition to the condenser E.M.F., which is the cause of that current. Also, as there is no resistance, there is no *resultant voltage*, and, therefore, at any instant, the sum of the two E.M.F.'s must be zero.

Let i be the current at a given instant; then the E.M.F. of self-induction is $L\frac{di}{dt}$.

Also, if Q be the quantity in the condenser at that instant, the condenser E.M.F. is $\frac{Q}{K}$, and $i=\frac{dQ}{dt}$, or $\frac{di}{dt}=\frac{d^2Q}{dt^2}$

$$\therefore\quad L\frac{di}{dt}+\frac{Q}{K}=0 \text{ (algebraically)}$$

$$\text{or}\quad L\frac{d^2Q}{dt^2}+\frac{Q}{K}=0$$

$$\text{or}\quad \frac{d^2Q}{dt^2}+\frac{Q}{LK}=0 \qquad (9)$$

This is a familiar type of differential equation, which is known to represent an undamped simple harmonic motion, whose periodic time, T, is given by

$$T = 2\pi\sqrt{LK} \tag{10}$$

Hence, we find that such a discharge is *oscillatory*. It is not difficult to understand the physical meaning of this result.

The charged condenser contains a certain amount of energy stored up in the form of an electrostatic field in the dielectric. During discharge, the static field disappears, but, in its place, a magnetic field comes into existence, and as, owing to the assumed absence of resistance, no energy is lost, the whole of the energy, at the instant the condenser is discharged, is stored up in this magnetic field. Then, the magnetic field begins to disappear, and in so doing sets up an E.M.F. in the *same* direction as the discharge current which is dying away, but which is thereby prolonged, and evidently this means that the condenser begins to charge up again, but in the *opposite* direction to its previous charge. Hence, the energy oscillates between the two forms of field, much as a spring oscillates when suddenly released. In actual practice, however, there must be *some* resistance, and, therefore, some energy (due to C^2r heat in the conductor) will be lost in each transformation, so that the oscillations will gradually decrease in amplitude until they disappear.

It is instructive to derive the same result from the fundamental principle for all vibrations, that, *assuming no loss*, the sum of the potential and kinetic energies is constant at any given instant. Now the potential energy, at any instant, is $\frac{1}{2}\frac{Q^2}{K}$, and the kinetic energy is $\frac{1}{2}Li^2$

$$\therefore \quad \frac{1}{2}\frac{Q^2}{K} + \frac{1}{2}Li^2 = \text{a constant.}$$

Differentiating with respect to time, we obtain

$$\frac{1}{2K}.2Q.\frac{dQ}{dt} + \frac{1}{2}L.2i.\frac{di}{dt} = 0$$

$$\text{but } i = \frac{dQ}{dt}$$

$$\therefore \quad L\frac{di}{dt} + \frac{Q}{K} = 0, \text{ as before.}$$

Discharge of a Condenser through a Circuit containing Self-induction and Resistance.—In this case, there is, at any instant, a resultant E.M.F. equal to iR, and the expression becomes

$$L\frac{di}{dt} + \frac{Q}{K} + iR = 0$$

$$\text{or} \quad L\frac{d^2Q}{dt^2} + \frac{dQ}{dt}.R + \frac{Q}{K} = 0$$

$$\text{or} \quad \frac{d^2Q}{dt^2} + \frac{dQ}{dt}.\frac{R}{L} + \frac{Q}{KL} = 0 \tag{11}$$

This is the characteristic equation of a damped vibration. Its solution is somewhat difficult, and need not be given here, but it leads to the result that the periodic time is given by

$$T = \frac{2\pi}{\sqrt{\frac{1}{LK} - \frac{R^2}{4L^2}}} \tag{12}$$

It will be seen that when $R=0$, this equation reduces to the same value as before (equation 10).

Again, when $\frac{1}{LK}=\frac{R^2}{4L^2}$, the discharge is just non-oscillatory.

It will be noticed that the effect of damping is to alter the period of vibration. This is a characteristic property of all simple harmonic vibration, and it is possible to express it in another way, by writing

$$T=2\sqrt{\pi^2+l^2}\,.\,\sqrt{LK}$$

when l is the logarithmic decrement.

The alteration in period due to damping is of very great importance in connection with wireless telegraphy.

Electric Oscillations.—The student should clearly realise that *both* inductance and capacity must be present in a circuit, in order that current oscillations may occur in it. As we have seen, when only one of these factors is present, the current rises and falls according to some exponential function of the time, but *without* oscillations. Similarly, in order that a material system may vibrate, it must possess *both* elasticity and inertia, and we have already seen (p. 584) that capacity (or rather its reciprocal) and self-induction represent analogous quantities in electric circuits.

Lord Kelvin, in 1853, first obtained mathematically the result given in equation (12), and from it, he inferred that under certain conditions electric discharges were oscillatory. Five years later, the fact was confirmed experimentally by Fedderson, who photographed the image of a Leyden jar spark in a rotating mirror, and found that it was drawn out into a series of images due to sparks following each other in rapid succession.

High-frequency Currents.—Alternating currents of very high frequency are most readily obtained by means of condenser discharges. A very convenient form of apparatus[1] for experimental purposes on a small scale is shown diagrammatically in Fig. 406.

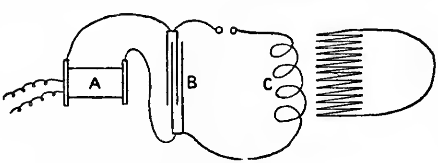


FIG. 406.

A is an induction coil—the more powerful the better—worked by a battery in the usual way. B is a condenser, consisting of one or more Leyden jars, in parallel with the coil; and a discharge circuit is provided through a spark-gap and an inductance, C. (The coil

[1] Although this apparatus is described here for the sake of convenience, such arrangements were not devised until after the discoveries of Hertz, mentioned subsequently.

shown on the right of C may be ignored for the moment.) The coil charges the condenser until a spark occurs across the gap—each discharge being oscillatory and of a frequency depending only upon the capacity, inductance, and resistance in the discharge circuit. The spark is short, but very bright and noisy, and the current strength (as read on a hot-wire ammeter) may be several amperes, very much greater than the current in the secondary of the induction coil. The *maximum* value of the current is of course not given by the ammeter, but it is remarkably great and may amount to 50–100 amperes with jars of quite moderate capacity. This is because the actual time of discharge is very small, and hence a comparatively small quantity of charge passing round the circuit may be equivalent to a large current during that time.

Owing to the high frequency, inductive effects can be produced with this apparatus in a very striking manner. For instance, let the inductance (C in Fig. 406) consist of 4 or 5 turns of well-insulated copper wire in the form of a square, with sides about 2 feet long, supported on a rough wooden frame; and let another similar coil be made with its circuit closed through a 10 or 20 volt glow-lamp. It will be found that the lamp lights up, when its coil is held parallel to the first coil, and anywhere reasonably near it. When they are brought quite close together, the lamp may be burnt out.

In a certain case, the joint capacity of two Leyden jars, which were used, was ·0038 microfarads. The coil consisted of 5 turns wound on a square frame, each side being 65 centimetres long, and its inductance was found to be 130,000 absolute units (often written 130,000 centimetres, as the dimensions of L are those of a length). Neglecting the effect of resistance, which would be relatively small, we have

$$T = 2\pi\sqrt{LK}$$

$$\text{also frequency} = n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LK}}$$

In substituting numerical values in this equation, L and K must be expressed in absolute units. It was shown on p. 583 that 1· microfarad $= 10^{-15}$ absolute units of capacity, and hence we have

$$n = \frac{1}{2\pi\sqrt{130{,}000 \times \frac{\cdot 0038}{10^{15}}}} = 230{,}000 \text{ alternations per second.}$$

With this arrangement, the oscillations are superposed upon a certain amount of non-oscillatory discharge derived from the coil, and for high-frequency currents, it is convenient to use a simple transformer of the kind introduced by Tesla. The primary may consist of a very few turns (perhaps less than a dozen) of thick copper wire, *without* an iron core, and the secondary is a single layer of fine insulated copper wire ending in well-insulated discharge terminals, the windings being immersed in oil to obtain sufficient insulation. The iron core is omitted because experience shows that its presence does not in any way increase the power of the coil. In fact, at these high frequencies, the effect of eddy currents in the iron is great enough to prevent the magnetic induction from following the rise and fall of the current with sufficient rapidity, and thus it becomes practically inert.

The two windings of this coil are shown on the right of Fig. 406, but the secondary is there shown short-circuited, whereas it would have been better to have introduced discharge terminals.

This transformer raises the voltage without altering the frequency, and, when in action, a highly oscillatory discharge, several inches long, passes across the secondary terminals. If a piece of glass be held between them, it does not interrupt the discharge, which behaves very much as if the glass were not there. The glass, however, soon becomes very hot, and has, in fact, been acting as a condenser of small capacity. It is merely an instance of the result obtained on p. 448, where it is shown that

$$C = 2\pi n KE = \frac{E}{\frac{1}{2\pi n K}}$$

In this case, although K is small, n is so great that the *impedance* of the glass is scarcely appreciable. The heating of the glass shows that some dielectric loss occurs in it at each charge and discharge (as pointed out on p. 447).

If the hand be brought near one of the terminals, the sparks are distinctly more painful than ordinary sparks of the same length, but if a piece of metal on which the spark is received be held in the hand, nothing whatever is felt. Short sparks can then be drawn from any part of the body of the operator; he can light a gas jet with his finger, without being insulated from the ground; and a vacuum tube merely held in the hand lights up more or less brilliantly. It is a remarkable fact, first demonstrated by Tesla, that at these high frequencies, very powerful discharges may be passed through the body without any sensation whatever being felt. Using the apparatus just described, with a 10-inch spark coil worked by six chromic acid cells, one of the writers has frequently performed the experiment of short-circuiting the secondary terminals of the Tesla coil through his body. If the terminals are touched whilst the discharge is passing, a shock will be felt at that instant. To avoid this, it is better to first bring the terminals into contact with each other (this does not injure the coil), and then they may be grasped in the hands and separated without anything being felt. Before releasing the terminals, they should be again brought into contact. To indicate that a discharge is really passing through the body, a small incandescent lamp (about 10 volts) may easily be introduced between one hand and a terminal; its filament then glows more or less brilliantly.

Impedance at High Frequencies.—In consequence of the high frequency, the "choking" effect of a few turns of wire is very great, and even straight conductors possess considerable impedance (for reasons explained on p. 434). For instance, when the secondary terminals were joined by a coil of well-insulated copper wire, having only 1·5 ohms resistance, the spark discharge, although shorter than before, still persisted, thus indicating that a very considerable P.D. still existed across the gap.

Surface Distribution of Current at High Frequencies.—At very high frequencies, however, another important effect comes into operation, which makes the resistance of a conductor greater than its actual value for steady currents. A current starts at the surface of a conductor, and takes an appreciable, though very short, time to penetrate to the interior. When the alternations are very rapid, it may never reach the centre at all, and the effective cross section of the conductor is thus decreased. In fact, at very high frequencies, a wooden rod covered with tinfoil conducts as well as copper.

The theory of this effect has been worked out by Lord Rayleigh, to whom the following formula is due:—

Let R = the resistance to steady currents in ohms,
l = the length of the conductors in centimetres,
n = frequency,
R_1 = the effective resistance,

$$\text{then} \quad R_1 = \sqrt{\pi n l R \mu}$$

This expression applies only to straight conductors.

Considering a round wire, let A = area of section $= \frac{\pi d^2}{4}$, and let s = specific resistance.

$$\text{Now} \quad R = \frac{l}{A} \cdot s = \frac{4l}{\pi d^2} \times s$$

$$\text{or} \quad l = \frac{R\pi d^2}{4s}$$

Substituting this value of l, we obtain

$$R_1 = \tfrac{1}{2}\pi d R \sqrt{\frac{n}{s} \cdot \mu}$$

which shows that the effect increases with the diameter of the conductor, a fact that we might expect. If $d=1$ centimetre, and $n=10^6$, R_1 is about 40 times R; but if $d=\frac{1}{40}$ centimetre, and $n=10^6$, R_1 is practically the same as R. Hence, all conductors for use with such currents should be laminated, *i.e.* made up of separately insulated small wires or strips. As permeability influences the result, this "skin effect" is much greater for iron than for non-magnetic metals. For instance, at a frequency of 100, a current will penetrate to a depth of about 26 millimetres in copper, but only about 2 millimetres in iron; and at a frequency of 10^6, these values become about $\frac{1}{15}$ millimetre for copper, and $\frac{1}{200}$ millimetre for iron.

Electro-magnetic Radiation.—Fitzgerald appears to have first pointed out that the oscillatory discharge of a condenser must set up electro-magnetic waves in the surrounding space, and on the basis of Clerk-Maxwell's theory, he inferred that such radiations must be identical with ordinary light, in all essential respects except frequency. Little progress was made, however, until, in 1888, Hertz brought the subject within the range of experiment by devising a convenient method of producing oscillations of extremely high frequency, and (what was even more important) a method of detecting the waves thereby radiated into space.

His "oscillator" took the form shown in Fig. 407. Two large metal plates (or two conducting spheres) are fitted with long metal rods or wires, between the ends of which is taken the spark from an ordinary induction coil. The capacity is mainly provided by the plates, and the inductances by the rods—both quantities being very small. The plates may, in fact, be regarded as the two coatings of a condenser separated by a very long air-space. The induction coil charges up the plates until the P.D. across the gap rises to the sparking value, and then the spark practically amounts to connecting the rods, very suddenly, by a path of low resistance, across which the discharge oscillates at a rate which (as already stated) does not in any way depend upon the induction coil.

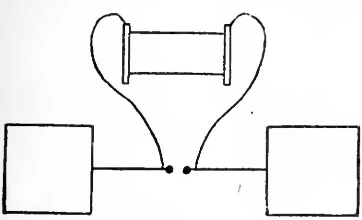


FIG. 407

The magnitude of the various quantities involved will be realised more clearly if we make a rough estimate of their value in a particular case. Suppose (merely for convenience of calculation) that the plates are circular and 50 centimetres in diameter, and that the rods are each 50 centimetres in length and ·5 centimetre in diameter. Now, we know that the capacity of a circular disc in static units is $\frac{2r}{\pi}$, where r is the radius. Hence, the capacity of each plate is $\frac{2 \times 25}{\pi} = 16$ static units. The two plates are really in series, so that the total capacity is $\frac{16}{2} = 8$ static units (assuming, as a first approximation, that all the capacity is in the plates)

It is somewhat difficult to calculate the self-inductance of a straight wire, and here it will be sufficient to give (without proof) the following expression for it—

$$L = 2l\left(\log_e \frac{4l}{d} - 1\right)$$

where l is the length and d the diameter of the wire.

We, therefore, have for a total length of 100 centimetres of wire—

$$L = 2 \times 100\left(\log_e \frac{400}{\cdot 5} - 1\right) = 1140 \text{ absolute units.}$$

As before, the frequency, n, is given by

$$n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LK}}$$

In this equation, L and K must be expressed in the same system of units (although it is immaterial which system we use). On p. 582, we have shown that the absolute unit of capacity in the magnetic system is v^2 times greater than the static unit, and hence K is $\frac{8}{(3 \times 10^{10})^2}$ magnetic units.

$$\therefore\ n = \frac{1}{2\pi\sqrt{\dfrac{1140 \times 8}{(3 \times 10^{10})^2}}} = \frac{1}{\dfrac{2\pi}{3 \times 10^{10}}\sqrt{1140 \times 8}} = 500 \text{ millions per second.}$$

This is the rate at which electric charges surge backwards and forwards in the plates and rods. To understand how such oscillations may give rise to wave motion, we must remember that during the very brief time required to charge the plates, an electrostatic field has been forming in the surrounding space, which must be regarded as extending without limit, although it is very weak except in the immediate neighbourhood of the plates. At the instant they are fully charged, we may think of these lines as extending in sweeping curves from one plate to the other, and, for the moment, as being stationary. Then, as the plates begin to discharge, the ends of these lines move towards each other along the rods (which is the same thing as saying that a current flows), and a magnetic field begins to form and to extend throughout all space, which, as already explained (see p. 600), must in its turn disappear, re-forming another electrostatic field reversed in sign of charge.

Under ordinary circumstances, *i.e.* at moderate frequencies, the energy represented by the two forms of field merely surges backwards and forwards without sensible loss; but at very high frequencies, a certain portion may never return to the circuit, but may be thrown off into space. If, for example, the two ends of an electrostatic line move together with sufficient rapidity along the rods and across the spark-gap, we may regard them as meeting and crossing *before* the line itself has had time to contract and disappear. The result will be the formation of a closed loop, which is detached and thrown off into space. Such a loop represents a certain amount of energy, although it is essentially unstable; it must immediately begin to contract, and the contraction must continue until finally it disappears. We have seen that the contraction and disappearance of an electrostatic field always produce a magnetic field linked with it. Hence, a magnetic field forms, which reaches its maximum strength at the instant the static loop vanishes, but this, in its turn, is unstable and must immediately begin to contract, with the formation of another static field. As no energy has been dissipated by resistance, the cycle of changes persists indefinitely, the result being an electro-magnetic wave.

It is now certain that ordinary light-waves are of the same nature, but of very much greater frequency. For instance, the smallest frequency of a visible light-wave is about 10 million times greater than that of the waves radiated from the Hertzian oscillator described above. Again, as the wave-length is obtained by dividing the velocity of propagation by the frequency, in the case worked out it will be $\frac{3 \times 10^{10}}{5 \times 10^{8}} = 60$ centimetres, whereas the wave-length of the longest visible light-wave is about $\frac{1}{14000}$ centimetre.

Damping Due to Radiation.—It is now evident that, even if oscillations occur in a path of no resistance, they will not persist indefinitely if radiation occurs, and the more powerfully the system radiates, the greater will be the damping. The rate of radiation of an oscillatory circuit does not depend merely upon the frequency, but also upon its form. Just as ordinary substances at a given temperature differ in their radiating powers for heat, so do different types of circuit differ as regards their readiness to emit electro-magnetic waves. Those circuits in which the plates of the condenser (or its equivalent) are widely separated, so that the lines of electrostatic force extend far into space, are the best radiators; and those in which the condenser plates are close together (as in the arrangement shown in Fig. 406, using ordinary jars) are poor radiators. In the first type, energy is rapidly lost and the oscillations die out quickly; in the second, the amplitude decreases much more slowly and the oscillations tend to persist. For this reason, in a Hertzian oscillator—which is a good radiator—only a few strongly damped vibrations occur at each spark, and there is a distinct interval between the sparks during which no oscillations are taking place. In fact, the actual time, during which oscillations are occurring and giving rise to radiation, is an extremely small fraction of the total time. Hence, although the amount of energy radiated per second may be quite small, the rate at which it is radiated is usually very great. In the particular case taken, it would probably be more than 50 horse-power.

Detectors of Radiation.—Hertz, by devising a form of detector for electro-magnetic waves, was the first to show that such radiation actually existed. His receiver took various forms, the simplest being a circle of wire, two or three feet in diameter, terminated in knobs which were separated by a small air-gap. Such a circuit has inductance and capacity—both, of course, small—and when held in a suitable position near the oscillator, as shown in Fig. 408, the radiation falling upon it sets up a rapidly alternating P.D. between the terminals, and the surging thus produced becomes great enough to spark across the gap. The best effect is obtained when the dimensions of the receiver are such that its natural period is the same as that of the oscillator, but, as a matter of fact, the period of the oscillator becomes somewhat indefinite on account of the excessive damping, and anything like exact "tuning" is impossible in practice, so that a very fair approximation to the exact dimensions, found by trial, gives good results, when the distance is not greater than a few yards. In the position shown in the figure, it will be evident that the inductive effect on the ring is entirely due to the magnetic component, for the direction of the static field is such that it cannot produce a P.D. between the knobs. If the ring were rotated through 90° in an anti-clockwise direction, the static component would also become effective, for then static lines would pass from knob to knob. But if it were to be tilted through 90° into a vertical plane, both components become inoperative, and the sparking disappears.

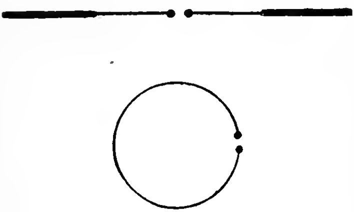


Fig. 408.

Such a receiver is very inconvenient and troublesome in practice, but with its aid Hertz was enabled to demonstrate that the radiations from his oscillator travelled at sensibly the same speed as light, and also to measure their wave-length and frequency. This was accomplished as follows: The vibrator was placed before a metallic reflector, so that the waves returned back upon themselves. In this case the interference between direct and reflected waves formed stationary nodes and vibrating segments, whose positions could be determined by recurrent maxima and minima of sparking in the resonator. The distance between two positions of maximum sparking being half a wave-length, the velocity of propagation could be easily found, when the number of vibrations per second was known. This gave a velocity nearly equal to that of light, although the conditions of experimenting were such as to permit of approximate measurements only.

Hertz also showed that the radiation was almost perfectly reflected by metallic surfaces, whereas it readily passed through non-conductors, such as paraffin-wax, pitch, or even a brick wall. He also showed that the ordinary laws of reflection and refraction were obeyed; for the latter purpose he used a large prism of pitch.

These researches, here very briefly described, supplied the experimental proof which was required to establish the theories of Maxwell.

Although Hertz's form of oscillator is not used for commercial purposes, little has been changed since his time in the method of producing electro-magnetic radiation, except in matters of detail. Perhaps the greatest advance has been made in the direction of obtaining undamped and persistent trains of waves by making use of certain properties of the arc, to be subsequently described. On the other hand, enormous progress has been made in devising delicate and convenient receivers or detectors of radiation, and it is chiefly owing to these improvements that wireless telegraphy on a commercial scale has become possible.

The "Coherer."—The second important step was the invention of the "coherer," which is based upon a peculiar microphonic behaviour of loose metal contacts to electric waves, detected some twenty years ago by Hughes, but first brought into prominent notice by Branly, and afterwards more thoroughly investigated by Sir Oliver Lodge, to whom the term "coherer" is due. Its action may be easily demonstrated by fitting, say, two pins into a narrow glass tube about an inch long, which is partially filled with iron or nickel filings.[1] If this tube be arranged in circuit with a single cell and a reflecting galvanometer—in which it is best to include, for convenience of adjustment, a steadying resistance of, say, 500 or 1000 ohms—there will possibly be no deflection, or, at any rate, a very slight one, owing to the almost infinite resistance offered by the filings. If, however, an electric spark be produced in the neighbourhood, the impact of its waves upon the filings enormously reduces in some way their resistance, and a sudden deflection of the galvanometer is produced, the coherer retaining its low resistance until its particles are disturbed by tapping or shaking. This simple form is not very reliable, and it was much improved by Marconi. His pattern consists of a narrow glass tube, about an inch and a half long, containing accurately-fitting silver terminals (A, B, Fig. 409) about half a millimetre apart, between which are the metal filings. The tube is partially exhausted before sealing—it is, in fact, a small vacuum tube.

FIG. 409.

Suppose now that an ordinary sensitive relay is inserted in the coherer circuit instead of the galvanometer. Then evidently the

[1] The best results are obtained with iron or nickel mixed with a small proportion of silver or gold filings.

impact of an ether-wave will close the relay, and thus actuate an independent battery working an ordinary telegraphic receiver. If to this an automatic tapper be added to "decohere" after each signal, we have a workable method of signalling.

The great drawback of this form of coherer lies in the fact that it does not automatically return to its original state. Modifications possessing this property have been devised, among which may be mentioned Castelli's form, consisting of a globule of mercury between two iron surfaces. Sir Oliver Lodge uses a steel disc, revolved by clockwork, which just dips into a vessel containing mercury covered with a thin layer of paraffin-oil. A cell is connected through some form of receiving instrument, such as a syphon recorder, to the disc and mercury respectively, but the layer of oil forms a bad contact and interrupts the circuit. When, however, electric waves fall on it, the oil film is pierced and a current flows of sufficient strength to give signals, the arrangement returning at once to its former state. With such detectors, it is now a common practice to dispense with ordinary telegraphic apparatus, and to receive the signals as clicks in a telephone suitably connected to it.

Wireless Telegraphy.—The general arrangements adopted for "sending" purposes in Marconi's system are indicated in Fig. 410.

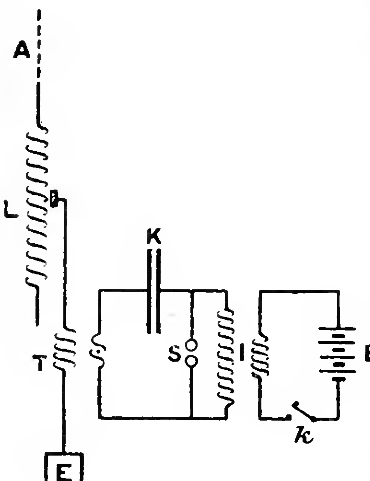


Fig. 410.

A is a long vertical wire, or group of wires, insulated at the upper end and earth-connected at the lower end. It is known as the "antenna" or "aerial." In it is inserted (1) an inductance, L, so arranged that the number of turns may be readily adjusted; (2) the secondary of a special kind of transformer, T, in which alternating currents of high frequency are produced. The primary of this transformer has only one turn. The aerial possesses a certain small capacity, and, therefore, also a natural period of oscillation, which may be adjusted within certain limits by varying the inductance, L. Its function is to radiate into space, in the form of waves, the energy supplied to it by means of the transformer, T. The earth connection is very important, because it ensures that the lines of electrostatic force are thrown off as loops closed through the earth, and hence the waves creep along the earth's surface, instead of being propagated in straight lines. Were it not for this fact, the rotundity of the earth would be a serious obstacle to long-distance telegraphy. It follows, however, that the conductivity of the earth's surface is an important factor. When it is a good conductor, as at sea, the best results are obtained, but over long stretches of dry or rocky ground signalling is difficult or impossible.

The aerial is usually made of copper or of aluminium; iron is not suitable because its magnetic properties damp down the oscillations too rapidly, although, as high-frequency currents are practically confined to the surface, it will serve if thinly plated with copper.

It will be seen that the remainder of the apparatus is essentially the same in principle as that already described (in Fig. 406).

B (Fig. 410) is a battery working an induction coil, I, the signals being sent by means of the key, *k*. The secondary is connected to a circuit containing a condenser, K, a spark-gap, S, and the primary of the transformer, T, which feeds the aerial. The capacity of the condenser is adjustable, and the frequency is thereby adjusted to bring it in "tune" with that of the aerial.

At the more powerful stations used for long-distance work, the most recent development is that of charging the condenser directly from a battery of 6000 accumulators in series, giving 11,000 to 12,000 volts, and the spark-gap is replaced by the "disc discharger" shown diagrammatically in Fig. 411.

B is the battery which feeds (1) the condenser through choking coils, *cc*, (to prevent backrush), whilst the discharge circuit contains the primary of the aerial transformer, T, and the disc discharger. This last consists of a metal disc, D, having a number of copper studs, SS, fixed at regular intervals around its periphery, and rotating at a very high speed between two other discs, *dd*, which rotate slowly in a plane at right angles to that of D. The studs just touch the side discs in passing and momentarily bridge

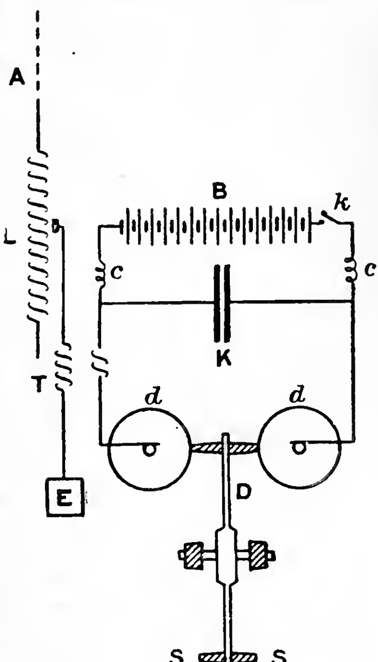


FIG. 411.

the gap. The sparks occur between the studs and side discs *before* contact, and the immediately ensuing contact enables oscillations to take place without loss of energy due to the resistance of the spark-gap. The latest form of condenser consists of insulated metal plates suspended in air, thus dispensing with the use of glass. This eliminates a serious loss of energy due to "dielectric hysteresis" produced in solid dielectrics; and is also economical from another point of view, for all condensers of the ordinary type soon deteriorate with use.

The arrangements at the receiving station differ considerably in detail, according to the form of detector adopted. In all cases, however, the essential parts are (1) a second aerial, which receives the incident radiation and converts it into a rapidly oscillating current; (2) a transformer, having its primary in series with the aerial, and its secondary in connection with the detector and auxiliary apparatus, the best results being obtained when the receiving aerial is tuned in unison with that of the emitting station. Fig. 412 shows one of the simpler arrangements adopted by Marconi. The aerial, A, is connected to earth through the adjustable inductance, L, and the primary of the transformer, T. The secondary of this transformer is in circuit with a condenser, K, and a coherer, C, from which wires, WW, are led off to some form of receiver. For instance, they may be connected to a cell and a sensitive relay governing ordinary telegraphic apparatus, or to a cell and a telephone, in which case a click is heard when the incidence of a wave reduces the resistance of the coherer and thus permits the cell to send a current through the telephone. With the telephone, however, as already stated, it is necessary to use some form of detector which automatically and instantly regains its original state.

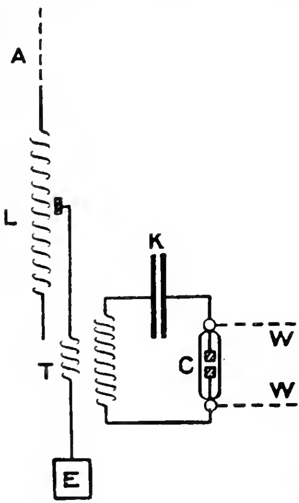


FIG. 412.

For very long-distance work, the aerials are now usually prolonged horizontally, as it has been found that this arrangement, instead of radiating in all directions alike as does the vertical form, tends to radiate most strongly in a direction *opposite* to that in which the aerial points, thus effecting a partial, but very useful, concentration in any desired direction.

Perhaps the most important applications of wireless telegraphy is in connection with signalling between vessels at sea, or between such vessels and shore stations. In ship installations, the details are simplified as much as possible to lessen the risk of a breakdown, and, by international agreement, the normal wave-length for that purpose is 300 metres (corresponding to a frequency of one million per second), although 600 metres is also permitted. Every coast station employs one or other of these wave-lengths. For transatlantic purposes a longer wave-length is usually employed.

Marconi has recorded certain important phenomena, which as yet cannot be fully explained. For instance, it is much more difficult to transmit signals over long distances by day than it is by night. Another very curious fact is that short wave-lengths are more affected than longer ones. The difficulty itself *may* be due to a partial conductivity of the air owing to ionisation produced by sunlight, but no satisfactory explanation has been suggested regarding the latter fact.

We have briefly outlined the methods adopted in one system of wireless telegraphy, but many other systems, based upon the same fundamental principles, are in use. Again, the number of detectors which have been or are actually employed in the various systems is so great, that we can only enumerate the more important, without attempting any explanation of their action. They may be regarded as devices for detecting very feeble alternating currents of high frequency.

Detectors of Electro-magnetic Waves.—

(1) Those depending upon the production of a visible spark. (The original Hertz receivers are instances. These are not very sensitive.)

(2) Those depending upon the coherer principle, *i.e.* upon a bad contact in some form.

(3) Magnetic detectors, depending upon the demagnetising effect of a feeble oscillatory current upon the magnetism of a steel bar. (Introduced by Rutherford, materially improved by Marconi—very reliable and very sensitive.)

(4) Electrolytic detectors. (A thick platinum wire dips into a vessel containing dilute sulphuric acid, and another extremely fine wire dips only a fraction of a millimetre into it. Owing to the back E.M.F. of polarisation, a single cell can send practically no current through this arrangement, but the polarisation is temporarily reduced by the passage of a feeble oscillatory discharge. This is very sensitive and convenient in use.)

(5) Thermal detectors, in which the oscillatory current produced in the receiving aerial is passed through an extremely fine wire, and is detected by its heating effect. In some forms, use is made of the change of resistance due to the heating; in others, a small thermopile is in contact with the wire. Duddell's thermo-galvanometer (see p. 298) can be used in this way, the current from the

aerial passing through the fine-wire heater. Such detectors are not so sensitive as either of the types (3) or (4).

(6) Rectifying detectors, which depend upon a difference in conductivity in different directions, especially marked in certain crystals. As a result, an oscillatory current can be partially transformed into a uni-directional current by stopping the flow in one direction. A cell in circuit with a telephone may be arranged to maintain a P.D. across the arrangement, the current then being increased by superposing an oscillatory discharge. Among these, we may mention (1) a carborundum crystal between two brass plates, (2) a corner of a fragment of zincite (native oxide of zinc) pressing against a piece of chalcopyrite (iron-copper sulphide), (3) a pointed piece of graphite touching the face of a crystal of galena. These receivers are very sensitive, and are largely used in actual practice.

(7) Those depending upon certain properties of an electric discharge in gases at low pressures. The most important of these is Professor Fleming's oscillation valve, which depends upon an effect discovered many years ago by Edison. It is really a carbon filament glow-lamp, containing also a metal plate. When the carbon filament is heated to incandescence by a local battery, a single cell will send a current in one direction between the cold plate and the hot filament, but not in the other direction. The cell must be joined up so that its negative terminal is connected with the negative end of the filament. When the aerial replaces the cell, the oscillatory currents induced in it are converted into uni-directional currents, and may be used to deflect a galvanometer or to work a telephone. This is an excellent detector and is much used.

Wireless Telephony.—This is a much more difficult problem, which up to the present has been only partially solved, the greatest distance as yet covered being about 200 miles. Without going into details, it may be pointed out that in order to transmit speech, and not merely audible signals, it is essential (1) that the sending apparatus should emit a continuous and uniform series of undamped waves of high frequency, and (2) that these waves must be controlled in duration and amplitude by some microphonic arrangement set in action by the voice. (3) The effect on the receiver must be *proportional* to the *strength* of the current induced in the aerial, *i.e.* it must respond to rapid and slight changes in the waves.

Undamped Waves.—Undamped wave-trains are also extremely useful in wireless telegraphy, on account of the readiness with which exact tuning can be obtained, the consequent sharpness of resonance between the receiving and the emitting circuits greatly reducing thereby the power required for signalling over a great distance. It is merely an instance of the well-known fact in ordinary mechanics that very feeble impulses, if only persistent and regularly timed, will set up vibrations of great amplitude in a body capable of vibrating with the same frequency, whereas very much more powerful impulses, if irregular, will produce only a comparatively small effect. At the same time, although several methods of producing undamped trains of waves are now known, they have not yet been very extensively employed in wireless telegraphy.

The most direct method of producing such waves would be to excite the aerial by means of an alternating dynamo designed to give the necessary high-frequency. Machines of this type have been made, but the practical difficulties are very great, and at present there is little prospect of overcoming them. The earliest successful method, due to Poulsen, is based upon the properties of Duddell's musical arc.

Duddell's Musical Arc.—Duddell in 1900 discovered that, if a continuous current arc, fed by some steady source of E.M.F. such as a battery of storage cells, was established between *solid* carbons

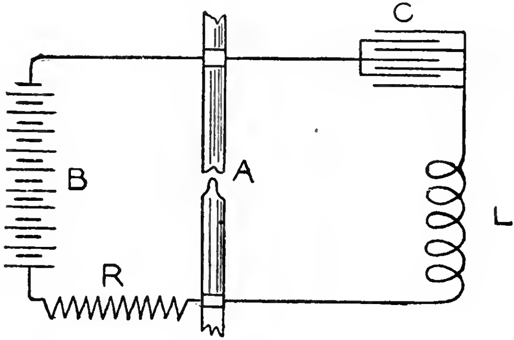


Fig. 413.

(ordinary carbons have a core of special composition running through them centrally), and shunted with a capacity and a self-inductance, persistent oscillations were set up in the shunt circuit, whose frequency depended entirely upon the constants of that circuit. When the frequency was suitable, the arc emitted a loud musical note, and by arranging a variable disposition of capacity and inductance controlled by a simple keyboard, it was possible, for instance, to make the arc play a simple tune. His arrangement is shown in Fig. 413 (taken from J. A. Fleming's *Electric Wave Telegraphy*), in which B is a battery supplying the arc A through a resistance R of about 40 ohms. C is a condenser, which may be from 1 to 5 microfarads capacity, and L a coil having an inductance of about ·005 henry and a small resistance of about ·5 ohm. According to Duddell's theory, such oscillations depend primarily upon the peculiar relation between the current through the arc and the P.D. across it (already alluded to on p. 553). Imagine a condenser to be suddenly connected across the arc. It at once begins to charge up, and in doing so slightly reduces the current through the arc. This reduction, however, implies a rise in the P.D. between the carbons, and therefore tends to charge the condenser still further. When the charging is completed, the current rises to its old value, and, in consequence, the P.D. between the carbons falls and the condenser begins to discharge. The result is an increase of current, which lowers the P.D., and thus facilitates the discharge.

Thus, the condenser alternately charges and discharges, and as it is part of a circuit possessing a natural period of vibration, these actions tend to set up oscillations of the corresponding frequency, which differ from those produced by a spark in being regular and persistent, instead of being merely a series of separate and damped wave-trains. The frequencies readily obtained in this way, vary from about 500 to 10,000, and are not great enough for the purposes of wireless telephony. It might be thought that any desired frequency could be obtained by giving suitable values to the capacity and inductance of the shunt circuit, but, as a matter of fact, the very small capacity required for high frequencies does not produce a sufficient change in the strength of the arc current to maintain the action properly.

In 1903, Poulsen of Copenhagen pointed out the great advantage of surrounding the arc with a hydrogen atmosphere, when very high frequencies are desired. In his form of apparatus, the arc is produced between a negative electrode of carbon and a hollow water-cooled positive electrode of copper. It is placed in a chamber filled with coal gas (or the vapour of any hydrocarbon), and a strong transverse magnetic field is brought to bear upon it, which tends to blow it out and thereby increases the P.D. required to maintain the arc between the electrodes. A small arc is used, taking 1½ to 2 amperes at about 220 volts. As a result, Poulsen obtained much higher frequencies—of from 100,000 to 1,000,000 per second.

Apparently these modifications facilitate the very rapid variations in the intensity of the arc current which are required—the hydrogen atmosphere by its cooling effect, and the magnetic field by its tendency to quicken the dying away of the arc by blowing it out.

As regards the working details of telephonic systems, the oscillatory circuit is linked with an aerial, in essentially the same way as that shown in Fig. 410, and a carbon microphone is often used as a transmitter. This may be connected up in several ways; for instance, it may be placed in series with the aerial between the latter and the earth, thus controlling the amplitude of the current induced in the aerial itself.

The arrangement of the receiving circuit is also essentially the same as that already described. It is, however, necessary that the detector should be of a kind which will respond *quantitatively* to the small changes in intensity of the current induced in the receiving aerial. When only the Morse code of dots and dashes has to be signalled, it does not signify whether the effect produced on the detector varies exactly with the current strength or not, as long as it responds at all; but for reproducing articulate speech, it is evident that such a condition is of the utmost importance. Fleming's oscillation valve or the electrolytic detectors are very suitable for the purpose.

Hitherto, the greatest obstacle to the practical development of wireless telephony has been the relatively great cost of the apparatus required as compared with that with ordinary telephony. It seems probable that this difficulty will be eventually overcome; in fact, at the time of writing, several new methods, of which particulars are not yet available, are being brought forward which appear to have been fairly successful over short distances.

The "Telefunken" or "Quenched Spark" System.—This is a "spark" method of producing wave-trains which are only moderately damped. To explain the principle involved, it is necessary to mention a well-known effect which occurs in all cases when two vibrating systems—mechanical or electrical—are tuned to resonance. For instance, let the aerial in Fig. 410 be tuned to the same frequency as the primary circuit containing the spark-gap. Then the oscillations induced in it react inductively again on the primary circuit itself, and the result is an interchange of energy between them, which leads to phenomena essentially similar to "beats" between musical notes, the amplitude of the vibrations waxing and waning alternately. It can be shown that this is equivalent to the production in the aerial of two independent vibrations of nearly equal frequency, and, as the receiving systems can only be tuned into unison with one of these frequencies, there is a consequent waste of energy.

Any two vibrating bodies, which can thus react upon each other, form a "coupled system." When the coupling is "loose"—*i.e.* when the arrangement is such that the motions of one only slightly affect the other—the above action is negligible, and thus only one frequency is induced when resonance occurs. Two electric circuits are "loosely coupled" when their mutual inductance is small, and "tightly coupled" when it is large. In the limiting case of exceedingly tight coupling—equivalent to a rigid connection—there is again only one induced frequency at resonance; but this is not, as a rule, a possibility.

This action and reaction between the aerial and the primary may be avoided by abruptly stopping the primary oscillations as soon as the maximum amount of energy has been transferred to the aerial; and in 1906, Wien showed that this could be done in a simple manner by using exceedingly short sparks between large masses of metal. With this arrangement, the resistance of the spark-gap is very quickly restored by cooling, and the circuit is thereby broken at the first or second oscillation. The result is equivalent to a single impulse given to the aerial, which then vibrates without further disturbance at its own natural frequency, and with very much less damping.

Fleming's Cymometer.—The actual frequency of the oscillations in an aerial or in any circuit, can be readily measured by the aid of this instrument. It consists of an inductance and a condenser in series with each other, the circuit being completed by a straight metal bar.

The inductance is a wire solenoid with a sliding contact, and the condenser is formed by one cylinder sliding inside another. The arrangements are such that the inductance and capacity can be simultaneously altered by the same motion of a handle, to which is attached a pointer moving over a fixed scale.

When the straight bar is placed near and parallel to any conductor in which oscillations are occurring, induced oscillations of identical frequency are produced in it (the coupling being designedly very loose), and the inductance and capacity can be adjusted until the maximum effect is obtained. When resonance occurs, the maximum P.D. exists between the coatings of the condenser; and a vacuum tube connected across them (preferably containing neon on account of its brightness), then lights up most brilliantly. Evidently the natural period of the cymometer circuit is then equal to that of the aerial, and the frequency is obtained at once from the scale reading and the constant of the instrument.

ANSWERS

EXERCISE III (p. 29)

1. No effect. **3.** (*a*) No effect; (*b*) partial collapse.
4. Parallel to the line joining the centres of the spheres. **5.** $+24$, -12 units.
6. 5 dynes, $\frac{1}{16}$ dyne. **7.** 29·3 and 20·7. **8.** ·1414 dyne. **9.** $\frac{140}{\sqrt[4]{6}}$
10 ·5 mg. **11.** 121·3. **12.** 4·9. **13.** ·151.
14. 37 : 13 with similar charges; 7 : 1 with opposite charges.

EXERCISE IV (p. 47)

1. 3 ergs. **2.** ·436. **3.** $23{\cdot}09\dot{3}$. **4.** $V_D=5{\cdot}414$. $V_O=8{\cdot}484$. Work $=3{\cdot}07$ ergs.
5. 45 units. **6.** 5·2. **7.** $7{\cdot}1\dot{3}$. **8.** 2·94 dynes. **9.** 36 and 12 units.
10. 18 and 2 units. **11.** 26 units. **12.** $1\dot{7}\dot{6}$. **13.** 1 : 14. **14.** 500 ergs.
16. 33, 5. **18.** 5 units. **21.** $6{\cdot}\dot{7}$. **22.** 250 dynes.
25. (*a*) The leaves of the electroscope diverge with electricity similar in kind to that on the knob of the Leyden jar; (*b*) on the potential of the knob and its distance from the end of the tube.
26. Quantities are as 1 : 5; densities are as 5 : 1.
27. When the two balls are placed near together, the potential of the charged ball depends on the quantity of its charge and on its capacity; the potential of the other depends on the distance of its centre from the first. When touched with the finger, it acquires zero potential. The difference of potential is thus increased, and a spark passes.
28. The leaves of an electroscope diverge when we have a difference of potential between the cap and the base. When the electrified rod is held under the can, there is a uniform potential over the can and throughout its interior, and the cap and base are at the same potential. When the cap is touched with the finger, it is brought to zero potential, thus producing the necessary difference of potential to cause the leaves to diverge.
29. (1) No electricity will pass, as the potential is the same as that of the conductor. (2) Electricity will pass when they are removed to a distance from it; they remove unequal charges, and as their capacities are equal, their potentials are different.
30. $\frac{2}{\sqrt[3]{2}}P$, *i.e.* $P\sqrt[3]{2^2}$. **31.** Second ball, 14; third ball, 24. **32.** 1 : 1·8.
33. -20 units. **34.** 4 cm. **35.** 144 ergs. **36.** $\frac{1}{10}$.
38. (1) A deflection; (2) leaves collapse; (3) a deflection—same as at first; (4) deflection unaltered; (5) deflection unaltered; (6) might be first a collapse and then a re-divergence more or less marked with a charge of opposite sign.

EXERCISE V (p. 70)

1. 3. **2.** From the larger jar. **3.** $6{\cdot}\dot{6}$ ergs. **4.** 26. **5.** $\frac{3}{5}$ V.
6. $636\frac{4}{11}$ static units or centimetres. **7.** 4 : 9. **8.** 25,000 ergs. **9.** 2 : 1.
10. Charge $=\frac{1}{2}$; potential $=\frac{1}{2}$; energy $=\frac{1}{4}$. **11.** 1 : 2. **12.** 50. **13.** 84 : 325. **14.** 7·2

15. The divergence of the leaves in connection with the charged plate diminishes, and that of the other increases, when the sulphur is introduced.
16. (*a*) The divergence will increase; (*b*) the leaves will collapse.
17. (*a*) The amount of induced electricity depends upon the amount of charge on the inducing body, the distance between the two bodies, and the specific inductive capacity of the dielectric. When the glass plate is introduced, the specific inductive capacity increases. (*b*) Move the plate away.
18. Heat$\propto\frac{1}{2}$QV. When the plates are brought in contact, their capacity is diminished, and the potential is therefore greater, which makes the heat greater. **19.** 1 : 9.
20. The heat produced by the discharge varies as the potential. When the ball is brought near the wall its capacity increases, and therefore its potential is less than it was in the first discharge. **21.** 5 turns. **22.** 625 ergs.
23. 144 units. **24.** 69·6. **25.** 233·$\dot{3}$. **26.** 4000 ergs.
27. Energy of discharge of A = 16·53; of B = 19·83.
28. Energy of A = 3×10^7; of B = $1{\cdot}5\times10^7$. The energy lost is represented by the spark which passes at contact.
29. 40. **30.** $566{\cdot}6\times10^3$ ergs. **31.** 16π lines per sq. cm.

EXERCISE VI (p. 101)

1. Depends on the *capacity* of the instrument. A gold-leaf electroscope can be made of smaller capacity than an electrometer, and therefore a given charge will raise its potential to a higher value.
2. 4·$\dot{2}$. **6.** 35. **7.** Field = 80π lines per sq. cm. V = 160π.
8. In second case the attraction is $(\cdot 55)^2 = \cdot 3$ of its first value.

EXERCISE VIII (p. 170)

1. $\frac{m_A}{m_B}=\frac{4}{1}$
2. Three times greater with the straight spring than with the bent one.
3. 2 sin a = sin b, where a is the angle between the long magnet and the meridian, and b the angle between the short magnet and the meridian.
4. ·707 : 1. **5.** 1 : 1·732. **6.** 1 : 2. **7.** 2·52 dynes. **8.** ·414. **9.** 205·4.
10. 84·5. **11.** 14·4 and 21·6 inches **12.** 230° **13.** 750°.
14. $\frac{M_A}{M_B}=\frac{7}{11}$ **15.** 293·3°.
17. No difference. Times depend on (*a*) moment of inertia of the magnet, (*b*) strength of earth's field, (*c*) magnetic moment of system. The first two are the same; the resultant couples in the third are also equal.
18. 1 : 1·21. **19.** 1 : 1·56 (nearly). **20.** 64 : 81 : 36 **21.** 1·76 : 1.
22. 1·75 : 1. **23.** 20 : 9. **24.** 3·26 : 1 **25.** 13·57 min.
26. 17 : 24. **27.** 64 : 125. **29.** 30°. **30.** ·01.
31. $\frac{H_G}{H_P}=\frac{1{\cdot}331}{1}$ **32.** ·143. **34.** 370·3. **33.** (1) $144\times11\times H$; (2) $\frac{99\times10^3\times H}{49}$
35. Field at A = earth's field × 6·25. **36.** 125 : 216. **37.** 24.
38. $20\sqrt[3]{2}$ cm. **39.** 150. **40.** 27 : 64.

EXERCISE IX (p. 193)

3. Equilibrium in any position. **5.** No difference.
8. (*a*) Needles must be parallel; similar poles in the same direction; if moments are unequal, similar poles need not be in the same direction. (*b*) Needles must be parallel, of equal moment, and similar poles in opposite directions. **11.** ·256 dyne. **16.** 205,000 nearly.

EXERCISE X (p. 215)

4. (i.) No effect; (ii.) leaves of electroscope connected with platinum end collapse, and those connected with zinc end diverge further; (iii.) leaves connected with zinc collapse, those with platinum diverge further.
5. (i.) No resultant difference of potential; (ii.) difference of potential which is maintained by the energy of the chemical action between zinc and acid.

EXERCISE XI (p. 227)

1. $\frac{1}{6}$ ampere. **2.** P.D. at terminals is reduced from 4 volts to 3 volts.
3. $\frac{1}{3}$ ohm. **4.** $\frac{1}{2}$, $\frac{1}{4}$. **5.** 6·6 ohms. **6.** ·4, ·36. **7.** ·088, ·142, ·1 (ampere).
8. ·1 ohm. **9.** ·055 ampere (nearly). **10.** In series. **11.** 2 : 1. **12.** $\frac{1}{4}$ volt.
13. 1·95 volt. **14.** 1·2 volt. **15.** 80, 50, 25.
17. (*a*) ·35 ampere, (*b*) 4 volts, (*c*) 1·43 ohm.
18. (1) deflection, (2) no deflection. **19.** ·183 ampere.
20. $C = \frac{115}{66}$ ampere, $C_1 = \frac{80}{66}$ ampere flowing backwards through weaker battery.
21. 4 : 3. **22.** $\frac{2}{7}$ of 35 feet from copper terminal. **23.** $1\dot{3}$ volt.
24. (1) ·0478, (2) ·0239 (ampere). **25.** $C = \cdot\dot{3}$ ampere. P.D. $= 1\cdot\dot{3}$ volt.
26. Four rows of six cells in series. **27.** $\frac{1}{3}$ ampere.
28. With P_1 the deflection is *d* and negative compared with the middle point of the wire; with P_2, no deflection; with P_3 the deflection is *d* in positive direction; with P_4 deflection 2*d* in positive direction.
29. 1 wrongly connected. **31.** 49 lamps.
32. 275 ohms; 60 watts in battery, 40 watts in leads, 1100 watts in lamps.

EXERCISE XII (p. 238)

1. Rise in temperature in cell with plates wide apart will be twice that in other cell.
2. The number of units of heat is the same in both cases, but they are distributed over a much greater weight of metal in the thick wire than in the thin one.
3. The relative amounts of heat per unit time are inversely as their lengths.
4. 15 : 24. **5.** 2·4 units. **6.** 108 units. **7.** (1) $\frac{H_a}{H_b} = \frac{2}{3}$; (2) $\frac{H_a}{H_b} = \frac{3}{2}$
8. 2 : 1. **9.** 2143 units. **10.** $1\frac{2}{3}$ ampere.
11. (*a*) 1 : 3, (*b*) 1 : 2. **12.** 10 : 1. **14.** 2×10^9 ergs per second. **15.** 1·9 ohm.
16. $4 \cdot 15 \times 10^3$ grams. **17.** ·77 ampere. **18.** ·9 volt; $\frac{1}{100}$ ohm. **19.** $4\frac{1}{2}$d. (nearly).
20. ·37 ampere, 2 cells, ·917 ohm.

EXERCISE XIII (p. 254)

6. (*a*) Becomes magnetised, (*b*) no effect.
7. The equilibrium will probably be disturbed, owing to attraction or repulsion of parallel currents.
8. Time required for 20 oscillations depends on the strength of the field, being smaller when field is stronger. If the current is flowing downwards, the field due to current and that of the earth strengthen each other on the west and weaken each other on the east, ∴ time of vibration is shorter on the west than on the east.

EXERCISE XIV (p. 280)

1. 3·13 ohms. **2.** 22·5 metres. **3.** 52·008 ohms. **4.** 1·08 mm.
5. 14·34 ohms (nearly). **6.** 1 : 100 **7.** 2 ohms. **8.** 5·955. 2·134.
9. $6 \cdot \dot{6}$ ohms. **10.** 9·1 ohms. **11.** 60 ohms. **12.** ·999 ohm. **13.** 52·5 metres.
14. The branches BA, AD, and BC, CD each give 2 ohms resistance. Their joint resistance is 1 ohm. There is no modification when A and C are connected by a wire.
15. ·62, ·31 (ampere). $\frac{H_1}{H_2}=\frac{2}{1}$
16. $C=\frac{1}{3}$ ampere. Potential at middle point of FA=0. Let A be the copper. Potential at $A=+\frac{1}{4}$ volt (above earth); at $F=-\frac{1}{4}$ volt (below earth); at $C=-\frac{19}{20}$ volt; at $B=-\frac{39}{20}$ volt. **20.** $\frac{100}{9}$ ohms, 1 ampere.
21. 70 divisions. **22.** $\frac{1}{7}$ ohm. **23.** $\frac{11}{80}$ ampere open or closed; ·141 ampere open; 1·337 ampere closed. **24.** 45·4° C. **26.** ·0058 sq. inch.

EXERCISE XV (p. 301)

1. 1 : $\sqrt{3}$.
2. Strength of current through large galvanometer is twice that of the small one. **3.** $\frac{A}{B}=\frac{1}{\cdot 78}$ **5.** ·716 ampere. **6.** ·034 and ·34. **7.** $\frac{1}{2}$ ohm.
8. ·083 metre. **9.** 30°. **10.** ·022 in direction of axis.
11. ·18 (nearly). **12.** $2\sqrt{3}$: 1.
13. Increased or decreased according to direction of current. **14.** 1 : $\sqrt{2}$.

EXERCISE XVI (p. 316)

3. 1·88 (nearly) **4.** $\frac{A}{B}=\frac{3}{5}$ **5.** ·24 ohm. **6.** 1·03 ohms. **7.** 30°.
8. 3·9 ampere. **9.** $\frac{A}{B}=\frac{10}{7}$

EXERCISE XVII (p. 341)

1. 6·78 amperes. **2.** 14·15 grams. **3.** 6·16 amperes. **4.** 1·4 ampere.
5. 2·3 **7.** $\cdot 00\dot{1}$. **8.** ·000828. **9.** $\cdot 000\dot{3}$.
10. Weight of ion in first case : weight of ion in second case : : 1 : $\sqrt{2}$.
11. ·0104 grs. **13.** (*a*) 195 grains, (*b*) 97·5 grains. **14.** 260 grains.
15. 1 : $\sqrt{2}$. **16.** 1·49 volt. **17.** 2·39 volts. **18.** 1·53 ohm. **19.** 84,175 watts.
21. Weaker in cell with platinum plates, because back E.M.F. is set up in that cell and not in the other.
22. Current is diminished in strength, as electrolysis is set up, giving an opposing E.M.F.
23. With one cell $C=\frac{E-e}{R}$, where e is the back E.M.F., which is smaller than E (the E.M.F. of a Grove's cell); with two cells $C_1=\frac{2E-e}{R+r}$, where r is the internal resistance of the added cell. Consider a numerical example: if $E=2$, $e=1 \cdot 5$, $R=1$ ohm, and $r=\cdot 2$ ohm, in case (i.) $C=\cdot 5$, and in case (ii.) $C_1=2 \cdot 08$.
24. 66 volts. **25.** 1·03 ampere in A, ·365 ampere in B.

EXERCISE XVIII (p. 366)

5. At middle part of both magnets, practically no current; when crossing the the poles, at a maximum.
6. The effect of the iron is practically to make the magnet longer. (i.) The magnet pole must be thrust further into the ring; (ii.) the ring must be moved towards the neutral line of the magnet.
7. From S to N in the upper moving side.
8. Greatest when axis of rotation is at right angles to the line of dip; least when axis is parallel to that direction.

EXERCISE XIX (p. 380)

1. $12{\cdot}57 \times 10^{-5}$ volts.
2. 250 dynes acting at right angles to the field and to the length of conductor.
3. ·0007 volt (nearly).
4. Force on coil produces a couple whose moment is $i.n.\pi.r^2.$H, where i = current in absolute units; n = number of turns; r = radius of coil; H = horizontal component.
5. (i.) With steady current, currents are as 4 : 3; (ii.) self-induction, acting as a momentary resistance, is set up, which will be greater in the wire coiled round the iron than in the zigzag wire. The data are not sufficient to give any ratio between the currents.
6. H = 5. Diameter has no effect.
7. $e = \mathrm{H}lv$. If conductor moves through distance s in t seconds, $v = \frac{s}{t}$;

$\therefore\ e = \frac{\mathrm{H}.l.s}{t}$, but $\mathrm{H}.l.s$ is number of lines cut in t seconds.

8. L = ·05 henry (nearly), M = ·018 henry (nearly). Reduced.
9. $4\pi i$ ergs, where i = current in absolute units.

EXERCISE XX (p. 423)

5. 2235 ampere-turns. **6.** 2511 ampere-turns. **7.** 100 lbs. (nearly).

EXERCISE XXI (p. 451)

1. 140·5 volts. **2.** ·028 henry. **3.** 3 ohms. **4.** 16π volts.
5. (a) $\frac{15}{\sqrt{2}}$ amperes, (b) 30 amperes. **6.** 60 turns (nearly). **7.** 28·6 sq. cm. **8.** 97·5 volts. **9.** $\frac{1}{3}$ ampere (nearly).
10. C = 1 ampere. D.P. across condenser = 800 volts (nearly). D.P. across resistance = 600 volts (nearly).

EXERCISE XXII (p. 479)

4. At the junction of A with antimony, heat is absorbed, and therefore junction is cooled; junction of B with antimony is warmed. The effect is proportional to the strength of the current (not to its square).
5. Not the same strength, as difference of potential depends not only on the difference of temperatures, but also upon the mean temperature.
11. E = ·00147 volt (nearly); neutral point = 223° C.

EXERCISE XXIII (p. 500)

6. $2\pi r \times 10^{-10}$ dynes, where r = radius of orbit ; $r = \frac{1}{\sqrt[3]{2\pi \times 10^{10}}}$ cm.

EXERCISE XXV (p. 545)

5. The rate of consumption of zinc decreases. Rate $\propto$ current, but $C = \frac{E - e}{R}$, where E = E.M.F. of battery, e = back E.M.F., and R = resistance of the circuit. Now, E and R are practically constant, while e is directly proportional to speed of engine ; $\therefore$ if e is increased, C is diminished.

10. 82·8 amperes. **13.** 5×10^6 lines.

EXERCISE XXVI (p. 556)

3. 288 amperes **4.** 2s. 4d. (nearly). **5.** 55 watts ; 25 watts.

EXERCISE XXVII (p. 572)

2. We notice that a current of $\frac{5}{500}$ or $\frac{1}{100}$ of an ampere deflects pointer to its full extent, and we must arrange matters so that a difference of potential of 50 volts sends $\frac{1}{100}$ ampere through the instrument ; hence, its resistance must be increased by r ohms, where $\frac{1}{100} = \frac{50}{500 + r}$; *i.e.* $r = 4500$ ohms. **3.** 14·7 divisions ; 19·09 divisions.

EXERCISE XXVIII (p. 591)

6. ·003 volt.
7. Electrical energy costs 226 times as much as heat energy.

INDEX

Printed by Ballantyne, Hanson & Co.
Edinburgh & London

6